A Fundamental Stress-Strain Pattern in Granular Materials Sheared with Small or No Volume Change

Relation contrainte-déformation fondamentale de matériaux granulaires durant cisaillement à volume constant

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SUMMARY

During constant-volume shearing tests on cohesionless granular materials, after particle reorientation, suppressed dilatancy usually causes stresses to increase parabolically with strain until near the C.V.R. locus. The volumetric moduli of sands appear to vary exponentially with porosity; their critical pressures also do, but far more rapidly until crushing begins.

Unconfined compression tests on siltstone, sandstone, saturated silty clay (undrained), stabilized marl, and hard coal have given partly parabolic stress-strain curves. Their granular structures may be hard particles in contact, or intact lumps between microfissures which close at higher pressures.

Natural or artificial viscous cement in voids often makes granular material appear "elastic" at lower stresses in normal laboratory tests; extremely slow straining reveals the effect of granular structure. Repeated slow loading of stabilized soil, simulating traffic action, makes elasticity much more parabolic.

SOMMAIRE

Au cours d'essais de cisaillement à volume constant effectués sur des matériaux granulaires non cohésifs, le fait de supprimer la dilatabilité produit une courbe contrainte-déformation de forme parabolique. Les modules volumétriques des sables et les pressions critiques semblent changer exponentiellement avec la porosité jusqu'au commencement du broyage.

Des essais sans contrainte latérale effectués sur des échantillons de grès fin, de grès, d'argile limoneuse saturée, de marne stabilisée et de charbon dur ont donné des courbes contrainte-déformation partiellement paraboliques. Ces matériaux peuvent être constitués de grains très résistants ou encore d'agglomération de grains séparés par des microfissures qui se referment à haute pression.

Sous des contraintes moins fortes, la cimentation visqueuse fait souvent paraître élastiques les matériaux granulaire lors des essais normaux de laboratoire; une déformation extrêmement lente révèle les effets de la structure granulaire. Le chargement répété d'un sol stabilisé, en simulant l'action de la circulation, rend l'élasticité beaucoup plus parabolique.

INTERACTION OF DILATANCY AND COMPRESSIBILITY

Though concave-upward stress-strain curves (in which the slope steadily increases during much of the test) were reported many years ago, possible causes for the generally parabolic shape of the concave section do not appear to have been discussed. Fig. 1 shows some constant-volume tests on a silt and two sands by other investigators. The initial consolidation pressure, of 5 lb/sq.in. (0.35 kg/sq.cm.) or over, caused the rapid initial rise.

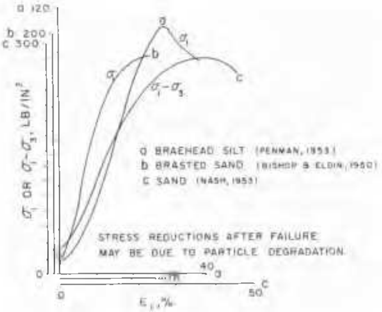

FIG. 1. Parabolic stress-strain curves in undrained triaxial tests on cohesionless soils.

Dilatancy, discovered by Osborne Reynolds (1885), means the tendency of a mass of particles to expand when sheared. This occurs below the critical stress, here defined as the mean stress giving a particular critical void ratio (Roscoe, *et al.*, 1958) during continued shearing. In triaxial compression tests, changes of mean pressure also cause volume changes; the over-all change is the sum of both effects, which cancel out in a constant-volume test.

Though a clay can contract easily without shear strains, a granular material needs shear strains to undergo large volume changes. At least for high relative porosities (i.e., when dense), granular materials generally dilate much more powerfully than inactive clays. Work by Frederick (1961, 1962) in the author's laboratory showed that dilatancy is far more sensitive to changes of relative porosity in highly rounded particles than in less rounded ones. These problems have also been discussed by Kolbuszewski and Frederick (1963).

The resistance of a granular material, at the same strain, varies rapidly with porosity. Fig. 2 shows results of tests on two sands (Chaplin, 1961a, 1961b) with curves from K_0 and isotropic tests by Fraser (1957) on Brasted sand. The volumetric modulus varies exponentially with porosity, so far as can be seen, and the slopes are remarkably similar. The K_0 curve for Brasted sand lies above the isotropic curve because dilatancy in the K_0 test has more effect than the freedom of grains to rotate; the curves tend

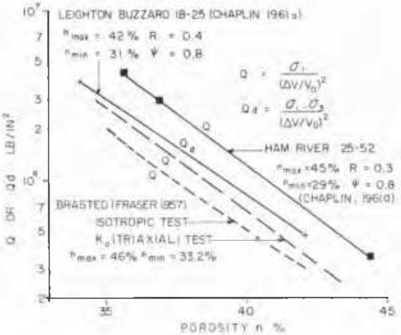

FIG. 2. Variation of parabolic compressibility modulus Q with porosity in three sands.

to converge at higher porosities because dilatancy tends to zero as the maximum porosity is approached. The formation of new contacts under increasing load, discussed previously (Chaplin, 1963), presumably had little effect in this sand.

In both K_0 and isotropic tests, deformation at grain contacts involves large indentations at higher stresses, with rolling in the K_0 test. Rolling and sliding are very localized in the isotropic test, and comparing the tests, σ_1 affects the volume changes far more than σ_3.

Real granular materials behave very differently from perfect spheres in regular packings, for which the load per contact only varies between 0.44 and 1.0 σD^2, where D is the diameter and σ the average isotropic stress (Chaplin 1961a, 1961b). In random packings at a low relative porosity (loose state), few contacts are suitably oriented to resist the principal stresses. If a grain turns, it is likely to lose nearly as many contacts as it gains. At a high relative porosity, many more contacts are near the best directions, and grain rotation is far more likely to make new contacts than break old ones.

One may deduce how stresses should increase with shear (or axial) strain in a constant-volume test (ignoring membrane penetration) by making three assumptions.

Assumption 1: A granular material tries to expand in proportion to the increase of shear strain, apart from crushing. Fig. 3 suggests how the constant of proportionality may vary in granular materials of different particle shape, except near the C.V.R. locus.

Assumption 2: The volumetric strain in a compressibility test at a low stress ratio varies linearly with the square root of pressure (Chaplin, 1961a, 1961b).

Assumption 3: The pressure increase needed to suppress a potential volume increase obeys the same power law (say 0.5 index) at both low and high stress ratios.

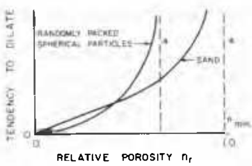

FIG. 3. Influence of relative porosity on the tendency to dilate.

We conclude that the square root of any of the stresses σ_1, σ_2, σ_3, $(\sigma_1 - \sigma_3)$, $(\sigma_1 + \sigma_2 + \sigma_3)/3$, etc., should increase linearly with strain. Fig. 4 gives some examples for cohesionless granular soils. A large isotropic consolidation pressure sets up particle contacts differing markedly from the preferentially oriented ones set up after K_0 consolidation. Fig. 4 also emphasizes how much shear strain is needed for non-spherical particles to develop new contacts to suit a changing stress pattern, unlike the implicit assumption by Rowe (1962, 1963) that none is needed.

When granular particles are placed at successively lower porosities before shearing, at some porosity the stresses at failure will cause crushing. This limits the strength attainable in undrained and drained tests, giving a fairly sharp discontinuity in the C.V.R. locus. Crushing excepted, the critical pressure seems likely to have an exponential form (Figs. 5a and 5b) for any soil with *either* a very small clay fraction *or* enough clay to keep the granular particles "floating" separately.

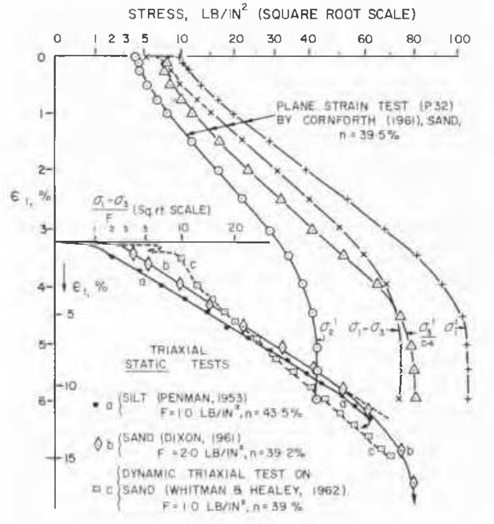

FIG. 4. Stress-strain curves for undrained compression tests on cohesionless soils.

It is very difficult to get reasonably uniform porosity in a small sample and measure it accurately, but the sand results (Fig. 5a) fit an exponential pattern very well. Though one sand did not crush even at the highest pressures, the other crushed slightly, and the silt apparently crushed at all pressures except the lowest. Presumably the silt had weak grains, for example, shell fragments, rather than very angular or flaky grains (they would probably have given much higher porosities). At low pressures it behaved like the two sands, because crushing did not occur.

Tests on spherical steel balls and glass beads, Fig. 5b, show that highly spherical particles can "lock" solidly if sheared without volume change below a narrow range of porosity, within which critical pressures vary extraordinarily rapidly with change of porosity. Indeed one would expect a given grading of spherical particles with a nearly infinite elastic modulus to have the same average critical porosity at

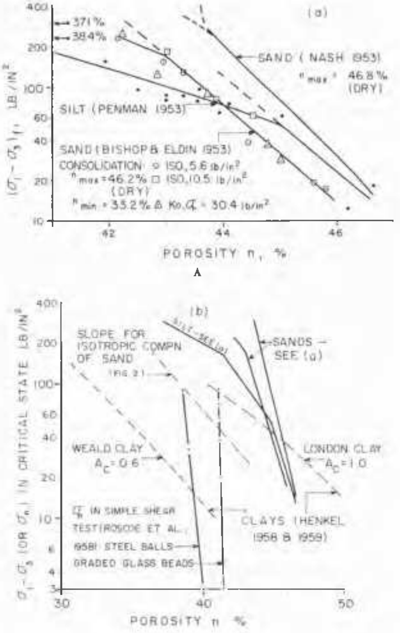

FIG. 5. Variation of strength in limiting state with porosity in (a) cohesionless soils (b) clays and spherical particles.

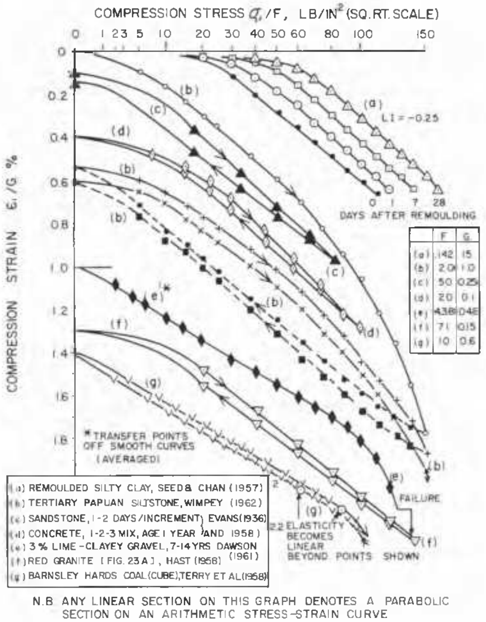

FIG. 6. Stress-strain curves of cohesive and cemented granular materials.

all pressures, and that the critical porosity would be independent of size.

The clays in Fig. 5b have critical pressures varying far more slowly with porosity than any granular material. The Weald clay (activity 0.6) and the more active London clay (activity 1.0) differ partly because of grading, but more likely because of unequal amounts of "soft ice" around particles which decrease the effective porosity. In clay soils at low pressures the effective friction is that of the "soft ice," but at higher pressures the amount of clay fraction makes a great difference. If there is a small clay fraction, at higher pressures the granular particles come into contact and form a separate structure. Their friction, which is probably higher than that of the "soft ice," increasingly controls that of the whole material as the porosity decreases.

The slopes in Fig. 5b for the two clays are close to that of the variation of volumetric stiffness for Fraser's isotropic compressibility tests on sand (Fig. 2). It seems reasonable to attribute much of the well-known exponential pattern of variation of clay properties with void ratio to the geometric properties of random packings, for which we can take the sand as a model.

STRESS-STRAIN CURVES OF COHESIVE GRANULAR MATERIALS

The curves in Fig. 6a show how strongly a granular structure can affect unconfined compression behaviour. The silty clay was tested at various time intervals after remoulding. The first test was affected by changes taking place during the test, but the others show identical behaviour typical of a granular material, here held at constant volume by the pore water. The large stress after a small strain is due to the suction present, which grows slightly and causes "sensitivity."

The energy for rebound, as in the siltstone test shown in Fig. 6b, comes from the energy stored at intergranular contacts. Fig. 6c shows a sandstone tested slowly enough for the cementing to add but little stiffness in shear, though preventing much expansion. Fig. 7 suggests a general pattern of behaviour in cemented granular materials at different strain rates. Fig. 6d shows that concrete under slow loading can behave as a typical granular material; there is a serious lack of tests at low strain rates on concrete of modern gradings.

An exceptionally well-matured 3 per cent lime-clayey gravel road base, Fig. 6e, gave a remarkably parabolic stress-strain curve. Though the cementing was sufficient to prevent much dilation during most of the shearing, its own shear stiffness was low enough not to mask the granular behaviour at low stresses.

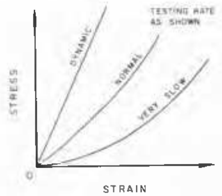

FIG. 7. Influence of testing rate on form of stress-strain curve for a cemented granular material.

Microfissured materials are represented by a granite (Fig. 6f) and a hard coal (Fig. 6g). The rebound of the coal suggests that at low porosities the expansion of the aggregations between microfissures will almost exactly equal the compression over the same stress range, whether the microfissures be open (parabolic elasticity) or closed (linear elasticity).

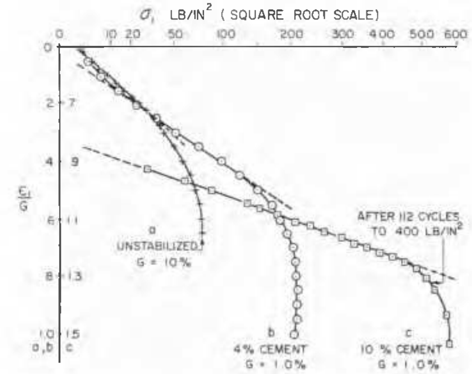

FIG. 8. Stress-strain curves for remoulded and stabilized Keuper marl.

Fig. 8 shows that even unstabilized remoulded Keuper marl, curve a, can behave as a granular material at low stresses. Repeated loading of stabilized Keuper marl, curve c, confirms the impression from Fig. 6e that the soil-cement, lean concrete, etc. used in roads may have a much smaller elastic modulus at small strains, after repeated loading by traffic, than normal laboratory tests apparently show.

In cohesive granular materials with cementing, long-term tests will show low strengths because the tensile strength of cement varies so greatly with time of loading. Even though the cement may contain enough hard crystals to itself be dilatant (e.g. Portland cement), during some intermediate time range it will yield enough in shear not to add much stiffness to the granular structure, but without failing in tension.

Carey (1953) gave a numerical definition of rheidity as the time within which 0.999 of the deformation of a particular material is plastic. The rheidity of a cementitious deposit with a granular structure would be affected by the eventual tensile breakdown of cementing material following its viscous flow, in addition to creep and solution effects at the intergranular contacts between mineral grains.

DEGRADATION BY CONTINUED SHEARING

Continued shearing changes the shape of particles in a granular material. Débris accumulates, and some or much of it lies loosely in the voids without taking part in resisting stress. The effective porosity increases, and the effective relative porosity may increase markedly (at least below crushing pressures) through sharp corners becoming rounded, so affecting the "limiting porosities" (unobtainable) of those particles currently in contact. Crushing can affect both undrained and drained stress-strain curves at higher stresses.

The new surface produced during shearing is roughly proportional in a given material to the energy absorbed. Shear strains under low pressures, as in a beach, if repeated often enough can cause great degradation, and one might expect traffic stresses to have a similar though far smaller effect. If the same sand is redeposited in a shear box after each test, after a few tests at the same porosity and moderate normal pressure (1 to 2 tons/sq.ft.) it can become markedly weaker, despite complete redeposition between tests.

CONCLUSION

Concave-upward stress-strain curves have been shown to be associated with the interaction of dilatancy and compressibility in materials with a structure of reasonably hard mineral grains. Cemented granular materials and microfissured solids in unconfined compression, also sands and silts in constant-volume tests, have given parabolic curves over part of the stress range. No clays other than very silty or sandy clays appear to do so.

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