Secondary Compression Effects during One-dimensional Consolidation Tests

Effets de compression secondaire durant des essais de consolidation unidimensionnelle

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SUMMARY

One-dimensional consolidation experiments with remoulded samples of two estuarine clays and two clay minerals are described. In one series of tests each load increment was maintained for approximately 24 hours and in a second series, for about one month. In all cases the excess pore water pressures at the base of the sample were measured by pressure transducer.

Using an extension of Terzaghi's theory as a first approximation, theoretical solutions have been obtained which clarify the effect of the flexibility of the pore water pressure measuring system during such tests.

The majority of the tests are not significantly affected by this and further analysis shows that they are in closer agreement with the theory of Gibson and Lo than with that of Terzaghi. However, in the long-duration tests, the rate of pore-pressure dissipation at the base is still higher than one would expect from the measurements of surface settlement.

SOMMAIRE

On décrit des expériences de consolidation unidimensionnelle avec des échantillons remainiés de deux argiles d'estuaire et de deux mineraux d'argile. Au cours d'une première série d'expériences chaque augmentation de charge fut maintenue pendant approximativement 24 heures et au cours d'une deuxième série pendant un mois environ. Dans chaque expérience la pression de l'eau interstitielle à la surface inférieure de l'échantillon fut mesurée avec un transducteur de pression.

En appliquant une extension de la théorie de Terzaghi pour obtenir une première approximation, des solutions théoriques ont été obtenues qui expliquent l'effet de la flexibilité du système de mesure des pressions interstitielles pendant ces expériences.

Cette flexibilité n'a pas d'effet significatif sur la plupart des expériences, et des analyses plus détaillées montrent que les résultats s'accordant mieux avec la théorie de Gibson et Lo qu'avec la théorie de Terzaghi. Cependant, dans les expériences de longue durée, la rapidité de dissipation de la pression interstitielle à la surface inférieure est encore plus grande que ne le feraient prévoir les mesures se rapportant au tassement de la superficie.

ONE OF THE FIRST ATTEMPTS at a rational revision of Terzaghi's one-dimensional theory of consolidation for clays was made by Taylor and Merchant (Merchant, 1939; Taylor and Merchant, 1940) who suggested that primary consolidation and secondary compression occur concurrently, with the component due to secondary compression developing at a rate which is proportional to the amount of residual secondary compression. In another paper (Christie, 1964) it has been shown that, provided certain errors in the published versions of Taylor and Merchant's theory are corrected, their theory is mathematically equivalent to a more recent one (Gibson and Lo, 1961) in which the compressibility of an element of the soil skeleton is represented by a rheological model comprising a Hookean spring in series with a Kelvin body.

In this paper some experiments are described in which an attempt was made to verify this theory, not only by settlement measurements during one-dimensional consolidation tests, but also by measuring the pore water pressure at the base of the sample during the tests.

The effect of the flexibility of the pore water pressure measuring system during such tests has been considered in some detail because this can give rise to misleading results.

OUTLINE OF EXPERIMENTS

Two series of one-dimensional consolidation experiments were conducted as follows:

1. Three remoulded soils (kaolinite, clay from Leigh-on-Sea, Essex, and silty clay from Grangemouth) were subjected to tests of normal duration with measurements of pore water pressure at the base of the sample throughout each test. Four samples of each soil were subjected to either five or seven pressure increments between zero and 105 lb/sq.in.

2. The clays from Leigh-on-Sea and Grangemouth and samples of bentonite were subjected to similar tests in which each pressure increment was maintained on the sample for approximately one month.

The results of some general classification tests on the four soils are given in Table I.

TABLE I. GENERAL PROPERTIES OF SOILS TESTED

Soil	Liquid limit (per cent)	Plastic limit (per cent)	Per cent by weight $<2\mu$	Predominant clay mineral
Kaolinite	80	43	98	Kaolinite
Wyoming bentonite	687	42	84	Montmorillonite
Grangemouth silty clay	60	24	26	Illite
Leigh-on-Sea clay	88	29	43	Illite

The consolidation tests were carried out in a cell which permitted free drainage of pore water from the top of the sample only. The pore water pressure at the base of the sample was measured by a transducer, incorporating a diaphragm and unbonded resistance strain wires. The soils to be tested were mixed under vacuum at water contents in the region of 90 per cent of their liquid limits.

FLEXIBILITY OF THE PORE-PRESSURE MEASURING SYSTEM

In some tests the measured pore water pressure increased gradually to a peak value, which was less than the pressure increment applied to the top of the sample, before decaying. A similar effect has been reported previously (for example, Geuze, 1957) and it has also been suggested that this might be due to the flexibility of the pore water pressure measuring system which allows partial drainage of pore water from the base of the sample and provides temporary storage for this (Whitman, Richardson, and Healy, 1961). Whitman, Richardson, and Healy did not produce a rigorous mathematical analysis of this problem but simulated the effect by means of a special-purpose electrical analogue. Similar problems, but with different boundary conditions, have been analysed by Gibson (1963).

In the analysis which follows it is assumed, as a first approximation, that each element of soil behaves in accordance with Terzaghi's one-dimensional theory of consolidation, with the coefficient of consolidation (c_v) having the same value in swelling as in compression, but the flexibility of the measuring system introduces an unusual boundary condition at the base of the sample (thickness H). In general, the flexibility of the measuring system comprises (a) compressibility of the fluid used to transmit the pore water pressure from the base of the sample to the measuring device; (b) expansibility of any pipes and valves containing the above fluid: (c) expansibility of the system due to deformation of the diaphragm of the pressure transducer (if used). These components are not considered separately in this analysis and the flexibility is assumed to be linear, i.e., the volume change of the measuring system per unit change of pressure within it is constant ($=\chi$).

If the initial pressure within this system (p_{mo}) is less than the initial excess pore water pressure throughout the soil (u_0), some pore water escapes from the base of the sample and is stored temporarily in the measuring system. Later, when the pore pressure at the base is higher than elsewhere in the soil sample, this water flows back into the soil. Throughout this process the pore water pressure at the base of the sample and the pressure in the measuring system (p_m) are equal and continuity of flow must be preserved at the base of the sample.

The partial differential equation to be solved is

$$\partial u/\partial t = c_v(\partial^2 u/\partial z^2) \qquad (1)$$

subject to the boundary conditions

(i) $u = 0$ at $z = 0$ $\qquad (t > 0)$

(ii) $u = u_0$ when $t = 0$ $\qquad (0 < z < H)$

(iii) $p_m = p_{mo}$ when $t = 0$

(iv) $u = p_m$ at $z = H$ $\qquad (t > 0)$

(v) $(Ak/\gamma_w)(\partial u/\partial z) + \chi(dp_m/dt) = 0$ at $z = H (t > 0)$

where A is the area of the sample and the remaining symbols have their usual significance in Terzaghi's classical theory of consolidation.

Using condition (iv) p_m can be eliminated from (v) to give

(vi) $(Ak/\gamma_w)(\partial u/\partial z) + \chi(\partial u/\partial t) = 0$ at $z = H \qquad (t > 0)$.

Introducing the Dirac delta function (see, for example, Sneddon, 1961) conditions (ii) and (iii) can be replaced by

(vii) $\partial u/\partial z = u_0\delta(z) - (u_0 - p_{mo})\delta(z - H)$ when $t = 0$.

Using the usual separation of variables technique we assume a solution to equation (1) of the form

$$u = e^{-c_v\beta^2 t}(D\cos\beta z + B\sin\beta z)$$

where D, B, and β are constants.
From condition (i), $D = 0$.
From (vi)

$$\frac{Ak}{\gamma_w}e^{-c_v\beta^2 t}B\beta\cos\beta H = \chi c_v\beta^2 e^{-c_v\beta^2 t}B\sin\beta H,$$

i.e. $\qquad \beta\tan\beta H = Ak/\gamma_w\chi c_v$

Or, putting $\alpha = \beta H$,

$$\alpha\tan\alpha = C \qquad (2)$$

where $\qquad C = AkH/\gamma_w\chi c_v = AHm_v/\chi$.

(The roots of equation (2) have been tabulated by Carslaw and Jaeger, 1959.) The series solution is therefore

$$u = \sum_{n=1}^{\infty} e^{-\alpha_n^2 T_v}B_n\sin\frac{\alpha_n z}{H} \qquad (3)$$

where $\qquad T_v = c_v t/H^2$.

Differentiating with respect to z and applying condition (vii),

$$\sum_{n=1}^{\infty}\frac{\alpha_n B_n}{H}\cos\frac{\alpha_n z}{H} = u_0\delta(z) - (u_0 - p_{mo})\delta(z - H)$$

$$\text{therefore } \frac{\alpha_n B_n}{H} = \frac{u_0\int_0^H\delta(z)\cos(\alpha_n z/H)dz}{\int_0^H\cos^2(\alpha_n z/H)dz} - \frac{(u_0 - p_{mo})\int_0^H\delta(z - H)\cos(\alpha_n z/H)dz}{\int_0^H\cos^2(\alpha_n z/H)dz}$$

$$= \frac{u_0 - (u_0 - p_{mo})\cos\alpha_n}{[H/2(\alpha_n^2 + C^2)](\alpha_n^2 + C^2 + C)}$$

$$\text{therefore } B_n = \frac{2(\alpha_n^2 + C^2)}{\alpha_n(\alpha_n^2 + C^2 + C)}\{u_0 - (u_0 - p_{mo})\cos\alpha_n\}$$

Substituting in (3), the final solution for excess pore water pressure at any point in the sample is therefore

$$u = 2u_0\sum_{n=1}^{\infty}\frac{(\alpha_n^2 + C^2)}{\alpha_n(\alpha_n^2 + C^2 + C)}e^{-\alpha_n^2 T_v}\sin\frac{\alpha_n z}{H} - 2(u_0 - p_{mo})\sum_{n=1}^{\infty}\frac{(\alpha_n^2 + C^2)}{\alpha_n(\alpha_n^2 + C^2 + C)}e^{-\alpha_n^2 T_v}\cos\alpha_n\sin\frac{\alpha_n z}{H} \qquad (4)$$

where α_n is a positive root of equation (2).
(Equation (4) can also be derived by means of the Laplace transform.)

Numerical solutions have been evaluated from special cases of equation (4) for pore water pressure at the base of the sample ($z = H$), for various values of C, assuming that the initial pressure in the measuring system is either equal

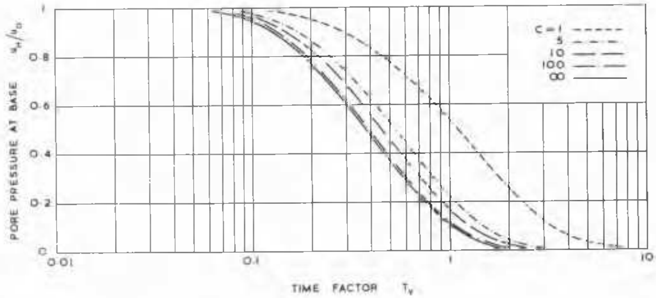

FIG. 1. Theoretical effect of flexibility of the pore pressure measuring system on pore pressure at base of sample, $p_{mo} = u_0$.

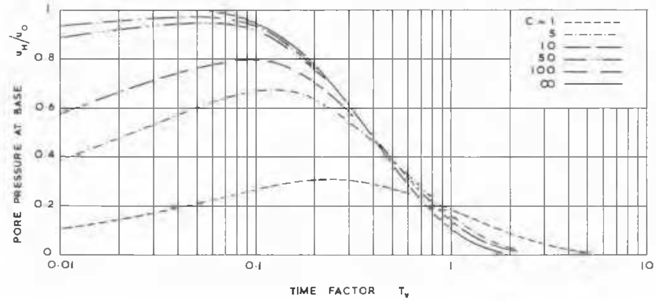

FIG. 2. Theoretical effect of flexibility of the pore pressure measuring system on pore pressure at base of sample, $p_{mo} = 0$.

to the initial excess pore water pressure in the sample or zero. These are shown in Figs. 1 and 2 respectively.

If $p_{mo} = u_0$, the effect of increasing the flexibility of the measuring system, and thereby decreasing C, is to prolong the consolidation process. If $p_{mo} = 0$, the pore water pressure at the base of the sample increases from zero to a peak value and then decays. In this case, decreasing C reduces the peak pressure and increases the time lag in the process of pore-pressure dissipation.

The effect of the flexibility of the measuring system on the settlement of the surface of the sample has also been evaluated. If $p_{mo} = 0$, a very flexible system causes unusually rapid settlement at small values of time but delays the settlement during the later stages of consolidation. For this case, the relationship between pore water pressure at the base of the sample and average degree of consolidation, for various values of C, is shown in Fig. 3.

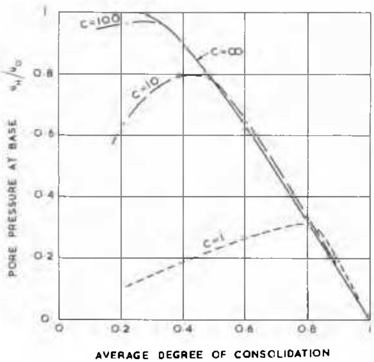

FIG. 3. Pore pressure at base of sample plotted against average degree of consolidation for various values of $C (= AHm_v/\chi)$. Initial pressure in measuring system, $p_{mo} = 0$.

OBSERVED EFFECT OF ENTRAPPED AIR

Taking the measured value of flexibility of the pore water pressure measuring system in conjunction with the minimum observed value of coefficient of soil compressibility (m_v), one obtains a value of C ($= AHm_v/\chi$) greater than 1,000. The measuring system itself is therefore effectively rigid in relation to the soils tested. A less satisfactory feature of the apparatus is the difficulty of placing a soil sample in the oedometer without enclosing air at the base of the sample. Air entrapped in this way has the effect of increasing the flexibility of the measuring system by an unknown amount.

Some typical curves of pore water pressure at the base of the sample plotted against time are shown in Fig. 4. For kaolinite sample K4, the corresponding plot of base pore water pressure against average degree of consolidation is included in Fig. 5. (The average degree of consolidation is defined as $U = (e_1 - e_a)/(e_1 - e_2)$ where e_1 and e_2 are the initial and final void ratios for an increase in effective stress from p_1 to p_2, and e_a is the average void ratio throughout

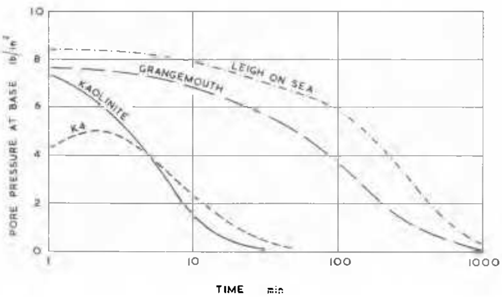

FIG. 4. Typical recorded curves of pore pressure dissipation at base of sample. (Pressure increment 8.75 to 17.5 lb/sq.in.).

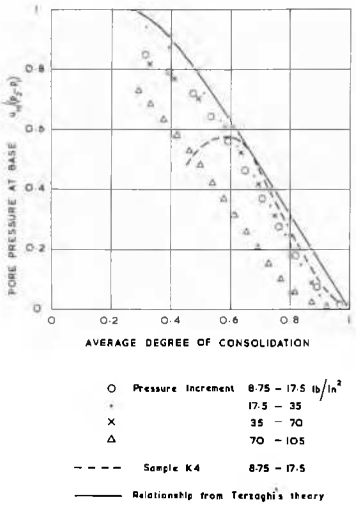

FIG. 5. Average results, from tests of normal duration with kaolinite, of pore pressure at base of sample plotted against average degree of consolidation.

the sample at time t). A comparison of the curves for sample K4 with the theoretical curves in Figs. 2 and 3 (and a similar comparison of the settlement curves, which are not shown here) indicates that all the symptoms of high flexibility of the measuring system are displayed in this case. At the conclusion of the test, the flexibility of the pressure measuring system for this sample was so large that it could no longer be measured with the equipment available.

This is evidently a case in which air was trapped at the base of the sample prior to testing. A few other tests were similarly affected and the results of such tests have been excluded from the subsequent analysis.

TESTS OF NORMAL DURATION

For the tests of normal duration, some deviations from Terzaghi's theory were indicated by the different values for c_v which were obtained by different curve-fitting methods.

These deviations are more clearly shown by a non-dimensional plot of pore pressure at the base of the sample against average degree of consolidation. Mean curves of this type for the kaolinite tests are given in Fig. 5. An examination of these curves, together with similar ones for Grangemouth and Leigh-on-Sea clays (Christie, 1963), revealed the following trends:

1. The deviations from Terzaghi's theory were least with kaolinite and greatest with the Grangemouth clay. A possible explanation of this is that the Grangemouth clay contains the smallest percentage clay fraction of the soils tested.
2. For the natural clay soils, the deviations were slightly greater at large values of applied pressure, or small void ratios.
3. In all cases the deviations were significantly larger for the last pressure increment, in which the pressure increment ratio $(p_2 - p_1)/p_1$, was reduced to one-half. This is in agreement with work elsewhere (Leonards and Girault, 1961).
4. For a given average degree of consolidation, the pore pressure at the base of a sample is always smaller than predicted by Terzaghi's theory.

With a suitable choice of parameters, curves based on Gibson and Lo's theory (Christie, 1964) would fit the experimental data somewhat better than Terzaghi's theory does. However, these parameters have not been evaluated from this series of tests because secondary compression was obviously incomplete at the end of 24-hour loading periods.

DETERMINATION OF PARAMETERS FROM LONG-DURATION TESTS

The parameters which are required to define Gibson and Lo's rheological model for an element of the soil skeleton are the compressibility of the Hookean spring (a), the compressibility and viscosity of the Kelvin body (b and $1/\lambda$ respectively), and the permeability (k). In place of λ and k the values of λ/b and $\theta, = k/(a\gamma_w)$, will be stated here. Gibson and Lo (1961) have described methods for determining these parameters from the settlement-time data for a particular test. To avoid the necessity for calculating strains, their procedure has been slightly modified by the author as follows.

At large values of time, when u is negligible, the surface settlement is

$$\rho_t \simeq (p_2 - p_1)H[a + b(1 - e^{-(\lambda/b)t})].$$

The final settlement is

$$\rho_f = (p_2 - p_1)H(a + b). \tag{5}$$

The residual settlement at time t is therefore given by

$$\rho_f - \rho_t \simeq (p_2 - p_1)Hbe^{-(\lambda/b)t}$$
$$\simeq d_f - d_t$$

where d_f and d_t are the dial gauge readings corresponding to ρ_f and ρ_t respectively.

Therefore $\log_{10}(d_f - d_t) \simeq \log_{10}[(p_2 - p_1)Hb] - 0.434(\lambda/b)t.$

A graph of $\log_{10}(d_f - d_t)$ is plotted against time and a straight line fitted to the points at large values of time. This line intercepts the vertical axis at $\log_{10}[(p_2 - p_1)Hb]$ and has a slope of $-0.434\ \lambda/b$. Hence b and λ/b are obtained and a follows from equation (5).

Gibson and Lo have shown that, at small values of time,

$$\rho_t \simeq 2a(p_2 - p_1)\sqrt{(\theta t/\pi)}.$$

The slope of the initial straight line in a graph of settlement against $\sqrt{t}$ is therefore $2a(p_2 - p_1)\sqrt{(\theta/\pi)}$, from which θ can be calculated.

RESULTS OF LONG-DURATION TESTS

The shape of the settlement-time curves in the long-duration tests can be defined more accurately by means of Gibson and Lo's theory than by Terzaghi's. The procedure described in the immediately preceding section has been applied to each of these tests and mean values of the parameters obtained in this way are given in Table II. It should be noted that these values are based on the settlement-time measurements only.

For the limited range of tests carried out, a and b decrease with increasing applied pressure. At the same time, the viscosity of the soil skeleton $(1/\lambda)$ increases so the ratio λ/b, which determines the rate of secondary compression, remains approximately constant for a given soil (Lo, 1961).

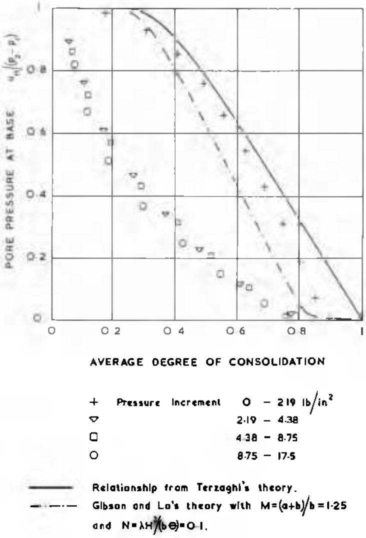

FIG. 6. Average results, from long-duration tests with Leigh-on-Sea clay, of pore pressure at base of sample plotted against average degree of consolidation.

Curves of pore pressure at the base against average degree of consolidation were plotted for each test and some average values for Leigh-on-Sea clay are shown in Fig. 6. Except for the first load increment, the deviations from the Terzaghi theory were very large. While the theory of Gibson and Lo gives better agreement with the experimental data, some discrepancy still exists. This is shown by the fact that the values stated in Table II (obtained from settlement measurements) lead to values of M $(=(a + b)/a)$ between 1.1 and 1.6 whereas much higher values would be required to fit some of the experimental curves in Fig. 6. In other words, the observed rate of pore-pressure dissipation is higher than one would expect from the settlement measurements.

REFERENCES

CARSLAW, H. S., and J. C. JAEGER (1959). *Conduction of heat in solids.* 2nd ed. Oxford University Press.

CHRISTIE, I. F. (1963). The consolidation of soils. Ph.D. thesis, University of Edinburgh.

——— (1964). A re-appraisal of Merchant's contribution to the theory of consolidation. To be published.

GEUZE, E. C. W. A. (1957). Discussion, Session 1. *Proc. Fourth International Conference on Soil Mechanics and Foundation Engineering,* Vol. 3, pp. 90–1.

GIBSON, R. E., and K. Y. LO (1961). A theory of consolidation for soils exhibiting secondary compression. *Acta Polytechnica Scandinavica,* 296/1961, Ci 10. (Reprinted by the Norwegian Geotechnical Institute).

GIBSON, R. E. (1963). An analysis of system flexibility and its effect on time-lag in pore-water pressure measurements. *Géotechnique,* Vol. 13, pp. 1–11.

LEONARDS, G. A., and P. GIRAULT (1961). A study of the one-dimensional consolidation test. *Proc. Fifth International Conference on Soil Mechanics and Foundation Engineering,* Vol. 1, pp. 213–18.

LO, K. Y. (1961). Secondary compression of clays. *Proc. American Society of Civil Engineers,* Vol. 87 (SM4), pp. 61–87.

MERCHANT, W. (1939). Some theoretical considerations on the one-dimensional consolidation of clays. S.M. thesis, Massachusetts Institute of Technology.

SNEDDON, I. N. (1961). *Special functions of mathematical physics and chemistry.* 2nd ed. Edinburgh, Oliver and Boyd.

TAYLOR, D. W., and W. MERCHANT (1940). A theory of clay consolidation accounting for secondary compression. *Jour. Mathematics and Physics,* Vol. 19, pp. 167–85.

WHITMAN, R. V., A. M. RICHARDSON, and K. A. HEALY (1961). Time-lags in pore pressure measurements. *Proc. Fifth International Conference on Soil Mechanics and Foundation Engineering,* Vol. 1, pp. 407–11.

TABLE II. MEAN VALUES OF PARAMETERS IN GIBSON AND LO'S THEORY FROM TESTS OF LONG DURATION

Soil	Pressure increment (lb/sq.in.)	a (sq.in./lb × 10^{-4})	b (sq.in./lb × 10^{-4})	λ/b (min^{-1} × 10^{-1})	θ (sq.in./min × 10^{-1})
Bentonite	0–2.19	965	223	0.29	1.11
	2.19–4.38	550	305	0.35	0.76
	4.38–8.75	507	169	0.53	0.46
Leigh-on-Sea clay	0–2.19	703	95	0.41	5.5
	2.19–4.38	202	55	0.57	3.8
	4.38–8.75	134	33	0.48	3.8
Grangemouth silty clay	0–8.75	190	22	0.96	19.2
	8.75–17.5	55	18.5	0.95	9.3
	17.5–35.0	32	7.2	0.41	7.2