Factors Influencing the Modulus of Elasticity of Dry Sand

Facteurs influençant le module d'élasticité du sable sec

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SUMMARY

Air-dry Ottawa sand in two significantly different states of density was subjected to confining pressures varying from 0.5 kg/sq.cm. to 5.0 kg/sq.cm. in a triaxial testing machine while various ranges of deviator stress were applied and cycled. Because of the inherently inelastic relationship between the normal stresses and the displacement of the individual grains, the importance of the history of the loading upon the modulus of elasticity is extremely significant. In addition to this conclusion, which could be deduced from theoretical considerations, the test results show that the modulus of elasticity of the sand was not a function of the maximum principal stress but definitely was a function of the confining pressure. The modulus of elasticity was significantly influenced by the magnitude of the range of deviator stress, the magnitude of the fixed stress upon which the deviator stress was superimposed, and whether the superimposition was additive or subtractive.

SOMMAIRE

Des échantillons de sable sec d'Ottawa de deux groupes de densité nettement différente furent soumis à des pressions isotropes variant de 0.5 kg/cm.ca. à 5 kg/cm.ca. dans un appareil triaxial en modifiant la contrainte du déviateur entre ces limites extrêmes. Vu la relation non élastique inhérente entre les contraintes normales et le déplacement des grains individuels, l'importance des conditions de charge, naturelles ou artificielles, antérieures aux essais sur le module d'élasticité est extrêmement marquante. Ajoutons à cette conclusion, qui aurait pu être déduite de considérations théoriques, que les résultats des essais montrent que le module d'élasticité du sable n'était pas fonction de la contrainte principale maximum, mais définitivement fonction de la pression isotrope appliquée. Le module d'élasticité était très influencé par la grandeur d'amplitude des contraintes du déviateur, par la grandeur de la contrainte fixe à laquelle celle du déviateur était superposée ainsi que du sens, positif ou négatif, de cette superposition.

ALTHOUGH THE CONCEPT of the modulus of elasticity has been of great use in the analysis of stress and strain in other materials, it has not been used extensively for soils where the amount of settlement, consolidation, stability of slopes, bearing capacity, etc., can be calculated with pertinent data from tests of earth samples. In recent years, however, a great amount of research has been conducted upon the dynamic properties of soils, their response to rapidly applied loads both as a single short-term loading of extremely high magnitude, such as a nuclear blast, and repeated loading of much smaller magnitude such as those induced by reciprocating machinery upon foundations. All of these studies have necessitated the formulation of theories involving the concepts of elasticity and consequently the modulus of elasticity of the soils.

The authors, involved in a study of compression and distortional waves in sand media, conducted some experiments to determine the effect of confining pressure and relative density of a sand upon the value of E, to evolve a theory which recognized the actual rather than the idealized behaviour of sand. This paper describes the results of this programme which, in addition to studying the effects of confining pressure, also included the effects of the range of deviator stress over which E was determined and the effects of cycling the range of deviator stress upon the value of E.

TEST PROCEDURE

All tests were conducted in a Geonor triaxial testing machine. The height and the diameter at various sections of the rubber membranes encasing the sand samples were measured prior to the start of the test in order to calculate the volume of the samples. The confining pressure, axial load, and axial deformation were measured in the usual manner by a Bourdon gauge, proving ring, and dial gauges, respectively.

The lateral deformations at the mid-points of the samples were measured by two optical micrometers which had verniers graduated in 140 divisions per mm, or 0.00715 mm per division. The optical micrometers were calibrated for the refraction effect of the water and surrounding plexiglass container by a finely graduated scale placed inside the container filled with water. One mm on the submerged scale consistently corresponded to 186 divisions on the optical micrometer.

An air-dry clean Ottawa sand having a uniformity coefficient of 1.3 was tested in two states of density, 101 lb/cu.ft. and 109 lb/cu.ft., corresponding to relative densities of 0 per cent and 88 per cent under confining pressures which ranged from 0.5 kg/sq.cm. to 5.0 kg/sq.cm. The loose state was obtained by gently pouring the sand into a thin rubber membrane encased by the conventional Geonor mould whose internal diameter was 3.60 cm. The dense state was obtained by tamping one-half inch layers of sand in the membrane. Densities calculated from the weight of sand placed into the membrane divided by the measured volume of the membrane were reproduced with a variation of ±1.4 per cent.

TEST RESULTS

Inelastic Deformations

Fig. 1 shows the variation in axial deformation with variations in deviator stress as a sample subjected to a

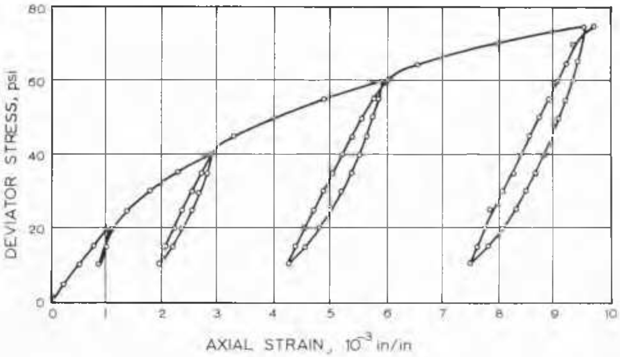

FIG. 1. Deviator stress *vs.* axial strain at 5 kg/sq.cm. confining pressure.

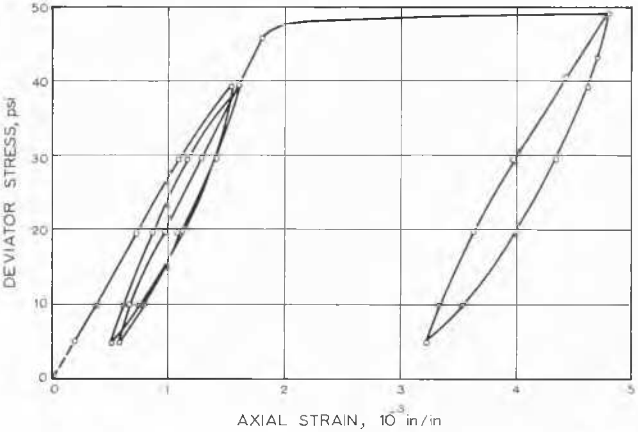

FIG. 2. Elastic-plastic behaviour at 2 kg/sq.cm. confining pressure.

confining pressure of 5 kg/sq.cm. was successively loaded, unloaded to 10 psi, and reloaded to higher values of stress. Large inelastic strains are evident when the deviator stress was reduced to 10 psi. The tangential compliance between the individual grains, which has been shown by Mindlin (1949) and Mindlin and Deresiewicz (1953) for an array of spheres to possess non-linear and inelastic characteristics, undoubtedly is responsible for most of the inelastic deformations shown in Fig. 1. Since the individual grains remained in approximately their displaced positions instead of rebounding to their original positions upon unloading, the sand then exhibited over most of its reloading curve the linear characteristics of a Hookean solid until the magnitude of the deviator stress was large enough to overcome the static friction between the grains. This value of deviator stress would necessarily be larger than the force required to overcome the dynamic friction accompanying the previous movement of the grains and accounts for the work-hardening characteristics exhibited by the curves. The work-hardening process could be continued by increasing the deviator stress until it eventually approximated the failure load, whereupon large axial and lateral deformations occurred with small increases in the magnitude of the deviator stress. This resulted, as shown in Fig. 2, in a deviator-stress-axial deformation curve that closely approximated the ideal elastic-plastic curve so frequently used in plastic analysis.

One of the most interesting characteristics of the sand tested in this programme is that the shape of the loading curve, after being loaded and unloaded, as shown in Fig. 1, appears to be what one would have expected of the virgin curve. This indicates that the modulus of elasticity of the sand corresponding to values of deviator stress greater than those which have been cycled has been unaffected by the previous cycling.

It was observed that the amount of inelastic deformation at a given value of deviator stress was less for higher confining pressures. Again, as one might expect, for a given confining pressure and deviator stress, the amount of inelastic deformation was less for a dense specimen than for a loose specimen. This behaviour is illustrated in Fig. 3 which shows the deviator stress-axial strain curves for the first and twentieth cycles of loading and unloading for various specimens at different confining pressures and densities.

In all of the tests performed in this study, most of the

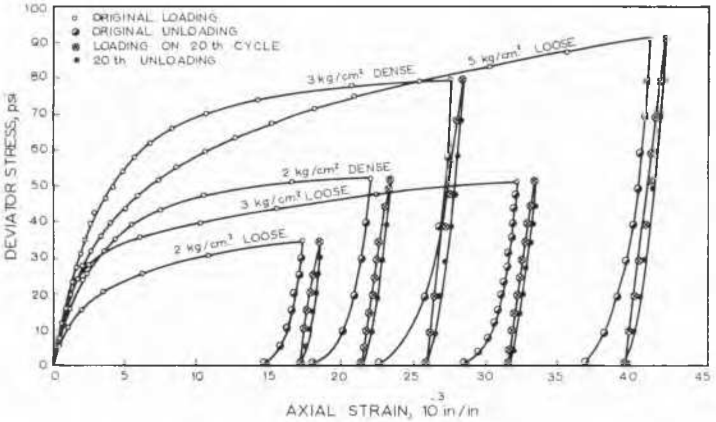

FIG. 3. Effects of confining pressures and repeated loading upon stress-strain relationship.

inelastic deformation occurred in the first cycle of loading and unloading, and increased with additional cycles at a decreasing rate. The width of the hysteresis loop also was greatest in the first cycle and decreased with subsequent cycles. Chen (1944) observed this also within the range of confining pressures of 0.5 to 1.0 kg/sq.cm.

The Effect of the Range of Deviator Stress upon the Modulus of Elasticity

A sample of sand in the denser state, relative density of 88 per cent, under a confining pressure of 5.0 kg/sq.cm., was cycled six times between 74 psi and 5 psi, and then loaded as shown in Fig. 4. The deviator stress was cycled twice over a range of 10 psi at progressively higher values of minimum stress. The slopes of the second reloading curve for each set of cycles are shown in the right hand portion of Fig. 4. The differences between these slopes undoubtedly would have been less if all the inelastic deformation had been removed but the increases in the slopes of the reloading curves as the load increased are so great that it obviously cannot all be attributed to increased stiffness of the sample caused by the earlier cycling. After the deviator stress had been cycled twice from a value of 69 psi, it was cycled once between 69 psi and various lower values of stress as shown in Fig. 5. Figs. 4 and 5 show the following effects upon the value of *E* caused by cycling the deviator stress:

1. The slope of every reloading curve was different and increased with increased cycling. After cycling six times between 74 psi and 5 psi, *E* was 35,700 psi; in subsequent loading between 12 psi and 20 psi, *E* was 35,400 psi; between 59 psi and 69 psi, *E* was 72,600 psi.

2. When the load was reduced from 69 psi to 20 psi, the reloading curve to 69 psi was linear over its entire length, with a value of *E* equal to 42,900 psi.

3. The influence of the range of stress upon the value of *E* is further demonstrated in Fig. 5 where the load was reduced to values of 20 psi, 40 psi, and 59 psi, and in each instance was again increased to 74 psi. In every case, the reloading curves were linear but with different slopes. The figure indicates that when the deviator stress is cycled from a constant maximum value, *E* is greater for smaller variations in stress than for larger variations in stress. At the end of the test, the deviator stress was brought back to 5 psi and increased to 74 psi. The resulting *E* was 37,200 psi, which was almost the same as the value of *E*, 35,700 psi, which was obtained before the incremental cycling shown in Fig. 4 began.

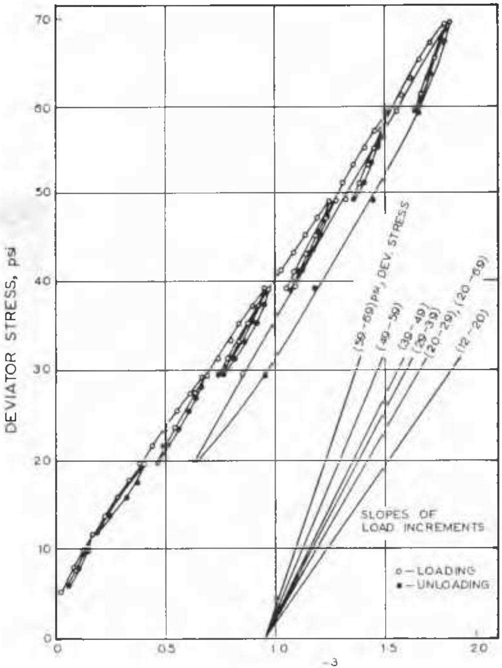

FIG. 4. Influence upon *E* of the lower limit of a constant range of deviator stress.

Some authors have contended that cycling a deviator stress in small increments gives an accurate measure of the modulus of elasticity because the small magnitude of the portion of the deviator stress being cycled will approximate the tangent to the over-all curve of deviator stress *vs.* axial strain. Figs. 4 and 5 indicate that this is not true.

The modulus of elasticity is shown definitely to depend upon the range of the deviator stress and the magnitude of either the maximum or minimum stress bounding the range. Fig. 4 shows that E increased in value as the range of deviator was held constant but the magnitude of the minimum stress was increased. Fig. 5 shows that with the magnitude of the maximum stress held constant, E decreased with increases in ranges of deviator stress and Fig. 1 shows that if the magnitude of the minimum stress is held constant, E does not change with changes in the range of deviator stress.

It should be pointed out that the loading conditions of Fig. 1 correspond to those most frequently encountered in practice where the dead load of a structure is the constant minimum deviator stress and the dead plus live load is the variable maximum value of deviator stress. Under this type of cycling, the value of E is for all practical purposes the same for all ranges of load after the inelastic deformations have been eliminated. However, this value of E over the range of cycled live load will be considerably greater than the value of E over the range of the stress caused by the dead load.

The case of cycling over a range of stress from some constant maximum value of stress depicted in Fig. 5 occurs less frequently in practice but does occur under footings of continuous structures when the proper spans are loaded with a live load which tends to produce uplift and thus reduce the resultant stress from the initial dead load value.

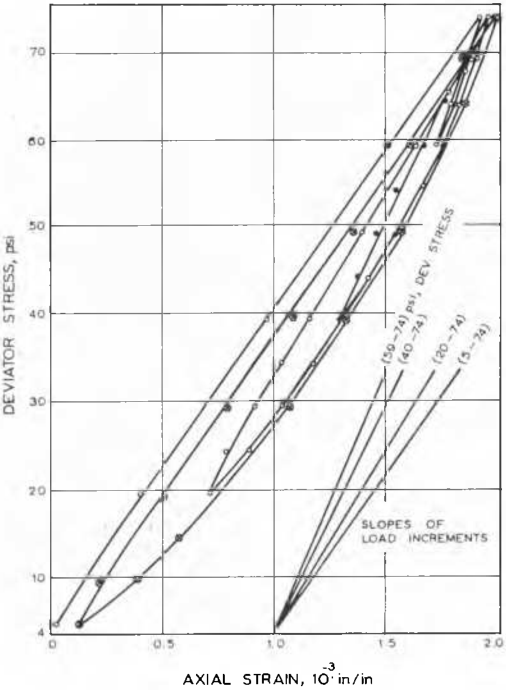

FIG. 5. Influence upon E of the range of deviator stress with a constant upper limit.

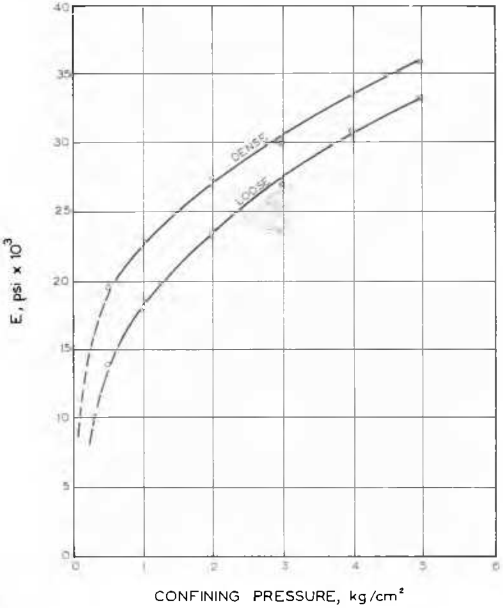

FIG. 6. E *vs.* confining pressure.

The Effect of Confining Pressure upon Modulus of Elasticity

Fig. 6 is the curve of E *vs.* confining pressure for the sand used in this study at two different relative densities. The upper two curves represent the values of the modulus of elasticity from samples subjected to different confining pressures; each sample was subjected to only one confining pressure and was cycled twenty times between zero deviator stress and a load very close to the failure load before the slope of the deviator stress-axial deformation curve was measured.

The effect that the confining pressure, especially the higher values of confining pressure, might have in altering the initial density of the samples was investigated by a study of the volume changes that occurred in the samples during testing. Since no discernible bulging of any of the specimens occurred during the tests, the measured diameter at the mid-height of the specimens was used with the measured changes in length to calculate the changes in the volume of the specimens. These data indicated that although some changes in volume did occur, the densities of the soils did not significantly change.

The curves in Fig. 6 illustrate the following:

1. The modulus of elasticity of sand definitely is a function of confining pressure. Some writers have suggested that E is a function of the maximum principal stress but as data in this study shows, Figs. 1, 4, and 5, the value of E is a constant at a specific confining pressure over a range of deviator stress and hence is constant over a range of the corresponding values of the maximum principal stress.

2. For confining pressures over 1.0 kg/sq.cm., the value of *E* does not vary much between the loosest and densest states; *E* for the densest state of the sand used in this study was approximately 3,000 psi higher than the *E* for the loosest state. This difference became larger for confining pressures less than 1.0 kg/sq.cm. *E* was not determined in this programme for confining pressures less than 0.5 kg/sq.cm. Sands having a greater range in density than the Ottawa sand used in this study might exhibit a greater difference in *E* in their loosest and densest states than shown here.

CONCLUSIONS

1. Sand in the loose or dense state exhibits strain-hardening characteristics over the range of deviator stresses and confining pressures used in this study. The modulus of elasticity of sands subjected to deviator stresses greater than those at which the deviator stress had been cycled appears to be approximately the same as the modulus of elasticity of sands that have not been cycled when subjected to the same deviator stress.

2. The inelastic strains in sand continue in a decreasing manner with continued cycles of loading and are less for higher confining pressures. No attempt was made in this study to cycle until they disappeared completely.

3. The modulus of elasticity of sand under some given confining pressure depends upon the range of stress over which it is determined and the magnitude of the deviator stress at either the lower or upper limits of the range of stress: (*a*) for different ranges of deviator stress that have the same minimum deviator stress, *E* will be for all practical purposes constant after the inelastic deformations have been eliminated; (*b*) for different ranges of deviator stress that have the same upper limit, *E* will be less for the larger ranges of stress and will increase in value as the range of deviator stress decreases; (*c*) for identical ranges of deviator stress, *E* will increase as the magnitude of the lower limit of the range of stress increases.

4. The modulus of elasticity increases with an increase in confining pressure. At confining pressures above 1.0 kg/sq.cm., the difference in *E* between the loosest and densest states of the sand tested in this study was approximately 3,000 psi.

5. Although the range in density possible with the uniform sand used in this study was not large, the results of this study indicate that the influence of the history of loading, the range of deviator stress, and the magnitude of either the upper or lower limits of the range of stress upon the magnitude of the modulus of elasticity of sand is more significant than its relative density.

REFERENCES

CHEN, L. S. (1944). Stress-deformation and strength characteristics of cohesionless soils. Doctoral thesis, Harvard University.

MINDLIN, R. D. (1949). Compliance of elastic bodies in contact. *Jour. Applied Mechanics, Trans. ASME,* Vol. 71, pp. 259–68.

MINDLIN, R. D., and DERESIEWICZ, H. (1953). Elastic spheres in contact under varying oblique forces, *Jour. Applied Mechanics, Trans. ASME,* Vol. 75, pp. 327–44.