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Effects of Grain Size and Grading on the Shearing Behaviour of Granular Materials

Influence de la granulométrie et de la dimension des grains sur le comportement au cisaillement des matériaux granulaires

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SUMMARY

In this paper the influence of grain size and grading on the shearing behaviour of granular materials is studied from the results of drained triaxial tests. The materials tested (sand and glass beads) were within the range of medium to coarse sand sizes, and in each material the particle shape and surface texture were uniform throughout this range. The influence on the angle of shearing resistance ϕ' of grain size alone is investigated. For this, samples containing particles virtually of uniform size were separated from graded materials. The results for each material, when compared at equivalent porosities, show that ϕ' increases as the grain size reduces. Tests were also performed on three artificial gradings made up from the size fractions of the sand. The results do not allow clear conclusions to be made on the influence of grading on the shearing resistance.

SOMMAIRE

La présente étude porte sur l'effet de la granulométrie et de la dimension des grains sur le comportement au cisaillement de matériaux granulaires à partir d'essais triaxiaux drainés. Les matériaux étudiés étaient du sable et des particules de verre dont la grosseur des grains variait de moyenne à grossière. De plus, la forme et la texture superficielle des grains étaient uniformes dans ces limites. En premier lieu, on a analysé l'influence de la granulométrie seule sur l'angle ϕ' de résistance au cisaillement. Dans ce but, des échantillons à grains essentiellement de même grosseur ont été préparés à partir de matériaux naturellement classifiés. Pour chaque matériau, les résultats, lorsque comparés à porosité équivalente, démontrent que cet angle ϕ' augmente à mesure que la grosseur des grains est réduite. De plus, quelques essais ont été faits sur trois grosseurs de grains classifiés artificiellement et préparés à partir des grains disponibles dans le sable. Toutefois, les résultats ne permettent pas de tirer des conclusions définitives quant à l'influence de la dimension des grains sur la résistance au cisaillement.

THE EXPERIMENTAL RESULTS presented in this paper have been accumulated over a period of years commencing in 1958. The work described is part of a wider research project on the basic properties of granular soils.

In the present investigation, shearing behaviour was interpreted from the results of drained triaxial compression tests. The standard procedure was used in these tests, the major principal stress being increased to failure while the minor principal stress was kept constant. The same cell pressure was used in all the tests of a series and the shearing properties were investigated over a range of porosity. Micrometer gauges were used in the measurements of sample dimensions and the usual corrections applied to the results. The results were evaluated by digital computer employing a simple programme.

Most of the work was performed on Leighton Buzzard sand. The effects of particle size were studied from tests on samples of six uniform particle sizes having diameters ranging between 2 mm and 0.3 mm. An investigation into the effects of grading was performed on the same material, three artificial gradings being made up from the uniform size fractions. The earlier triaxial tests were performed on 1½-in.-diameter samples. Recently, however, the series of tests on the Leighton Buzzard sand was repeated using 4-in.-diameter samples. This latter series was performed in an attempt to reduce the scatter of the results observed in the 1½-inch-diameter samples and also to provide confirmation of the previous findings on this material.

As a check to determine whether the trend of results obtained for the uniform sizes of the Leighton Buzzard sand could also be observed in materials with different particle shapes and surface roughness, a limited number of additional tests was performed on uniform particle size samples of glass beads.

EFFECTS OF PARTICLE SIZE

Nature of Materials

Samples consisting of particles of uniform size were separated from the graded materials by thorough sieving between closely spaced mesh sizes in the British Standard

TABLE I. DATA FOR UNIFORM-SIZED GRANULAR MATERIALS

Size group	Range of size (mm) (sieve aperture)	Mean particle size (mm) (arithmetic)	Limiting porosities max. %	min. %
		Leighton Buzzard sand		
LB1	2.00 to 1.68	1.85	46	33
LB2	1.40 to 1.20	1.30	46	33
LB3	1.00 to 0.85	0.93	46	33
LB4	0.71 to 0.60	0.65	46	33
LB5	0.50 to 0.42	0.46	46	33
LB6	0.42 to 0.36	0.39	46	33
		Glass beads		
G1	1.58 to 1.20	1.44	41	33
G2	1.20 to 0.71	0.96	40.5	33.5
G3	0.42 to 0.30	0.36	41	35

range of sieves. Details of the samples of sand and glass beads obtained in this way are shown in Table I.

Each of the samples was subjected to specific gravity tests and was examined under the microscope to check that the mineralogy and particle geometry were uniform throughout the range of sizes. It was for this reason that the range of sizes of Leighton Buzzard sand investigated had to be restricted to samples coarser than the 0.3-mm size. The finer sizes were found to contain a higher content of ferruginous material and to possess an appreciably greater particle specific gravity than the value of 2.66 measured on the coarser fractions. The results of tests on these fine sizes could not therefore be justifiably compared with the results obtained from the coarser material. Triaxial tests subsequently performed on sizes finer than 0.3 mm showed them to possess a considerably higher frictional component of strength than the other sizes. The particles of the sand in each of the size groups included in the investigation were found to have a high degree of sphericity and a medium roundness.

For the tests on glass beads four fractions were originally removed from graded materials. The size intervals between the limiting sieves used in the separating process were somewhat greater than those used for the Leighton Buzzard sand but the mean particle sizes of three of these corresponded approximately to size groups LB2, LB3, and LB6. Microscope examination of these three samples showed them to be composed almost entirely of spherical particles, only about 5 per cent being of oval shape. The remaining sample had a mean diameter of 3.6 mm but had to be excluded from the investigation into particle size, since it was found to contain about 60 per cent oval-shaped particles and 40 per cent spheres and was therefore appreciably different from the other three glass bead samples. The specific gravity of the smallest size of glass beads was 2.80 whereas that for the remaining sizes was 2.89.

Limiting Porosities

The limiting porosities for the sand were found by the methods suggested by Kolbuszewski (1948). The method for obtaining the minimum porosity (severe compaction by Kango hammer in a Proctor mould), although used, was not considered to be entirely suitable. With this method there was some evidence of grain fracturing. Consequently the grading and the particle geometry were subject to some alteration during the process of compaction. To prevent grain fracturing, this method for finding the minimum porosity was not used for the glass beads. In this case the minimum porosity was found by vibrating the material in a mould. Although the results obtained in this way are unlikely to be comparable with those of other methods, the data obtained from each of the three glass bead samples quoted here can be compared one with another.

The maximum and minimum porosities found for the uniform particle sizes of the Leighton Buzzard sand were 46 per cent and 33 per cent respectively, the figures being approximately the same for all particle sizes. Slichter (1897) showed, from a simple analytical approach, that the theoretical limiting porosities for spheres of uniform size are 47.6 per cent and 26 per cent, the figures being independent of the diameter. When dealing with particles having shapes other than spherical these values do not apply. However it is not unreasonable to expect that, for materials having particles of uniform shape, the limiting porosities of fractions containing a uniform size of particle are the same irrespective of the size.

The introduction of particles of different size into a sample will naturally alter the limiting porosity values, the change being dependent on the range of sizes present and on the grading. Effects of this nature seemed to be present in the glass bead samples (Table I) where the range of sizes in each of the three size groups was greater than in the size groups of the sand.

Triaxial Test Results

Triaxial results are illustrated in the form shown in Figs. 1 and 3 (and also for graded materials in Fig. 5). The angle of shearing resistance is determined from the peak point data on the stress-strain curve employing the normal Mohr-Coulomb interpretation.

The frictional component ϕ_f of the angle of shearing resistance shown in Figs. 1b and 5b was calculated from the principal stress ratio at the point of minimum volume of the sample during the test. This assumes that the frictional resistance of the grains must be overcome before the material can proceed to dilate. Arguments for making this assumption are provided by Kirkpatrick (1961).

A number of expressions for calculating the dilatancy contribution in the triaxial tests have been suggested since the well-known expression was proposed by Bishop and Eldin (1953). The majority of these agree that the dilatancy contribution is a function of the factor $d\Delta v/d\Delta L$ (where

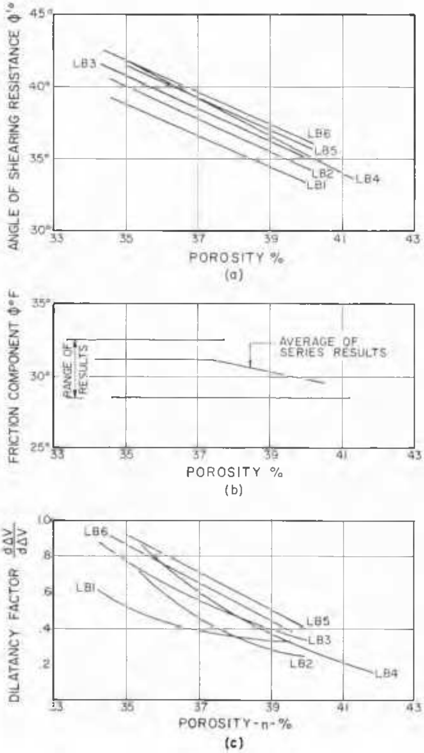

FIG. 1. Shearing properties of the uniform sizes of Leighton Buzzard sand.

$d\Delta v$ is the change in volumetric strain corresponding to an increase in axial strain of $d\Delta L$). Rather than show a preference for one of these methods the dilatancy contribution has not been determined as such in this paper. Instead, the dilatancy effects are illustrated by plotting the factor $d\Delta v/d\Delta L$, which is called the dilatancy factor, at peak point conditions against porosity or relative porosity.

The triaxial tests on the uniform particle size samples of the Leighton Buzzard sand were each performed at a cell pressure of 50 lb/sq.in. A base of porosity was used in plotting the results (Fig. 1) because the limiting porosity values were found to be the same for all size groups. For the sake of clarity, the experimental points are not shown in Fig. 1. The scatter in ϕ' values provided by the results, which were obtained from 4-in.-diameter samples, was small, however, and allowed the positions of the lines in Fig. 1a to be clearly established.

The variation in ϕ' with porosity for the individual size groups is, in general, linear over the range of porosity tested, and an increase in ϕ' at a given porosity is observed as the grain size reduces. The variation in ϕ' with the particle size is further illustrated in Fig. 2. In this figure values of

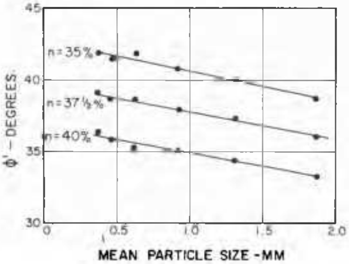

FIG. 2. Relation between ϕ' and particle size for Leighton Buzzard sand.

ϕ' taken from Fig. 1a are plotted against mean particle size for three different porosities. It is seen that the increase in ϕ' is approximately linear as the size reduces for results at constant porosity. This increase in ϕ' is accompanied by an increase in the dilatancy factor (Fig. 1c), the frictional component ϕ_f being independent of size (Fig. 1b).

It is possible that the frictional component may be nearly constant throughout the range of porosity for a given material, and results quoted previously, Kirkpatrick (1961), would indicate this to be so. The determination of the principal stress ratios at the point of minimum volume, however, is subject to inaccuracy, especially in the samples of low porosity, since the major principal stress is changing rapidly at this stage of the test. Such inaccuracy may have affected the results in Fig. 1b where there is a tendency for ϕ_f to decrease at higher porosities.

Fig. 3 shows the results of the trixial tests on the uniform particle size samples of glass beads. In order to save space only the ϕ' values are shown and these are plotted to a base of relative porosity. The results, which were obtained from 1½-in.-diameter samples at a cell pressure of 20 lb/sq. in., show some scatter. The scatter is due mainly to errors in the estimations of porosity. Such errors arise inevitably when dealing with 1½-in.-diameter samples even when refined techniques are used to measure sample dimensions. The results, however, fall into three distinct zones in the figure.

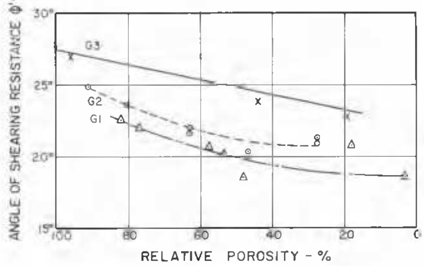

FIG. 3. Relation between ϕ' and relative porosity for glass beads.

As with the Leighton Buzzard sand an increase is observed in ϕ' as the grain size reduces. The frictional component ϕ_f was the same for all three sizes of glass beads and was constant over the ranges of porosity. This finding would justify the inclusion of size G3 in the comparison. It will be recalled that this material had a lower specific gravity than the other fractions of glass beads and therefore must be different in chemical composition. The particle shape and surface friction properties are however identical. The variation in dilatancy factor with particle size and porosity for the beads was similar to that noted previously for the uniform particle size samples of Leighton Buzzard sand.

The influence of particle shape, surface roughness, and probably also mineralogy can be noted when the results of the tests on the glass beads are compared with those of the uniform particle size samples of Leighton Buzzard sand. For the beads the strength components are appreciably lower, ϕ_f being approximately 16°, and the range of dilatancy factors was between 0 and 0.4.

It was originally intended to include a glass bead sample, having a mean particle diameter of 3.6 mm, in the analysis of the effects of grain size. This sample, however, was found to have a high percentage of oval particles (approximately 60 per cent) and the variable of particle shape prevented it from being used. Results of triaxial tests on this sample are, however, interesting since they indicate further the

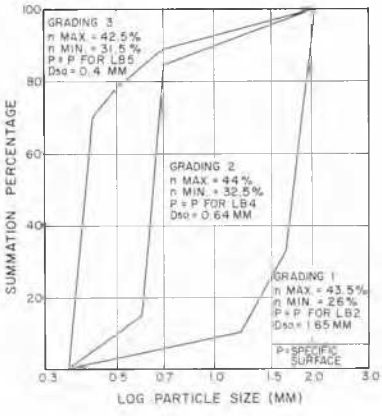

FIG. 4. Size distribution curves for the three artificial gradings of Leighton Buzzard sand.

effect of particle shape on the shearing resistance. The ϕ' relative porosity line for the 3.6-mm size is approximately the same as that shown in Fig. 3 for size group G3. It therefore does not fit with the results for spherical samples, relating particle size to ϕ'. ϕ_f was, however, the same but the dilatancy factors were somewhat higher than those measured on any of the spherical samples.

EFFECTS OF GRADING

Nature of Material

To study the effect of grading on shearing behaviour, the three artificial gradings illustrated in Fig. 4 were made up, using the size fractions of the Leighton Buzzard sand. Mean particle diameter has less significance in the case of graded materials than in the case of materials of uniform particle size. Specific surface was considered to be a better parameter and it was hoped that in using this a correlation might be obtained between the results from the graded sand and those from the sand samples containing a uniform size of particle. The gradings were therefore selected to give specific surfaces approximately the same as three of the uniform particle size samples.

Limiting Porosities

The maximum and minimum porosities were found using Kolbuszewski's methods. The values are shown, together with the other data on the graded sand, in Fig. 4. The limiting porosities are approximately the same for gradings 2 and 3. The presence of the small quantity of fines in the predominantly coarse make-up of grading 1 has had an appreciable influence on the minimum porosity value for this material.

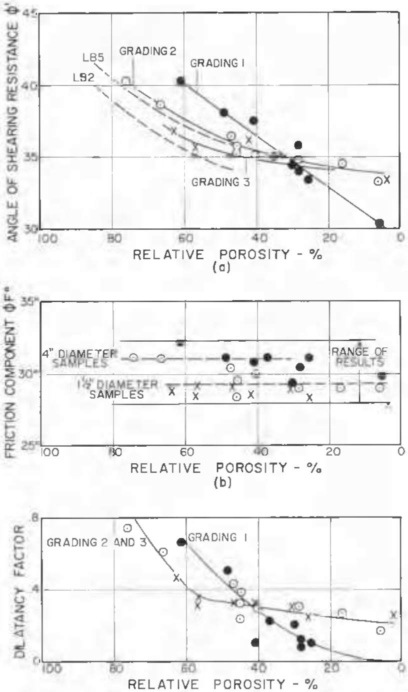

FIG. 5. Shearing properties for the artificial gradings of sand.

Triaxial Test Results

Both 4-in.-diameter and 1½-in.-diameter triaxial samples were used in this analysis and the tests were conducted at the same cell pressure as that used in the tests on the uniform particle size samples of the Leighton Buzzard sand. In the test results, shown in Fig. 5a, the experimental points for the 4-in.-diameter samples lie close to the lines whereas those for the 1½-in.-diameter samples provide a scatter similar to that described previously for this size of sample. The ϕ'–relative porosity curves for gradings 1 and 3 intersect each other and the curve for grading 2 lies slightly below that for grading 3. The results for the graded sands show no clear pattern, and they cannot be correlated with the uniform size samples. The intersection of the curves for gradings 1 and 3 prevents any general conclusions being made on the influence of specific surface or mean diameter on the shearing behaviour of the graded sands. In this analysis, based on relative porosity, much depends on the measured limiting values of n. Small errors in the estimation of these values can alter appreciably the relative position of the ϕ'–relative porosity lines. The different limiting porosity values which are caused by the differences in grading introduces an additional variable which cannot be accounted for simply in the comparative analysis attempted here.

Approximately the same range of values of ϕ_f was measured on the graded sands and on the uniform particle size samples of the Leighton Buzzard sand. In Fig. 5b no change in the value of ϕ_f can be observed over the porosity range. The values for the 4-in.-diameter samples tend to form about a line higher in value than for the 1½-in.-diameter samples. This is probably the result of using a relatively more accurate volume change measuring burette with the smaller samples. Variations in the values of ϕ_f can be noted when comparing grading 1 with gradings 2 and 3 and also when comparing these gradings with those measured on the uniform particle size samples of the sand. The variations are not large compared with the accuracy with which ϕ_f can be measured and it is reasonable to assume that ϕ_f is a particle property, independent of size and grading. As in the previous cases the increase in ϕ' is accompanied by an increase in dilatancy factor.

CONCLUSIONS

The results of the tests have shown that the frictional component of strength ϕ_f, calculated by the method described in this paper, is independent of particle size and grading. It is tentatively suggested that ϕ_f is also independent of particle shape. The magnitude of ϕ_f appears to depend on surface roughness and mineralogy.

It has not been possible to conclude from the results whether factors other than friction and dilatancy contribute to the strength. It was found that the dilatancy contribution, when calculated using the expression of Bishop and Eldin (1953), could not entirely account for the difference between ϕ' and ϕ_f. This could be due to experimental or theoretical shortcomings and does not necessarily mean that other factors are present. It can be stated, however, that the variations in the angle of shearing resistance ϕ' are caused, at least in part, by a varying contribution provided by dilatancy.

The influence of particle size on the shearing resistance has been investigated for particle sizes within the range 0.3 mm to 2 mm. Where grading effects are not present and other particle properties are uniform, it has been found that ϕ' decreases as the particle size increases.

No clear conclusions can be made on the influence of grading on the angle of shearing resistance. The grading influences the limiting porosity values and hence an additional variable is introduced, which prevents a simple comparison of the results from being made. The conclusions made here on the influence of particle size on the shearing resistance are contradicted to a degree by results reported by Kolbuszewski and Frederick (1963). These results were obtained from tests on three samples of closely graded glass beads (ballotini). The mean particle diameters of these were within the relatively narrow range of 0.48 mm to 0.86 mm. The method of determining the minimum porosity in the tests was different from that used for the glass beads in the present paper. Since the values assumed for the limiting porosities greatly influence the relative positions of the ϕ'–relative porosity lines, it is questionable to what extent the two sets of results can be compared. Kolbuszewski and Frederick, in a plot of ϕ' against relative porosity, found that the largest particle size gave ϕ' values appreciably higher than for the remaining sizes. The results of the two smaller sizes agree with the findings of the present paper to the extent that the smallest size gave a higher ϕ'–relative porosity line than the middle size over about 60 per cent of the relative porosity range.

The ϕ values quoted by Kolbuszewski and Frederick are in general much higher than those shown in Fig. 3 of this paper. This shows an important difference in the properties of the materials used in the two series of tests.

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