Tests Concerning Compaction and Displacements Performed on Samples of Sand in the State of Plane Deformation

Essais de compaction et de déplacement sur des échantillons de sable dans un état de déformations plane

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SUMMARY

For the analysis of plane deformation problems, particularly with respect to foundations on sandy soils, a new testing apparatus has been developed which secures a free state of distortion. The purpose of the work is the development of the plane distortion-due-to-stress law, including the failure condition. A series of seventy-three tests with dry sand is described and compared with earlier published results of three-dimensional distortion tests. The test values are used to compute volume changes, distortions and deformation moduli, and to represent them. The results show a clear deviation from the three-dimensional case. It is striking that there are linear relationships between the distortion modulus and the stress. Existing theories are compared with the measurements and the approach to a new theory is indicated, which, however, requires further confirmations with other sands.

SOMMAIRE

Pour étudier les problèmes de déformations planes, en particulier ceux des fondations en terrain sablonneux, un nouvel appareil d'essai a été développé qui n'impose aucune restriction aux déformations planes. Le but de ces travaux est développement d'une loi de déformation plane par suite de la contrainte, incluant la rupture. Une série de soixante-treize essais, exécutés avec du sable sec, est expliquée et comparée avec les résultats de recherches de déformations à trois dimensions publiées antérieurement. A l'aide des valeurs enregistrées, les changements de volume et de forme et les modules de déformation sont calculés et représentés. Les résultats démontrent une déviation marquée de ceux du cas de trois dimensions. Il est frappant de trouver qu'il y a des relations linéaires entre le module de déformation et la contrainte. Des théories existantes sont comparées avec les résultats mesurés et une voie est indiquée qui pourra conduire à une nouvelle théorie, mais qui a encore besoin de démonstrations par des essais avec d'autres types de sable.

LARGE-SCALE TESTS, performed particularly in Berlin, proved clearly that the determination of bearing capacity on the basis of theories of failure does not satisfy the real conditions. From the detailed reports (Muhs, 1961; Lorenz, 1962) only two facts will be mentioned here: (*a*) the theories of failure do not allow the calculation of deformations; (*b*) refinements of the theories of failure increase the discrepancy between the theoretical results and those obtained by large-scale tests. It is not advisable, therefore, to apply the failure theories, particularly those for sandy soils, in cases when the loads are near the surface. Further, the occurrence of failure planes in a soil mass is always preceded by distinctive settlements and dislocations which may be precarious for the foundations. Therefore, a simple practical case of a continuous footing on sand was considered for theoretical calculations of the deformation properties of a soil element, which can be further extended to determine the displacements of the footing (settlements) and of its surrounding area (subsidences and upheavals) in the same way as the displacements of the soil element. The case of a continuous footing is clearly one of plane deformation in a three-dimensional state of stress. Until now, the behaviour of a granular soil under these conditions of stress and strain has not been clarified sufficiently, and the logical application of the results from triaxial tests to the case in question was not possible. Therefore an apparatus to produce plane deformations has been designed on the lines of an apparatus developed by Kjellman (1936) which had not been used, as far as the authors know, to perform systematic analyses.

DESCRIPTION OF THE APPARATUS

The cubic sample (10 by 10 by 10 cm) is wrapped in a rubber mould. A system of 2 by 64 vertical rods (point loads) and 4 by 8 horizontal hammers (linear loads) provides purely normal surface stresses. Fig. 1 shows only one hammer and one rod for each face. The loading arrangement provided equal loads on 8 hammers regardless of their individual displacements.

TEST PROGRAMME

The tests were performed with dry sand having the following grain size distribution: 10 per cent < 0.1 mm, 60 per cent < 0.2 mm, and 90 per cent < 0.9 mm. Three degrees of compaction, dense, medium, and loose, corresponding to the porosities $n = 0.312$ to 0.320, $n = 0.350$ to 0.356, and $n = 0.383$ to 0.393 were utilized for tests. The loosest porosity of the test sand was $n_0 = 0.411$ and the densest $n_d = 0.288$. After the sand surface was covered with a rubber sheet and the upper vertical rods were brought into position, the horizontal stresses σ_1 and σ_3 of equal values were applied in equal steps of 0.5 kp/sq.cm. This process

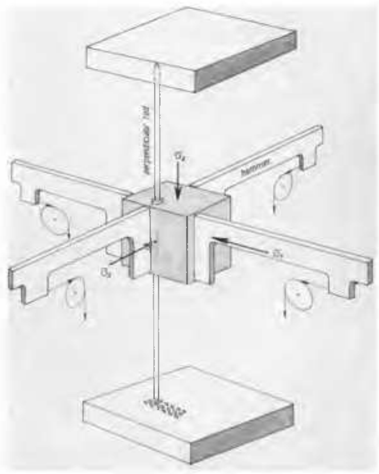

FIG. 1. Test cube with hammers and vertical rods.

is subsequently referred to as "hydrostatic loading." In the loading stage between 0.5 and 3.5 kp/sq.cm., specified for these tests, the horizontal stress σ_1 only was increased in progressively smaller load steps; this process is referred to as "deviatory loading." In this manner, loads were increased until failure. The failure is characterized by the attainment of the maximum value of σ_1/σ_3. Twenty-five such standard tests with dense samples, 23 with medium samples, and 25 with loose samples were used for the analysis. In addition to these standard tests, some samples were subjected to special tests, by alternately loading in the hydrostatic and deviatory manners under steadily increasing σ_1 stress, to determine the influence of the stress path.

The measurements after each load increment were the changes in horizontal spacings between two correspondingly opposite hammers, and the induced vertical stress σ_2 by means of strain gauge blocks. No influence of time could be found. In order to determine the possible dispersion, each test was performed at least twice.

EVALUATION

Values of ϵ_1 and ϵ_3 were first calculated from the measurements by taking the mean of the measured spacings. The use of the mean is justified since, in spite of the free mobility of the hammers in every stage of loading, the sample remained rectangular in the $\sigma_{1,3}$-plane, contrary to expectations. The stresses were reduced according to the proportions of the surfaces. Since ϵ_2 and, therefore, $\Delta\epsilon_2 = 0$, the entire state of stress and deformation is known for each stage of load. The following values were computed: volume change, defined as $\epsilon_v = \epsilon_1 + \epsilon_3 - \epsilon_1 \cdot \epsilon_3$; distortion, defined as $\epsilon_D = \epsilon_1 - \epsilon_3$; a value corresponding to the Poisson's ratio in case of uniaxial load, $\mu_P = -\Delta\epsilon_3/\Delta\epsilon_1$; a value corresponding to the coefficient of rigidity in case of universal distortion $E_P = \Delta\sigma_1/\Delta\epsilon_1$. These values were plotted for each test against σ_1 and/or $\sigma_1 - \sigma_3$. From the curves of ϵ_v and ϵ_D for uniform tests, mean value curves were determined graphically. These may be used as statistical mean values, since the dispersion did not exceed 5 per cent.

ANALYSIS OF THE TEST RESULTS

A characteristic set of test results for dense, medium-dense, and loose sands is shown in Fig. 2. For the purpose of explanation, a schematic illustration of the form of the sample is given for the horizontal section of some typical

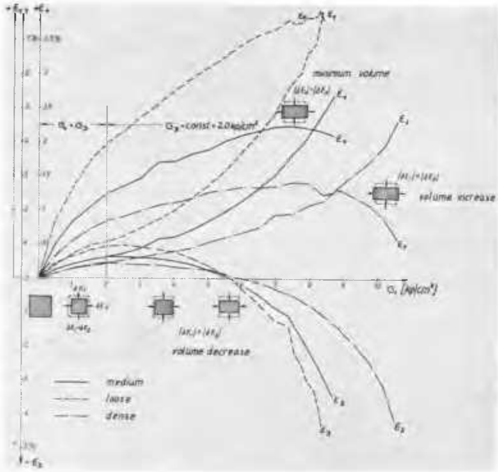

FIG. 2. Typical experimental results.

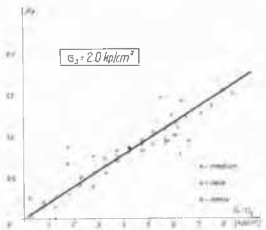

FIG. 3. Poisson's ratio ($\mu_P = -\Delta\epsilon_3/\Delta\epsilon_1$) within the deviatory loading range (σ_3 = constant).

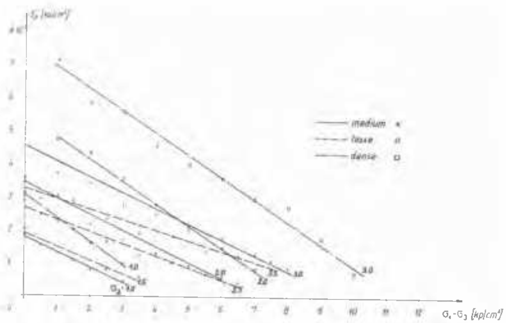

FIG. 4. Modulus of rigidity $E_P = \Delta\sigma_1/\Delta E_1$ *vs.* $\epsilon_1 - \sigma_3$.

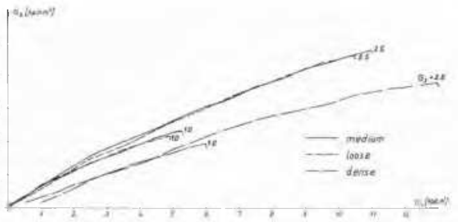

FIG. 5. Vertical stress σ_2 *vs.* σ_1.

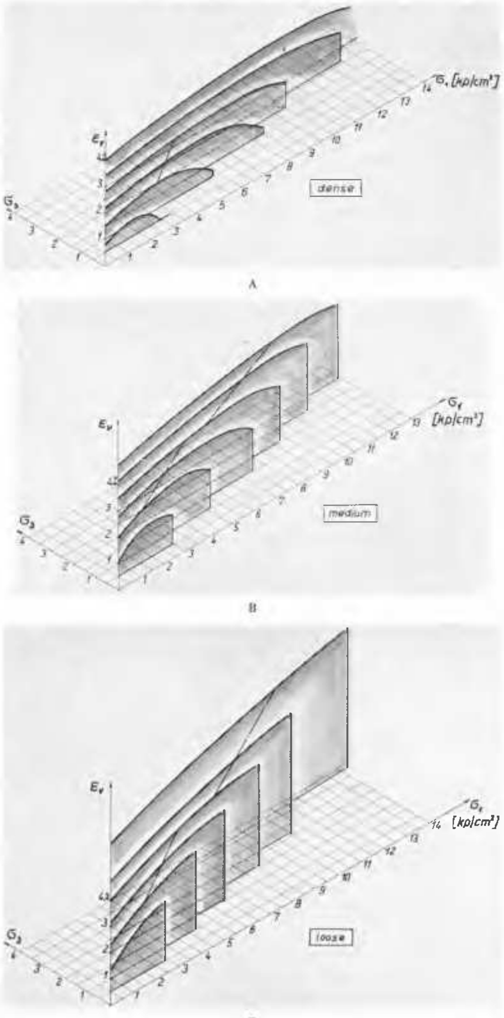

FIG. 6. Volume change ϵ_v *vs.* σ_1 and σ_3: A, dense sand; B, medium sand; C, loose sand.

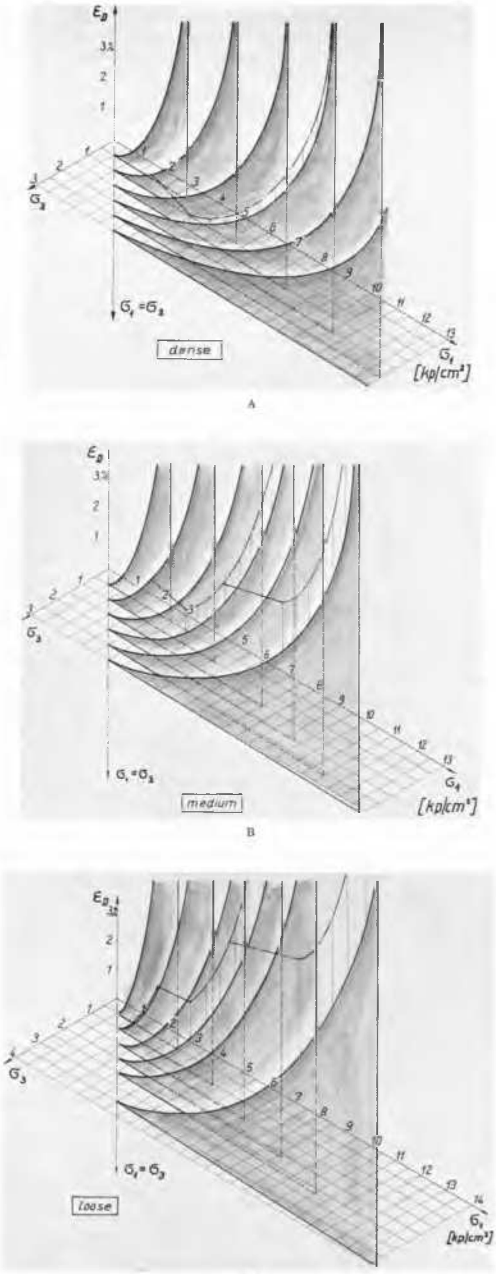

FIG. 7. Distortion ϵ_D *vs.* σ_1 and σ_3: A, dense sand; B, medium sand; C, loose sand.

stages of load. The arrows indicate the direction of the changes in deformation $\Delta\epsilon_1$ and $\Delta\epsilon_3$. Additional explanations as to the trace of the ϵ_v curves can be seen on the diagram. The values μ_P and E_P above $\sigma_1 - \sigma_3$ for some tests with use of dense, medium-dense, and loose sand are shown in Figs. 3 and 4. It will be seen that, regardless of the porosity, μ_P increases linearly from 0 to approximately 1.5 and that E_P, which is slightly dependent on the porosity, decreases from an initial value along an almost straight line to zero until the failure. Since $\epsilon_2 = 0$, $\mu_P = 1$ is the constancy of volume. The dispersion of these values is considerable, as they have been obtained by differentiation. The measured vertical stress σ_2 from some tests with varying porosities is shown in Fig. 5. Due to imperfections of the measuring device, the dispersion of the measured values is important. For this reason, further evaluation of the vertical stress had to be abandoned. The value of σ_2 is remarkably low. Failure is marked by a clear drop of σ_2. Figs. 6 and 7 represent the mean values determined from test results as spherical curves above the $\sigma_{1,3}$ plane in an axonometric way. The paths of deviating stresses are marked by dotted lines, and their trace is not exactly parallel to the σ_1 axis because of the reduction of σ_3. The special test results with a differing stress path have been entered dash-dotted. The curves end at the vertical lines above the points of failure. The volume change ϵ_v (see Fig. 6) increases in a sublinear way to a maximum value, and then drops again. Contrary to triaxial tests, however, no volume expansion, compared with the initial density, is reached. While the porosity n increases, the extreme value moves towards the point of failure. From the abscissa of the maximum value of ϵ_v an angle of friction, $\phi_P = \arcsin\,(\sigma_1 - \sigma_3)/(\sigma_1 + \sigma_3)$, can be computed that is nearly independent of the porosity n and of σ_3 (dense: $\phi_P =$ 34 to 35°; medium-dense: $\phi_P =$ 35 to 37°; loose: $\phi_P =$ 35 to 37°). The distortion ϵ_D (Fig. 7) begins at the $\sigma_1 = \sigma_3$ axis from the value 0 and ascends in a uniformly superlinear way with the increase of the $(\sigma_1 - \sigma_3)$ value. The asymptote values are given by the points of failure.

As Bernatzik (1947) has proved that for triaxial tests the stress path has no influence, some additional tests have been performed in order to check the validity for the plane case. Indeed, there were considerable deviations as can be seen from the dash-dotted ϵ_D curves. The tests are not yet adequate to form general conclusions. For this reason, the following information applies to the standard tests only. For the use of numerical computations, approximate formulae have been developed to represent the results within the given dispersion range: $\epsilon_v = c_1\sigma_1^{c_2} - c_3(\sigma')^{c_4}$. $\epsilon_D = c_5(\sigma')^{c_6}$ in case of $\sigma' \leqslant 1$, and $\epsilon_D = c_7[1 - c_8 \ln(c_9 - \sigma')]$ in case of $\sigma' \geqslant 1$, with $\sigma' = (\sigma_1 - \sigma_3)/\sigma_3^{0.82}$ and the constants for ϵ in per cent and σ in kp/sq.cm. The parabola for ϵ_v in

TABLE I. CONSTANTS FOR VARIOUS VALUES OF n

Porosity n (per cent)	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9
31.2 to 32.0	.62	.451	.00496	3.43	.247	1.31	3.63	.733	4.57
35.0 to 35.6	1.00	.472	.00406	3.66	.670	1.42	4.16	.793	3.88
38.6 to 39.6	1.31	.536	.001	3.75	1.00	1.74	5.11	.886	3.47

hydrostatic loading is known from triaxial tests. It has, however, a lower power because of $\epsilon_2 = 0$ and $\sigma_2 < \sigma_1 = \sigma_3$. The distortion is no longer a function of $(\sigma_1 - \sigma_3)/\sigma_3$ but of $(\sigma_1 - \sigma_3)/\sigma_3^{0.82}$.

THEORETICAL DISCUSSION

It is useful to start the theoretical discussion from the differential distortion-due-to-stress laws. It is to be noted in this case that the distortion-due-to-friction values of the sand are discontinuous. For the statistic characteristics, however, positively continuous laws may be applied. Hoshino (1961) developed a general theory that starts from the hypothesis that the distortion moduli are proportionate to the distortion energy. His method can be applied (using the symbols herein) to the case of hydrostatic loading of the tests as follows: For the volume change under a load of $\sigma_1 = \sigma_3 = \sigma$ let the differential law $d\epsilon_v/(1 - \epsilon_v) = d\sigma/K$ be valid. K is a modulus of compaction. According to Hoshino, $K = cA_v$, where $A_v =$ volume change energy. Since according to the definition: $dA_v = \sigma d\epsilon_v/(1 - \epsilon_v)$, $dA_v = \sigma d\sigma/cA_v$, and by integration $A_v^2 = A_0^2 + c\sigma^2$,

$$K = c\sqrt{(A_0^2 + c\sigma^2)},\ \text{i.e.}\ \frac{d\epsilon_v}{1 - \epsilon_v} = \frac{d\sigma}{c\sqrt{(A_0^2 + c\sigma^2)}}$$

After integration

$$\epsilon_v \doteq -\ln(1 - \epsilon_v) = c^{-3/2}\operatorname{Arsh}\frac{\sqrt{c}}{A_0}\sigma\,.$$

An attempt has been made to determine the constants c and A_0 from the test data in such a way that A_0 depends upon the porosity whereas c remains a true constant. First the scale $\sqrt{c}/A_0$ for σ has been determined that is dependent upon the porosity n and furnishes an optimum congruence of the curves $\epsilon_v = f(\sigma)$; then $c^{-3/2}$ has been found by balancing. By use of the results $c = 25$; $A_0 = 1.6$ (loose), 3.2 (medium-dense), 7.6 (dense) the measured values of ϵ_v (in per cent) are represented satisfactorily. For the deviatory loading Hoshino's theory does not furnish any applicable formula.

EFFORTS TO FIND A NEW THEORY

The straight-line increase of μ_P and the nearly straight-line decrease of E_D under deviatory loading conditions were not anticipated. These findings are in clear contrast to the behaviour in cases of three-dimensional distortion (see Jakobson (1957), Figs. 2 and 3) and indicate the way to a new theory. From the hypothetical equations:

$$E_D = E_3 - a(\sigma_1 - \sigma_3) \qquad \mu_P = b(\sigma_1 - \sigma_3)$$

that is
$$\frac{d\epsilon_1}{1 - \epsilon_1} = \frac{d(\sigma_1 - \sigma_3)}{E_3 - a(\sigma_1 - \sigma_3)}$$

and
$$\frac{d\epsilon_3}{1 - \epsilon_3} = -\frac{d(\sigma_1 - \sigma_3)}{E_3 - a(\sigma_1 - \sigma_3)}\,b(\sigma_1 - \sigma_3).$$

Formulae in good compliance with the test values are obtained by integration. The behaviour during the failure is thereby included with $\epsilon_1 = 1$ and $\epsilon_3 = -\infty$. These values explain the fact that no failure seams have been observed. Further tests with other sands, however, are required to prove whether or not this hypothesis may be made a basis for a distortion-due-to-stress law of general validity.

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