2/9

Effects of Rotation of the Principal Stress Axes and of the Intermediate Principal Stress on the Shear Strength

Effets de la contrainte principale intermédiaire et de la rotation des axes des contraintes principales sur la résistance au cisaillement d'une argile remaniée

B. B. BROMS, PH.D., *Associate Professor, Cornell University, Ithaca, U.S.A.*
A. O. CASBARIAN, PH.D., *Shell Development Company, Houston, U.S.A.*

SUMMARY

Three series of consolidated-undrained triaxial tests were carried out on hollow cylindrical specimens of a remoulded kaolinite clay. In series I and II the effects of rotation of the principal stress axes and of the intermediate principal stress were investigated separately. The combined effects were investigated in series III. Series I and II indicated that both the rotation of the principal stress axes and the intermediate principal stress have an appreciable effect on the deviator stress, the friction angle ϕ', and the pore-pressure coefficient A_f. Series III indicated that the shear-strength and pore-pressure parameters of a soil can be predicted for any rotation of the principal stress axes and for any value of the intermediate principal stress by the principle of superposition.

SOMMAIRE

Trois séries d'essais triaxiaux non-drainés ont été faites sur des échantillons creux et cylindriques de kaolin remoulé. Dans les séries I et II les effets de rotation des axes des contraintes principales et les effets de la contrainte principale intermédiaire ont été examinés. Ces effets combinés furent examinés dans la série III. Les séries I and II indiquent que la rotation des axes des contraintes principales et que la contrainte principale intermédiaire ont un effet sensible sur la déviateur ($\sigma_1 - \sigma_3$), l′ angle ϕ' et la pression interstitielle. La série III indique que les caractéristiques de la résistance d'un sol peuvent être prédites pour chaque rotation des axes des contraintes principales et pour chaque valeur de la contrainte principale intermédiaire par le principe de superposition.

ROTATION OF THE PRINCIPAL STRESS AXES frequently takes place under field conditions such as during the construction of earth dams. This rotation may have an appreciable effect on the shear-strength and pore-pressure parameters of a soil and hence may affect appreciably the design of such structures. Failure of many types of foundation structures, such as sheet pile walls and long footings, as well as slope failures of cuts and fills frequently takes place under the condition of plane strain. No lateral movements take place along the axes of these structures and the principal stress which acts in the direction of the axis of these structures will attain a value intermediate between the major and minor principal stresses.

The effects of the intermediate principal stress on the shear-strength and pore-pressure parameters of cohesive soils have been studied by Rendulic (1936), Habib (1953), Taylor (1955), Henkel (1958), Hirschfeld (1958), and Wu, Loh, and Malvern (1962) by means of compression and extension tests. However, the results reported by these investigators are conflicting. This paper presents the results of an investigation concerned with the effects of the rotation of the principal stress axes and of the intermediate principal stress on the shear-strength and pore-pressure parameters of a saturated remoulded kaolinite clay. Hollow cylindrical specimens were tested to failure under consolidated-undrained conditions at different rotations of the principal stress axes and at different values of the intermediate principal stress. The pore pressures developed during the undrained part of each test were measured.

EXPERIMENTAL PROGRAMME

Test Apparatus and Loading Unit

Hollow cylindrical test specimens were used in this investigation. The dimensions of the test specimen are shown in Fig. 1. The hollow clay cylinders were enclosed between an inner and an outer rubber membrane and were placed in a modified triaxial cell. Filter paper drains were placed along the outside and the inside perimeters to facilitate

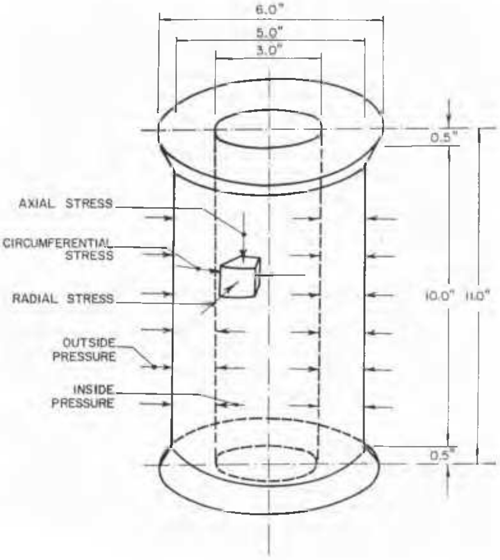

FIG. 1. Dimensions of test member.

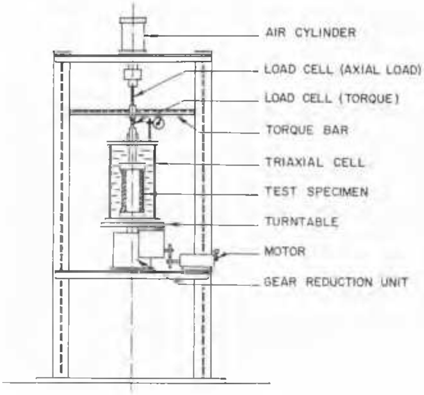

FIG. 2. Test arrangement.

drainage and to improve pore-pressure equalization. The pressures within the inside and outside chambers could be varied independently, as well as the axial load. The test specimens were loaded by the unit shown in Fig. 2. Axial load was applied by an air cylinder and kept constant by a sensitive air pressure regulator. Torque was applied by rotating the triaxial cell which was rigidly attached to a turntable. The applied axial load and the applied torque were measured by load cells to an accuracy of at least one per cent.

Stress Analysis

The inside and outside chamber pressures were chosen so that the average radial stress (Fig. 1) was equal to the intermediate principal stress σ_2 and that the major and minor principal stress axes were contained in the plane of the axial and circumferential stresses. The major and minor principal stresses (σ_1 and σ_3, respectively) have been calculated from the average axial stress, the average circumferential stress, and the average shear stress. Stress analyses based on theory of elasticity and theory of plasticity have indicated that the difference between the average stress and that determined from a more accurate analysis is small (Casbarian, 1964).

The torque measurements were corrected for the restraining effects caused by the rubber membranes, the filter paper drains, and the friction between the piston and the bushing of the triaxial cell. Similar corrections were not made for the axial load.

Material

A kaolinite clay of medium plasticity and of medium dry strength was used. Some of its index properties are as follows: liquid limit = 57 per cent; plastic limit = 32 per cent; clay fraction (< 0.002 mm) = 39 per cent; specific gravity = 2.68; coefficient of consolidation = 7.9×10^{-3} sq.cm./sec; and coefficient of permeability = 1.97×10^{-4} cm/sec.

Preparation of the Soil

The clay, which was obtained in powder form, was mixed in ten batches at a water content of 48.5 per cent. Each batch was stored separately in a humid room for at least four weeks before testing and each batch was sufficient to form at least five test specimens. The hollow specimens were fabricated by compacting the remoulded clay with a tamper between an inner and an outer mould. Special care was taken to eliminate all voids.

Test Procedure

After moulding, the test specimens were consolidated isotropically at a confining pressure of 7.0, 25.0, or 70.0 lb/sq.in. The time required for 90 per cent consolidation was approximately 20 minutes. However, the test specimens were allowed to consolidate for at least eight hours. At the end of the consolidation phase, de-aired water was circulated through the porous stones and through the pore-pressure lines to remove any trapped air. During the undrained phase of the consolidated-undrained tests a back pressure of 20 lb/sq.in. was used to ensure full saturation. All tests were stress controlled except for those in which rotation of the principal stress axes occurred. These tests were strain controlled. The loading rate was adjusted to allow at least 95 per cent equalization of pore pressures during all phases of the test programme. Approximately eight hours were required for the undrained part of each test.

The effects of the rotation of the principal stress axes on the shear-strength and pore-pressure parameters were investigated in series I. The soil samples, which were initially isotropically consolidated, were tested to failure under undrained conditions. During the undrained phase the intermediate principal stress and the radial stress were maintained constant at a value equal to the consolidation pressure. Both compression and extension tests were carried out. For the compression tests the axial stress was increased and the circumferential stress was decreased by the same amount. For the extension tests the circumferential stress was increased while the axial stress was decreased. For tests in which rotation occurred, the samples were thereafter tested to failure through the application of torque.

The rotation of the principal stress axes has been expressed by the angle α. This angle is equal to the rotation of the major principal stress axis which takes place during the undrained phase. A positive value of the angle α indicates a compression test while a negative value indicates an extension test.

In series II the effects of the intermediate principal stress were investigated. No rotation of the principal stress axes was allowed. During the undrained phase the intermediate principal stress (equal to the average radial stress) was kept constant at a value equal to the consolidation pressure. The major and minor principal stresses, σ_1 and σ_3, respectively, were adjusted in such a way that at failure the ratio N, equal to $(\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$, varied between 0 and 1.0. A value of $N = 0$ corresponds to the standard compression test. For this test the intermediate principal stress σ_2 was equal to the minor principal stress.

In series III the combined effects of rotation of the principal stress axes and of the intermediate principal stress were investigated. Both compression and extension tests were carried out. In this series the direction of the major principal stress axis rotated between 0° and ± 45° and the ratio N varied between 0 and 1.0. During the undrained phase, the test samples were subjected initially to an increase or decrease of the axial stress. Thereafter, for the tests in which rotation occurred, the samples were loaded to failure by applying a torque.

TEST RESULTS

Series I

The test results from series I are tabulated in Table I. The relationships between the pore-pressure coefficient A_f,

TABLE I. TEST RESULTS

Test no.	Consolidation pressure, p_c (lb/sq. in.)	Rotation of stress axes α (deg.)	Stress ratio N	Pore-pressure coefficient, A_f	Deviator stress $(\sigma_1 - \sigma_3)_{max}$ (lb/sq. in.)	Friction angle, ϕ' (deg.)
Series I						
A(1)	7.0	0	0.5	0.74	8.2	
A(2)		19	0.5	0.69	7.9	
A(3)		30	0.5	0.81	6.5	
A(4)		45	0.5	0.84	6.2	
A(5)		61	0.5	0.82	6.8	
A(6)		70	0.5	0.81	8.5	
A(7)		90	0.5	0.79	7.8	
B(1)	25.0	0	0.5	0.76	25.6	
B(2)		23	0.5	0.80	23.0	
B(3)		32.5	0.5	0.85	19.0	
B(4)		45	0.5	0.91	20.0	
B(5)		57	0.5	0.90	19.2	
B(6)		71	0.5	0.88	20.3	
B(7)		90	0.5	0.80	24.0	
C(1)	70.0	0	0.5	0.82	62.0	35.7
C(2)		17	0.5	0.89	54.6	32.0
C(3)		29.5	0.5	0.99	48.0	29.9
C(4)		45	0.5	1.11	41.0	28.2
C(5)		61	0.5	1.05	46.2	29.9
C(6)		73	0.5	0.98	52.8	32.0
C(7)		90	0.5	0.84	61.4	35.7
Series II						
A(21)	7.0	0	0.00	0.42	8.10	
A(22)		0	0.25	0.52	7.87	
A(1)or(7)		0	0.50	0.74	8.20	
A(24)		0	0.73	0.85	7.70	
A(25)		0	1.00	1.04	6.70	
B(21)	25.0	0	0.00	0.53	24.6	
B(22)		0	0.21	0.59	28.4	
B(1)or(7)		0	0.50	0.80	24.0	
B(24)		0	0.74	0.94	22.8	
B(25)		0	1.00	1.14	18.5	
C(21)	70.0	0	0.00	0.67	60.0	29.2
C(22)		0	0.25	0.77	63.0	34.4
C(1)or(7)		0	0.50	0.84	61.4	35.7
C(24)		0	0.70	0.97	50.0	35.9
C(25)		0	1.00	1.17	47.5	36.0
Series III						
A(21)	7.0	0	0.00	0.42	8.1	
A(32)		29	0.24	0.56	7.9	
A(4)		45	0.50	0.84	6.2	
A(34)		70	0.83	0.89	7.4	
A(25)		90	1.00	1.04	6.7	
B(21)	25.0	0	0.00	0.53	24.6	
B(32)		17	0.19	0.70	23.0	
B(4)		45	0.50	0.91	20.0	
B(34)		58	0.72	0.95	18.2	
B(25)		90	1.00	1.14	18.5	
C(21)	70.0	0	0.00	0.67	60.0	29.2
C(32)		23	0.15	0.83	57.5	30.6
C(4)		45	0.50	1.10	41.0	28.2
C(34)		74	0.90	1.10	45.0	31.8
C(25)		90	1.00	1.17	47.5	36.0

the maximum deviator stress $(\sigma_1 - \sigma_3)_{max}$, the friction angle ϕ' with respect to effective stresses, and the angle of rotation of the principal stress axes as obtained at a confining pressure of 70 lb/sq.in. are shown in Figs. 3a, 4a, and 5a respectively. It can be seen that the relationships shown in these figures are symmetrical about the 45° axis. This symmetry indicates that the samples were initially isotropic without any preferred orientation of the individual kaolinite particles, and that the test results were not affected by end restraint.

The pore-pressure coefficient A_f increased with increasing rotation of the principal stress axes (Fig. 3a). The coefficient A_f, when the principal stress axes rotated 45°, was approximately 1.40 times the pore-pressure coefficient when no rotation took place. Rotation of the principal stress axes caused a reduction of the maximum deviator stress (Fig. 4a).

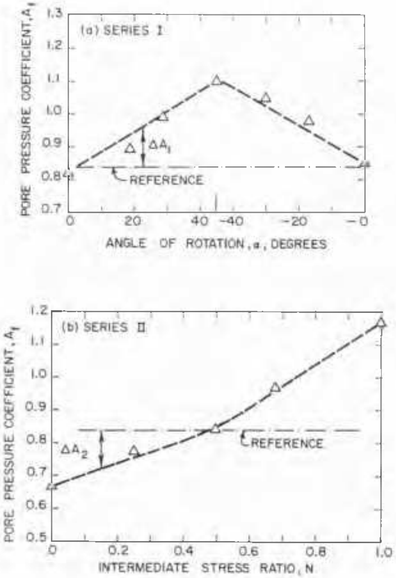

FIG. 3. Pore-pressure coefficient A_f.

An increase in rotation of 45° reduced the maximum deviator stress by one-third. It can be seen from Fig. 5a that rotation decreased the friction angle ϕ'. This friction angle decreased by 7° when the rotation of the principal stress axes increased from zero to 45°.

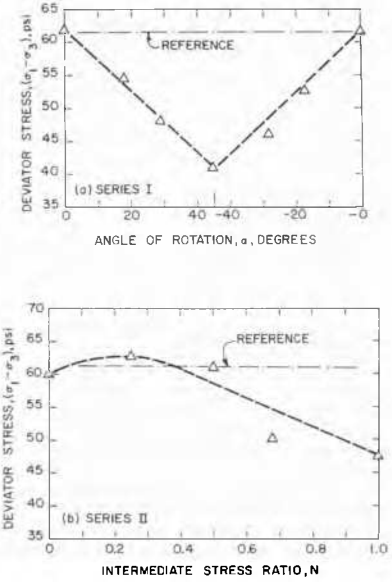

FIG. 4. Deviator stress $(\sigma_1 - \sigma_3)_{max}$.

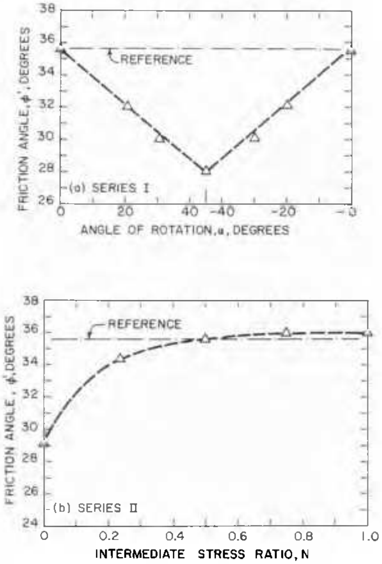

FIG. 5. Angle of internal friction ϕ'.

The effects of rotation of the principal stress axes can be attributed to the reorientation of the individual clay particles which takes place initially during the undrained phase of each test. It has been observed (Lambe, 1958; Hvorslev, 1960) that flat clay particles have a tendency to orient themselves perpendicular to the direction of the major principal stress. If, for example, the direction of the major principal stress has rotated through an angle of 45° at failure, then the orientation of the failure surface will almost coincide with the orientation of the clay particles which rotated during the initial phase of each test. The tendency of the individual clay particles to align themselves parallel with the final failure plane will increase with increasing rotation of the principal stress axes. Thus it can be expected that the maximum deviator stress and the friction angle ϕ' will decrease with increasing rotation of the principal stress axes. Such a decrease was observed. Due to the alignment of the individual clay particles and the resulting decrease of particle interlocking, it is also expected that the pore pressures at failure at a given void ratio will increase with increasing rotation of the principal stress axes.

Series II

The measured relationships between the pore-pressure coefficient A_f at failure, the maximum deviator stress $(\sigma_1 - \sigma_3)_{max}$, the angle of internal friction ϕ', and the intermediate principal stress ratio N are shown in Figs. 3b, 4b, and 5b, respectively.

It can be seen from Fig. 3b that the pore-pressure coefficient A_f increased approximately linearly with increasing stress ratio N. Fig. 4b indicates that the maximum deviator stress $(\sigma_1 - \sigma_3)_{max}$ increased with increasing N and reached maximum at $N = 0.25$. At further increases of the intermediate principal stress ratio the maximum deviator stress decreased. The maximum deviator stress obtained for the compression tests ($N = 0$) was approximately 25 per cent

TABLE II. COMPARISON WITH TEST DATA (SERIES III)

Test no.	A_{test}	A_{calc}	$(\sigma_1 - \sigma_3)_{\text{test}}$ (psi)	$(\sigma_1 - \sigma_3)_{\text{calc}}$ (psi)	ϕ'_{test} (deg.)	ϕ'_{calc} (deg.)
Test C(32)	0.83	0.84	57.5	53.0	30.6	28.6
Test C(34)	1.10	1.20	45.0	42.5	31.8	31.7

larger than that obtained for the extension tests ($N = 1$). These results are in agreement with those reported by Taylor (1955), Henkel (1958), and Hirschfeld (1958). The friction angle ϕ' (Fig. 5b) increased rapidly with increasing intermediate principal stress ratio for values of N less than 0.25. At values of N larger than 0.25 the increase in ϕ' was small. The difference in the value of ϕ' between axial compression and axial extension tests was approximately 7°.

Series III

The combined effects of rotation of the principal stress axes and of intermediate principal stress were investigated in series III. The test data, presented in Table I, indicate that the combined effects of these two parameters can be predicted by the principal of superposition. The results obtained from tests 1 or 7 are common to both series I and II and hence can be used as reference data, as illustrated in Figs. 3, 4, and 5.

As an example, the coefficient $A(\alpha = 23°, N = 0.153)$ corresponding to $N = 0.153$ and $\alpha = 23°$ can be predicted as the reference coefficient $A(\alpha = 0, N = 0.5)$ corresponding to $N = 0.5$ and $\alpha = 0$ corrected for rotation of the principal stress axes and the intermediate principal stress ratio as expressed by the equation:

$$A(\alpha = \alpha_1, N = N_1) = A(\alpha = 0, N = 0.5) + \Delta A_1 + \Delta A_2.$$

The deviation ΔA_1 from the reference coefficient ($N = 0.5$, $\alpha = 0$) caused by a 23° rotation of the principal stress axes can be determined as shown in Fig. 3a. It is equal to the difference between the coefficient A_f corresponding to $\alpha = 23°$ and $N = 0.5$ and the reference value. Similarly the deviation ΔA_2 caused by the intermediate principal stress ratio ($N = 0.153$) can be evaluated as the difference between the coefficient A_f corresponding to $\alpha = 0$ and $N = 0.153$, and the reference value. The reference value of the coefficient A_f ($\alpha = 0$ and $N = 0.5$) is equal to 0.84. The quantities ΔA_1 and ΔA_2 determined by the proposed method are equal to 0.12 and −0.12 respectively. Hence the predicted value of A_f corresponding to $N = 0.153$ and $\alpha = 23°$ is therefore equal to 0.84. This value compares well with the measured value of 0.83 as shown in Table II. In a similar manner the maximum deviator stress and the angle of internal friction can be calculated for any rotation and for any value of the intermediate principal stress ratio. The predicted values of the pore-pressure coefficient A_f, the deviator stress, and the friction angle ϕ' agree closely with those determined experimentally from series III as shown in Table II.

The test data indicate that the principle of superposition is applicable and that the shear-strength and pore-pressure parameters can be predicted for any value of rotation of the principal stress axes and for any value of the intermediate principal stress ratio from two series of tests where the effects of these two parameters have been evaluated separately.

ACKNOWLEDGMENT

This study was sponsored by the National Science Foundation (Grant NSF-G21833).

REFERENCES

CASBARIAN, A. O. (1964). Effects of rotation of the principal stress axes and of the intermediate principal stress on the shear strength of a saturated remolded clay. Ph.D. thesis, Cornell University, Ithaca, New York.

HABIB, P. (1953). Influence de la variation de la contrainte principale moyenne sur la résistance au cisaillement des sols. *Proc. Third International Conference on Soil Mechanics and Foundation Engineering*, Vol. 1, pp. 131–6.

HENKEL, D. J. (1958). The correlation between deformation, pore water pressure and strength characteristics of saturated clays. Ph.D. thesis, University of London, England.

HIRSCHFELD, R. C. (1958). Factors influencing the constant volume strength of clays. Sc.D. thesis, Harvard University, Cambridge, Massachusetts, 398 pp.

HVORSLEV, M. J. (1960). Physical components of the shear strength of saturated clays. *Proc. Research Conference on Shear Strength of Cohesive Soils* (Boulder, Colorado), pp. 169–273.

LAMBE, T. W. (1958). The structure of compacted clay. *Proc. American Society of Civil Engineers*, Vol. 84, No. SM2, Paper 1654.

RENDULIC, L. (1936). Relation between void ratio and effective principal stresses for a remolded silty clay. *Proc. First International Conference on Soil Mechanics and Foundation Engineering*, Vol. 3, pp. 48–51.

TAYLOR, D. W. (1955). Review of research on shearing resistance of clay. Report submitted to the United States Army Engineers Waterways Experimental Station.

WU, T. H., A. K. LOH, and L. E. MALVERN (1963). Study of failure envelope of soils. *Proc. American Society of Civil Engineers*, Vol. 89, No. SM1, pp. 145–81.