Primary and Secondary Consolidation of a Spherical Clay Sample

Consolidation primaire et secondaire d'un échantillon d'argile sphérique

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SUMMARY

A spherical sample is tested in a double cell apparatus under all-round pressure in order to study volume compression without distortion. Test results are compared with computations of primary consolidation. Samples loaded beyond the preconsolidation load show a considerable secondary volume compression.

SOMMAIRE

On décrit un essai sur un échantillon sphérique dans une cellule double, qui permet d'étudier la compression de volume en évitant la distorsion. Les résultats expérimentaux sont comparés avec le calcul de la consolidation primaire. Des échantillons chargés au-dessus de la charge de préconsolidation montrent des changements de volume secondaires considérables.

WHEN THE BEHAVIOUR of a soil mass under general three-dimensional loading has to be described, it is convenient to separate the deformation of the skeleton into two components: volume compression (without distortion) and distortion (without volume changes). Both modes of deformation are time dependent. However, the retardation for the two is different, because the reason for delayed reaction is not the same.

In the primary consolidation the free water opposes changes of volume. The gradual loss of pore water in impermeable soils causes a retardation which was described mathematically by Terzaghi (1923) and can be computed by his theory of hydrodynamic consolidation. Distortions are not hampered by the free water. In the three-dimensional extension of the hydrodynamic consolidation theory (Biot, 1941) shear deformations are supposed to follow instantaneously the loading of the soil by shear stresses.

For the secondary consolidation no theory is available

$$\frac{u_2}{p} = 2m\sum_{j=1}^{\infty} \frac{\{x_j \cos[x_j(1-\lambda)] - \sin[x_j(1-\lambda)] - x_j\lambda\} \exp(-x_j^2 c_v t/R_2^2)}{[m(2+\lambda)-\lambda^2(1-\lambda)]x_j \cos[x_j(1-\lambda)] - [m(1-\lambda)x_j^2 + 2m + \lambda(1+\lambda^2)]\sin[x_j(1-\lambda)]}$$

$$\frac{\Delta V}{V} = -\frac{p}{K+\frac{4}{3}G\lambda^3} + \frac{3p\lambda}{2G(1-\lambda^3)}\sum_{j=1}^{\infty} \frac{\{(1-\lambda)x_j \cos[x_j(1-\lambda)] - (\lambda x_j^2 + 1)\sin[x_j(1-\lambda)]\}\exp(-x_j^2 c_v t/R_2^2)}{\{[m(2+\lambda) - \lambda^2(1-\lambda)]x_j \cos[x_j(1-\lambda)] - [m(1-\lambda)x_j^2 + 2m + \lambda(1+\lambda^2)]\sin[x_j(1-\lambda)]\}x_j^2}$$

where $m = \dfrac{K+\frac{4}{3}G}{4G} = \dfrac{1-\nu}{2(1-2\nu)}$; $\lambda = R_1/R_2$;

and the x_j*'s are the solutions of* $\operatorname{tg}[x(1-\lambda)] = x\dfrac{mx^2+\lambda(1-\lambda)}{(m+\lambda^2)x^2+\lambda}$

FIG. 1. Formulae for excess pore pressure at outer boundary (u_2) and volume change (ΔV) of spherical sample, subjected to a constant all-round pressure p, according to the linear theory of three-dimensional consolidation.

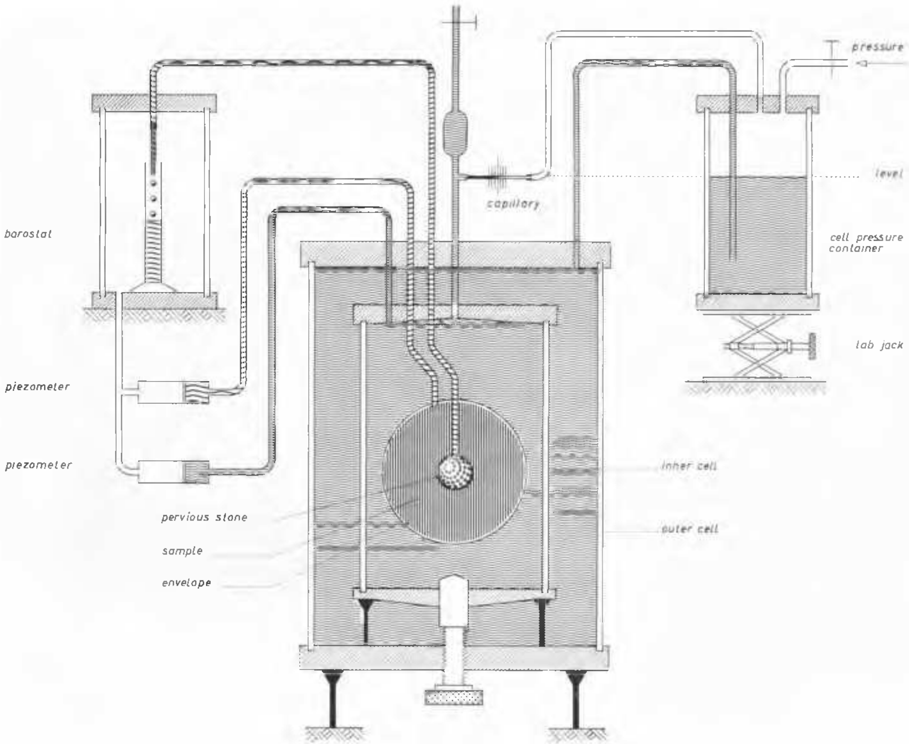

FIG. 2. Double-cell test set-up for measuring volume change, excess pore pressure and discharge from spherical sample under all-round pressure.

which describes mathematically the physical mechanism of deformation after excess pore pressure has vanished. Keverling Buisman (1940) suggested a logarithmic time law, which was based on settlement experiments in oedometer tests with lateral confinement.

In such a test one-third of the settlement is due to volume change and two-thirds due to distortion. From the test results it is not possible to determine the contribution of either mode of deformation to secondary consolidation. If the secondary motions are due to viscous retardation of particle displacements with respect to each other it could be conjectured that distortion is accompanied by more retardation because distortion requires more mutual displacement. However, other mechanisms are possible, for instance, the pore system may contain canals of different sizes. In that case excess pore pressure measured may indicate the pressure in the coarser canals only. Secondary consolidation can then be attributed to a hydrodynamic consolidation in the finer network. This kind of secondary consolidation can be expected to retard volume compression more than distortion.

Since every undrained test with uniform stress distribution allows the study of distortion without volume change, it was the object of this study to devise a test which permitted the study of the time dependence of volume compressibility without distortion.

A homogeneous, isotropic body of arbitrary form will not distort if subjected to an all-round pressure, because throughout the body the normal stresses are equal to this all-round pressure and all shear stresses are zero. However, during primary consolidation in clay the effective stresses are not uniform because pore water drainage is not uniform over the sample but localized at the draining boundaries. This non-uniformity entails deviator stresses and therefore it is impossible to devise a test in which shear stresses and distortions are prevented during primary consolidation. In the secondary period distortions are absent if the load is an all-round pressure.

In order to minimize the difficulties of test interpretation the form of the sample chosen was a sphere. The test device was arranged to measure excess pore pressures, discharge of drained pore water, and volume changes caused by an all-round pressure on the sample.

PRIMARY CONSOLIDATION THEORY

Spherical symmetry is the simplest case displaying the effects of three-dimensional consolidation. The test can be arranged in such a way that wall friction is absent. Primary consolidation can be described by a few variables. All concentric spheres remain concentric and conserve their spherical form during deformation. The only variables are their radial displacements v during compression or expansion and the excess pore water pressure u. Both are constant over each concentric sphere and are a variable of the radius R and the time t only. Spherical symmetric consolidation was treated by Cryer (1958), Gibson, *et al.* (1963), Josselin de Jong (1953, 1963, 1964), and Verruijt (1965) for different conditions of drainage and loading. The boundary conditions have to consist of conditions imposed on the pore pressure and stresses or displacements of the soil skeleton.

In the test set-up the sample with radius R_2 is surrounded by an impermeable sheet that is loaded by an all-round pressure p at $t = 0$. In the centre of the sample a rigid pervious sphere of radius R_1 is connected by a small tube to a zero pressure; water is drained out of the sample through this pervious sphere. Excess pore pressure can be measured at the outer boundary of the sample, just inside the envelope. Before $t = 0$ everywhere in the sample ($R_1 < R < R_2$), all variables are zero, $u = 0$, $v = 0$, $\sigma_R = 0$, where $\sigma_R = \sigma'_R - u$ is total normal radial stress (negative for pressure). After $t = 0$ the boundary conditions are:

$$R = R_1 : u = 0, v = 0 \quad (1)$$

$$R = R_2 : \partial u/\partial R = 0, \sigma_R = -p. \quad (2)$$

In the case of instantaneous elastic response of the soil skeleton, volume compressibility is expressed by the bulk modulus K, and the relation between deviator stress and distortion is given by the shear modulus G. By means of these moduli the effective stress σ'_R can be expressed in the displacement v. Assuming a linear Darcy's law with permeability k, the coefficient of consolidation $c_v = k(K + \frac{4}{3}G)/\gamma_w$ is constant. Then the primary consolidation is described by the following equations

$$\partial\epsilon/\partial t = c_v \nabla^2\epsilon \quad (3)$$

$$\nabla^2 u = (K + \tfrac{4}{3}G)\nabla^2\epsilon \quad (4)$$

where ϵ = volume strain = $R^{-2}[\partial(vR^2)/\partial R]$ and $\nabla^2\epsilon = R^{-1}[\partial^2(\epsilon R)/\partial R^2]$.

The differential equations (3), (4) can be solved with operational calculus for the boundary conditions (1), (2) giving expressions for u and v as functions of R and t. During the tests the excess pore pressure at the outer boundary u_2 and the volume change of the total sample $\Delta V = 4\pi R_2^2 v_2$ are recorded. These quantities are given in Fig. 1.

In order to plot u_2 and ΔV as functions of t it is necessary to adopt a value for m, which amounts to an assumption of Poisson's ratio ν. The figures are drawn for $m = 1$ (i.e., $\nu = \frac{1}{3}$). Different values for ν do not change the shape of the curves enough to be a critical measure for a determination of ν.

In order to account for deviations between theoretical curves and observed test data, it is necessary to introduce time-dependent or non-linear stress-strain relations. These alterations are liable to complicate the differential equations to such an extent that the solution by mathematical means is prohibited. For that case a solution by finite differences is prepared that permits the application of a graphical solution by Schmidt's method, or the use of a computer. The advantage of the graphical procedure is the insight gained into the behaviour of the variables during elaboration. The graphical procedure can easily be extended to non-linear problems (Verruijt, 1963).

TEST SET-UP

The test set-up is represented in Figs. 2 and 3. The volume change of the sample is measured by observing the water discharge from a double cell system. The pressure in the water in both cells is equal, so the inner cell transmits no pressure differences. Consequently there is no water loss from the inner measuring cell by leakage. Calibration showed that, besides the compressibility and temperature dilatation of the water, the response of the inner cell to pressure variations contained only a small reproduceable non-linear effect. The order of magnitude of this small correction corresponds to

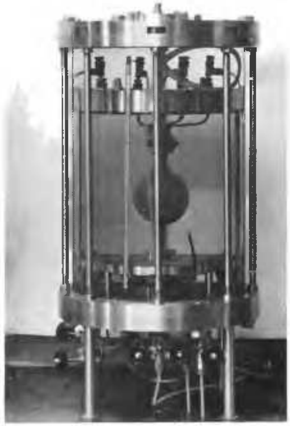

FIG. 3. Double-cell apparatus containing spherical sample.

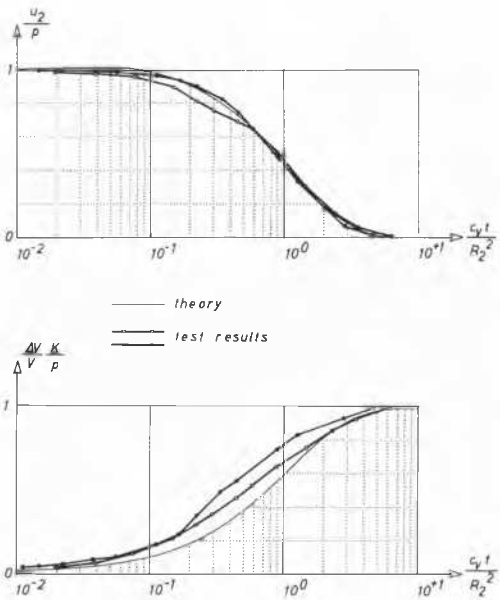

FIG. 4. Excess pore pressure at outer boundary (u_2) and volume change of sample (ΔV) as functions of the time (t) for loads below the preconsolidation load.

the experience of Kolbuszewski and Frederick (1963) with a similar double cell apparatus. The cell pressure is applied by air pressure in the container to the upper right in Fig. 2 that is connected to both inner and outer cells. This container can be adjusted vertically by a lab jack which is so operated that the water level in the container coincides with a capillary tube connected to the inner cell. Volume changes of the water in the inner cell are read from the motion of the water meniscus in this capillary tube without pressure changes. (We are indebted to ir. E. W. Taconis, LGM, Delft, for this device.) The meniscus in the capillary tube can be set at any desired position by the volume adjustment screw at the bottom of the inner cell. Accuracy is about 40 cu mm for a sample of 180 cu cm.

Pore water drained from the sample through the central porous stone is brought to the barostat (Fig. 2, upper left) and the discharge is observed in a calibrated tube. The barostat has an air pressure which is used as reference for the electronic piezometers which measure cell pressure and pore water pressure at the outer boundary of the sample. By using a barostat, fluctuations in atmospheric pressure are eliminated.

The sample was completely saturated in order to simplify test result interpretations. This was achieved by consolidating the clay starting from a suspension of clay particles in deaired water (water content about 120 per cent). The spherical envelope was filled with this suspension out of a closed circuit. When enough suspension was admitted, the supply tube was closed and water was drained from the centre through the porous stone by applying an all-round pressure. Final water content during experiments was 33 to 40 per cent. A verification of the degree of saturation was obtained by measuring excess pore pressure caused by an all-round pressure in undrained condition. Since the excess pore pressure was equal to the load and the compressibility of the sample equal to that of water, it was concluded that the degree of saturation was 100 per cent.

INTERPRETATION OF TEST RESULTS

The tests consisted of changing the all-round pressure by steps. Response was measured by determining: (*a*) volume change of sample by measuring the amount of water entering or leaving the inner cell, (*b*) discharge of pore water through porous stone, (*c*) excess pore pressure at outer boundary of sample. For completely saturated samples items (*a*) and (*b*) are to be equal and therefore give only one independent variable. (They were nearly equal in the experiments.) Item (*c*) gives the second independent variable.

Unloading and reloading to a preconsolidated value gave test results for u_2 and ΔV, plotted in Fig. 4. The time scale of the test results was adjusted in such a way that the test data and theoretical curve coincide at the moment that u_2 has reduced by 50 per cent. In the time parameter $c_v t/R_2^2$ then, t and R_2 (the size of the sample) are known and c_v can be computed. Different values for c_v as a function of ν are listed in Table I. The u_2 and ΔV curves fit quite closely,

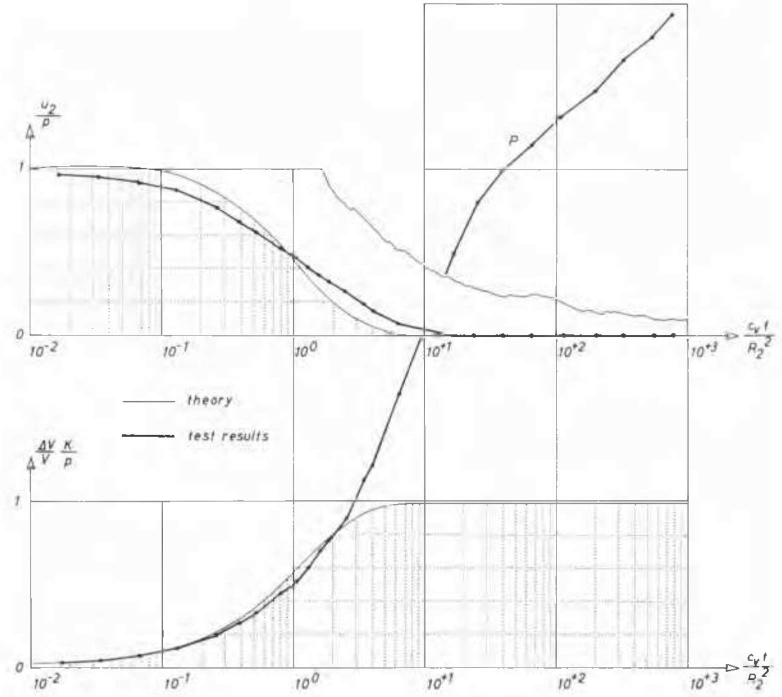

FIG. 5. Excess pore pressure at outer boundary (u_2) and volume change of sample (ΔV) as functions of the time (t) for loads beyond the preconsolidation load.

TABLE I. VALUES FOR THE CONSOLIDATION COEFFICIENT c_v AS FOUND FROM TEST RESULTS

Type of test	c_v(cm²/sec)		
	$\nu = 0.000$	$\nu = 0.333$	$\nu = 0.450$
Consolidation	3.41×10^{-3}	1.73×10^{-3}	1.32×10^{-3}
Swelling	1.51×10^{-3}	0.76×10^{-3}	0.58×10^{-3}
Reconsolidation	1.99×10^{-3}	1.01×10^{-3}	0.77×10^{-3}
Oedometer rests	varying from 0.62×10^{-3} to 2.86×10^{-3}		

showing that the clay possesses linear elastic properties in the primary consolidation period if preconsolidated. This is in accordance with test results recorded by Gibson, *et al.* (1963).

In Fig. 5 a less close fit is obtained for loads surpassing the preconsolidation load. The shape of the curve for u_2 is flatter and requires a further refinement of the theory of primary consolidation. The volume change ΔV on logarithmic time plot shows an S-shaped curve followed by a nearly straight line at a slope. If the S-shaped parts of the curves have to be attributed to a hydrodynamic consolidation process, the normal theory is not applicable since the complete S-curves for u_2 and ΔV are not developed in the same time interval. For a consolidation mechanism whose description requires only one constant value for c_v both S-curves would coincide. But if the canal system consists of pores of different sizes, the computation of the response would require a succession of consolidation processes with a spectrum of gradually varying values for c_v. The introduction of such a mechanism may be expected to result in a flatter slope of the curve for u_2 and a prolonged change in volume after the excess pore pressure has vanished. Also it is not impossible that a rheological model, as proposed by Tan (1954), will show a similarly shaped curve. See for instance the curves obtained by Gibson and Lo (1961).

The straight line beyond the point P in Fig. 5 is not liable to continue as a straight line indefinitely, but may be expected to bend towards the horizontal after an as yet unspecified time interval. This would result in a plot for ΔV consisting of two or more waves, which cannot be described by a simple logarithmic time relationship as proposed by Keverling Buisman (1940).

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