Effects of Vibrations on Sand and the Measurement of Dynamic Properties

Effets de vibrations sur le sable et mesure des propriétés dynamiques

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SUMMARY

Part A gives the results of a study on the effects of wave propagation in saturated sand, creating dynamic stability, compaction, or liquefaction.

Part B concerns dynamic tests of the subsoil. Oscillations of a concrete block and of the surrounding ground caused by the impact of a hammer make it possible to calculate the elasticity modulus, damping, and absorption.

SOMMAIRE

La partie A donne les résultats de l'étude de la propagation des ondes dans un sable saturé produisant la stabilité dynamique, le compactage ou la liquéfaction du sable.

La partie B donne les résultats d'essais dynamiques sur le sous-sol. L'auteur démontre qu'on peut calculer le module d'élasticité, l'amortissement et l'absorption en utilisant les vibrations d'un bloc de béton et du sol environnant causées par le choc d'un mouton.

A. VIBRATION OF SATURATED SAND

Previous Research

Vibration produces four effects in saturated sand: dynamic stability, compaction, liquefaction, and motion of sand. These effects are attained in the order named as acceleration increases from zero to a value of several gravity accelerations. Until now the studies (mostly for dry sand) comprised the limit of dynamic stability, defined by critical acceleration (Maslov, 1957), compaction (Mogami and Kubo, 1953; Barkan, 1959), liquefaction (Ivanov, 1962), and motion (Kroll, 1954). Nevertheless the results were sometimes contradictory. The attempt of the author (Bažant, 1961) to solve the effects of vibrations in a theoretical way was not confirmed experimentally. It was found that compaction is not the function of either velocity $f \times A$, or acceleration $f^2 \times A$, where f is frequency, A is amplitude (Bažant, 1964). It is necessary to take into account simultaneously the acceleration, frequency, height, and volume of the vibrating sand.

Wave Propagation in Saturated Sand

Vibration originating at the bottom of a layer of sand propagates from underlying to overlying grains and also through water. Vertical vibration of the base causes raising of the first layer of grains in the first phase; the upper layers do not move and compression of the sand results. Then, successively all the layers rise. In the second phase the base begins to fall and the first layer of grains follows. Up to this time the upper layers are rising and consequently dilatation of sand develops.

Alternate compression and dilatation can be negligible for small acceleration and dynamic stability is retained. This should be the case for machinery foundations. Greater acceleration causes compaction. This holds for ordinary vibrating equipment. For increasing acceleration further liquefaction of sand results, a transitory state followed by large compaction. This principle is used in vibroflotation. For acceleration attaining several gravity accelerations, motion of sand ensues; this is the steady state ending only at the end of the vibration. Vibratory motion of sand is used for handling operations on vibratory conveyors and for mixing purposes in vibrating mixers.

Wave propagation adds to the static state of stress pulsating dynamic stresses in grains and water. The effective dynamic normal stress is given by

$$\sigma_d' = \sigma_s + \sigma_d - u_s - u_d, \tag{1}$$

where subscript s denotes static value and d dynamic value. From the Mohr envelope S for static case (Fig. 1) we obtain the static tangential stress

$$\tau_s = \sigma_s' \tan \phi, \tag{2}$$

where $\sigma_s' = \sigma_s - u_s$. Stress conditions for the dynamic case can be derived (Bažant, 1964), under the assumption that the Mohr envelope D at constant acceleration α is curved (L'Hermite, 1948; Kutzner, 1962) (Fig. 1) and that the

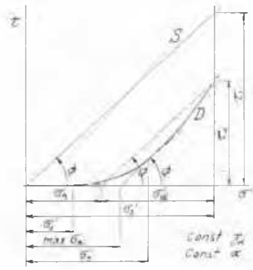

FIG. 1. Mohr's diagrams of saturated sand for static and dynamic case.

angle of shearing resistance ϕ does not alter during vibration and from these we obtain the dynamic tangential stress

$$\tau_d = \sigma'_d \tan \phi. \tag{3}$$

Dissipation of wave energy is neglected and it is assumed that the wave energy is sufficient for undamped wave propagation in the whole height of the layer, a case strictly holding for earthquake effects. Consequences of these assumptions are given below.

Dynamic Stability

Sand, which is sufficiently dense for a given acceleration, cannot be compacted by vibration, because its dilatation due to wave propagation is so small that the grains cannot move. Such sand has dynamic stability. For a given unit weight expressed by dry density γ_d, it is possible to determine the critical acceleration α_c, which is the upper limit of acceleration, creating in sand only elastic movements and therefore retaining the sand in a condition of dynamic stability.

Critical acceleration α_c was measured on a vibrating table moving vertically, on which a glass container filled with saturated sand was attached. After vibration the final vibration dry density γ_{dv} was measured, and the acceleration, through which it was obtained, was considered as critical acceleration α_c. This was confirmed by repeated vibrations, which proved that the sand of density γ_{dv} had reached dynamic stability. This dynamic stability holds for vertical vibration and for sand sedimented in the same way as in the glass container. For another direction of vibration and another kind of sedimentation, even if γ_{dv} was reached, a certain compaction can be expected but it is relatively small.

Stress conditions for dynamic stability were studied previously (Bažant, 1964). They require

$$\sigma_p \to 0. \tag{4}$$

σ_p is expansion pressure, which is defined by

$$-\sigma_p = \sigma_d - u_d \tag{5}$$

where σ_d is the dynamic total stress and u_d the dynamic excess pore pressure over hydrostatic pressure u_s. σ_p becomes negative because $u_d > \sigma_d$ and so it can be termed expansion pressure.

Variables of the problem were obtained from the assumption that α_c is a function of the dynamic effective stress given by Eq (1), which can be written as the sum of the static effective stress and expansion pressure, created in saturated sand during vibration:

$$\sigma_d' = \sigma_s' - \sigma_p. \tag{6}$$

Static effective stress is a function of

$$\sigma_s' = f(\gamma_{dv}, \gamma_w, H), \tag{7}$$

where H is the height of the vibrating sand layer, and expansion pressure

$$\sigma_p = f(\gamma_{dv}, \gamma_w, A, g, V), \tag{8}$$

where A is the amplitude. Expansion pressure was supposed to be a function of the wave equation, which governs the one-dimensional wave propagation in the prism of sand of volume V. Taking further into account the variation of wave velocity with frequency f (so-called dispersion) and considering the natural frequency f_n, the general form of the critical acceleration (or compaction) function was found to be

$$f(\gamma_{dv}, \gamma_w, f, f_n, A, g, H, V) = 0. \tag{9}$$

Applying the dimensional analysis the dimensionless function of five variables was obtained

$$f(\gamma_{dv}/\gamma_w, f/f_n, \alpha_c/g, H^3/V, A/H) = 0 \tag{10}$$

where γ_{dv}/γ_w is the relative dry density, f/f_n relative frequency, α_c/g relative critical acceleration, H^3/V form factor, A/H relative amplitude.

Assuming A/H to be negligible, it is possible to determine the relative critical acceleration α_c/g from Fig. 2 as the

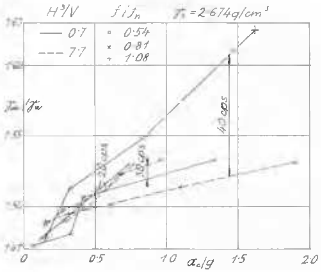

FIG. 2. Relative critical acceleration.

function of $(\gamma_{dv}/\gamma_w, f/f_n, H^3/V)$. The irregular shape of curves is inevitable, because for each combination of amplitude and frequency the value of neglected transversal and torsional wave propagation changes.

The sand tested had grain sizes of 0.2 to 0.5 mm, $\phi = 35°$, $\gamma_s = 2.674$ g/cu.cm., initial $\gamma_d = 1.47$ g/cu.cm., $n = 45$ per cent, $Rd = 0.19$, $f_n = 37$ cps.

Compaction

From Fig. 2 it is also possible to predict the value of compaction. Sand undergoes compaction until γ_{dv}, corresponding to a given acceleration α, is reached. Vibration density γ_{dv} can be found from Fig. 2 assuming $\alpha = \alpha_c$. It is of course necessary to take into account the relative frequency f/f_n and form factor H^3/V depending on the height H and volume V of the vibrating sand layer. Accuracy of the compaction prediction from Fig. 2 is estimated to be $\pm$ 0.02 γ_{dv}/γ_w.

Liquefaction

The stress condition for liquefaction is

$$\sigma_d' = 0, \tag{11}$$

i.e., the dynamic effective stress should be zero. This holds after Eq (6) for

$$\sigma_s' = \sigma_p. \tag{12}$$

Liquefaction is otherwise developed, if acceleration reaches the value of the liquefactional acceleration α_z, which causes the expansion pressure σ_p to annul the given static effective stress σ_s'. Liquefaction can then be characterized by Eq (10) also, because it is the function of the same variables as the dynamic stability. Liquefaction follows after loss of dynamic stability and therefore occurs for $\alpha_z > \alpha_c$. Generally only the upper layer of soil may be in the condition of liquefaction, as liquefaction is the function of effective stresses which are increasing with depth.

Results of tests of liquefaction are given in Fig. 3 for the same sand as used in tests of Fig. 2. Liquefaction was studied by the author especially as the effect of earthquakes (Bažant, 1965).

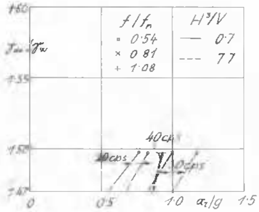

FIG. 3. Relative liquefactional acceleration.

Model Laws

Model laws require the same density, frequency, and acceleration in model prototypes. Dimensions of the sand layer should be geometrically similar. The neglected ratio A/H would require geometric similarity of amplitudes, which it is not possible to fulfil, because acceleration does not alter.

B. DYNAMIC INVESTIGATIONS OF SOILS AND ROCKS BY IMPACT OF THE TESTING BODY

Dynamic tests of the subsoil are usually performed by means of a vibrator whose revolutions and eccentric force are adjustable. Such tests are expensive and require a trained staff. Therefore a simplified method of dynamic testing was investigated. Rectangular concrete blocks with bases 2,500 sq.cm. and 5,000 sq.cm. are thrown into oscillations. The oscillations in the vertical direction are produced by axial impact of a wooden hammer. Wood is more suitable than steel which causes considerable local deformation of the concrete. The horizontal blow is performed by a steel cylinder, suspended on a rope and acting like a pendulum (Fig. 4). The stroke of the horizontal hammer and local deformation induced by impact are relatively small allowing the use of a steel body.


FIG. 4. Dynamic test on a concrete block in a trial gallery with vibrographs and steel hammer for horizontal impact.

The displacements of the block and of the surrounding ground surface due to oscillations are recorded by vibrographs. One or two of them are fixed on the top of the block, several others are placed on the ground surface. In order to obtain distinct free oscillations of the block, the hammer must not drop again after the first rebound. This requirement may be fulfilled either by a mechanical device or simply by catching the hammer by the hand at the instant of rebound.

The reliability of measurements can be improved if at least two different sizes of testing blocks, two hammers of different weights (20 and 30 kg), and four heights of drop (10, 20, 30, 50 cm) are used. The height of rebound is also recorded in order to determine the coefficient of impact.

The vibrograms give the displacement amplitudes and frequency of oscillations. The initial waves are usually distorted for reasons to be explained below. It is the middle and final part of the record from which the frequency of free oscillations and the damping ratio are derived.

The common solution of the impact is used for calculations. The block is considered as a body with one degree of freedom supported by elastic subsoil. With W as the weight of the block and C as the spring constant, the displacement is $y_0 = W/C$. The frequency of free oscillations $n_0 = (C/m)^{1/2}/2\pi = (Cg/W)^{1/2}/2\pi = (g/y_0)^{1/2}\pi \approx 5/y_0^{1/2}$ where y_0 is given in cm. The constant C and elasticity modulus E may be calculated from the measured frequency n_0 and known weight W.

The problem may be simplified for further considerations

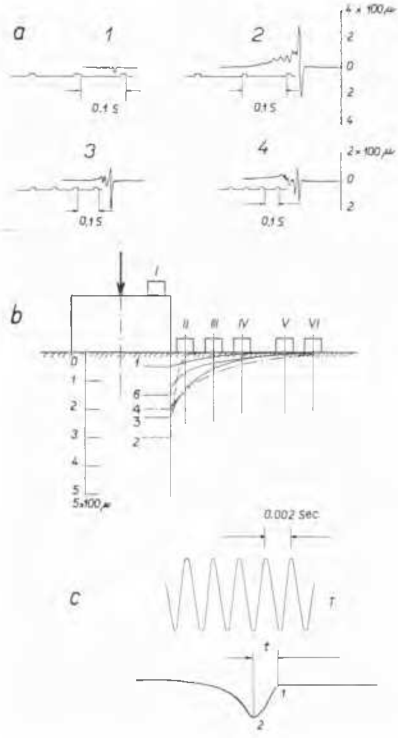

FIG. 5. Dynamic test in vertical direction. Numbers correspond to Table I. a, Records of the oscillations of the block: 1, quartzite (obliquely to the foliation); 2, quartzite, after the shear test; 3, sandy gravel; 4, dry sandy loam ($I_c > 1$). b, Course of displacements of the block and of the surface. c, Oscillogram of the impact: 1, closing; 2, opening of the electricity circuit; t, time of the impact; T, time marking.

as an impact of two free bodies—hammer and block—with weights B and W. The velocity v of the block after the impact of the hammer with a velocity v_1 is given by the known formula $v = v_1(1 + k)(B + W)$. The coefficient k can be obtained from the relation $k = (h'/h)^{1/2}$ where h and h' are the height of drop and rebound respectively. The force P effected by impact on the base of the block is at its maximum simultaneously with the maximum displacement a. The accomplished work is $A_1 = Pa/2$. Since $P = aW/y_0$, it follows that $A_1 = Wa^2/2y_0$. The energy of the drop is $A_2 = mv^2/2 = Wv^2/2g$. Further $A_1 = A_2$ or $Wa^2/2y_0 = Wv^2/2g$ and $a = v(y_0/g)^{1/2}$.

This expression enables the measured displacement amplitudes to be checked, particularly if the height of drop is gradually increased. In this case the first amplitude is decisive. The procedure also allows the estimation of the volume of subsoil involved in oscillations.

These considerations are valid in the case where the time of impact is negligible as compared with the frequency n_0, i.e., if the oscillations of the block start after the impact has been finished. The impact time was measured by electric contact. One output was on the hammer, the second one on the block. The contacts were formed by two copper sheets. As an alternative a stripe of conductive paint (Czechoslovak patent) prepared from silver pigment was applied on the block. The oscillogram of the impact in Fig. 5c was recorded with a running speed of 7 m/sec. The time of the impact is about 0.002 sec. which warrants the above-mentioned calculation for $n_0 < 50$ to 70 cps. Otherwise a distortion of the first oscillation takes place. The eccentricity of the horizontal impact should be small enough so that it could be neglected, which simplifies the calculation.

The determined moduli of elasticity and damping ratios are compiled for vertical impacts in Table I. At test 1 the moduli correspond with static moduli of deformation M. δ is the logarithmic decrement of damping. The mat of sand had no perceptible influence on values of E, whereas D was increased from 18 to 40 per cent.

TABLE I. MODULI OF ELASTICITY AND DAMPING RATIOS FOR VERTICAL IMPACTS

No	Soil or rock	E kg/cm² $\times 10^{-3}$	$D = \delta/2\pi$
1	Quartzite, direction oblique to foliation	56–79	0.061–0.098
2	Quartzite, direction oblique to foliation, base disturbed by shear test	2.8–11.2	
3	Dense sandy gravel	1.08–2.12	0.05–0.10
4	Weathered clayey slate	0.52–1.05	0.064–0.10
5	Weathered clayey slate, with a mat of sand 3 cm thick	0.55–1.02	0.081–0.118
6	Sandy loam, consistency index $I_c > 1$	0.40–0.90	0.043–0.080
7	Sandy loam ($I_c > 1$), with a mat of sand 3 cm thick	0.43–0.92	0.054–0.112

Some characteristic records of vertical oscillations of the block (Fig. 5a) show larger displacements and lower frequency of vibrations for a more compressible subsoil. The course of vertical amplitudes of the block and of the surrounding ground surface are shown in Fig. 5b. The weight of the wooden hammer is 30 kg, the height of drop is 20 cm. The oscillogram of the vertical impact of the 30-kg wooden hammer on the block is shown in Fig. 5c. The impact lasted approximately 0.002 sec.

The horizontal displacements of the ground surface are largest at the block wall opposite the place of impact; smaller displacements are at the sides and in front of the block, where the hammer is acting.

The described tests are very simple and the dynamic moduli of elasticity roughly correspond to the static ones.

AUTHORSHIP

Part A of this paper is by Z. Bažant and Part B is by A. Dvořák.

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