A New Experimental Method for the Determination of Hvorslev Strength Parameters for Sensitive Clays

Une Nouvelle Méthode expérimentale de determination des paramètres de Hvorslev pour la résistance au cisaillement des argiles sensibles

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SUMMARY

A new experimental procedure is proposed for the determination of the Hvorslev strength parameters, c_e and ϕ_e, for sensitive clays. The procedure involves the use of undrained triaxial tests with pore-water-pressure measurements performed on pairs of samples initially consolidated anisotropically. In each pair, one sample is sheared undrained. On the second specimen, the anisotropic stress condition is released without change in water content and the sample is then sheared undrained. From a comparison between the effective stresses of the two samples at failure, the parameters, c_e and ϕ_e, may be determined. Test data are presented to illustrate the procedure.

SOMMAIRE

Un nouveau procédé expérimental pour la détermination des paramètres de Hvorslev, c_e et ϕ_e, pour des argiles sensibles est proposé. Le procédé comporte des essais au triaxial non drainés, avec mesure de la pression interstitielle, sur des paires d'échantillons initiallement consolidés anisotropiquement. Dans chaque paire, un échantillon est soumis, sans drainage, au cisaillement. L'état anisotrope de contraintes du second échantillon est relaché sans aucun changement dans la teneur en eau et l'échantillon est alors soumis, sans drainage, au cisaillement. En comparant les contraintes effectives, à la rupture, des deux échantillons, les paramètres, c_e et ϕ_e, peuvent être déterminés. Afin d'illustrer le procédé, des données experimentales sont presentées.

SHEAR STRENGTH PARAMETERS, c_e AND ϕ_e

THE STRENGTH OF A CLAY is often expressed by two components: a physical component attributed to frictional resistance and interlocking, and a physicochemical component, which is referred to as cohesion. Experimental studies by Hvorslev (1937 and 1960) and other investigators have shown that the cohesive component of shear strength is a function primarily of the void ratio in the plane of failure at time of failure, while the frictional component depends on the normal effective stress on the plane of failure at failure. Therefore, at a given void ratio,

$$\tau_{ff} = c_e + \sigma'_{ff} \tan \phi_e. \tag{1}$$

In this equation, τ_{ff} and σ'_{ff} are shear stress and normal effective stress on the plane of failure at failure, respectively; c_e and ϕ_e are the Hvorslev's strength parameters.

EXPERIMENTAL DETERMINATION OF c_e AND ϕ_e

Several experimental procedures for the measurement of Hvorslev cohesion and angle of friction are based on the establishment of a Mohr failure envelope for a series of samples of the same soil at identical water contents, but with different effective stresses at failure. In a method proposed by Terzaghi (1938), c_e and ϕ_e may be deduced from the results of a series of drained direct shear tests on normally and overconsolidated samples (Fig. 1). Points A and B on the shear-strength diagram represent the shear stress at failure for two samples with identical water contents, w_1, and fix the position of the "true" envelope of failure for this particular water content. Skempton and Bishop (1954) extended Terzaghi's method for the determination of the "true" strength parameters by means of triaxial compression tests (Fig. 2). If undrained tests are performed on two samples, A and B, and the pore water pressures measured, the Mohr circles for the effective stresses at failure can be drawn. Since the water content, w_1, for the two samples is identical, the difference in their strength is due to unequal normal effective stresses at failure on the planes of failure of the two samples. Thus, the envelope to the two circles will determine ϕ_e and c_e for the water content, w_1.

In another method proposed by Bishop and Henkel (1957), a series of isotropically consolidated undrained tests with pore water pressure measurements is conducted on samples using different confining pressures. For a series of tests of this type, it may be readily shown that the strength of the specimens is expressed by the relationship

$$\frac{(\sigma'_1 - \sigma'_3)_f}{2p_e} = \frac{c_e}{p_e}\left(\frac{\cos \phi_e}{1 - \sin \phi_e}\right) + \frac{\sigma'_3}{p_e}\left(\frac{\sin \phi_e}{1 - \sin \phi_e}\right) \tag{2}$$

where σ'_1 and σ'_3 are the major and minor principal effective stresses at failure; p_e is equivalent consolidation pressure corresponding to the water content of the specimen; and, c_e and ϕ_e are the Hvorslev strength parameters. Since the ratio c_e/p_e, has been found to be essentially constant (Hvorslev, 1937), the ratio $(\sigma'_1 - \sigma'_3)_f/2p_e$ may be plotted against σ'_3/p_e. The slope of the resulting line is sin ϕ_e from which ϕ_e may be obtained; the intercept, d, of the line equals $c_e \cos \phi_e/p_e(1 - \sin \phi_e)$ from which c_e/p_e may be calculated.

Assuming that Hvorslev's failure criterion is valid at any axial strain, ϵ, Schmertmann and Osterberg (1960) presented a new testing procedure (CFS test) for the evaluation of c_ϵ and ϕ_ϵ at different strains and at an essentially constant void ratio. Although the definition of c_ϵ does not assume constant cohesion at constant void ratio (Schmertmann,

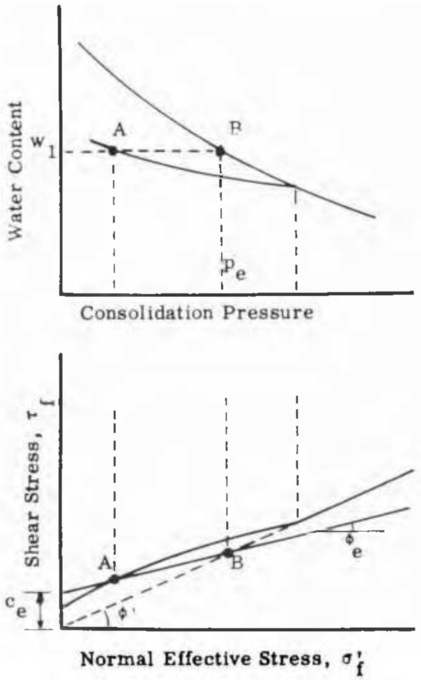

FIG. 1. Determination of c_e and ϕ_e by drained direct shear tests.

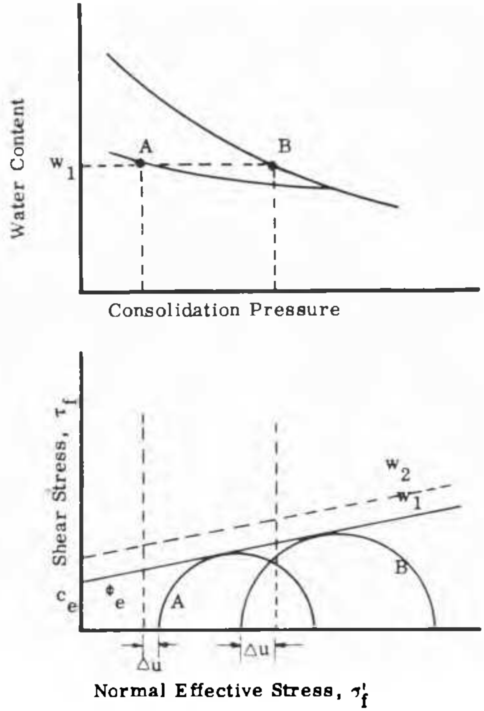

FIG. 2. Determination of c_e and ϕ_e by consolidated-undrained triaxial shear tests.

1961), at large strains, $c_\epsilon = c_e$ and $\phi_\epsilon = \phi_e$ (Schmertmann, 1963).

A practical difficulty in most of these procedures for the measurement of the Hvorslev parameters lies in ascertaining that the normally consolidated and the rebound samples have the same water content at failure. It can also be argued that there must be a considerable difference in structure between the normally consolidated and overconsolidated samples. The method proposed here minimizes these difficulties and is simple in that it does not require preparation of overconsolidated samples.

THEORETICAL BASIS FOR THE NEW METHOD

Consider a sample of saturated clay anisotropically consolidated in a triaxial cell. Throughout consolidation the stresses are applied in small increments and $\sigma_{3c}/\sigma_{1c} = K$ is maintained constant. A back pressure of u_0 is used to assure complete saturation. Fig. 3a illustrates the total, pore water and effective stresses at the end of consolidation. At this stage, the drainage valve is closed, and the stresses σ_1 and σ_3 are simultaneously removed. Consequently, the soil tends to expand, thereby causing a decrease in pore water stress of Δu. The stress condition after the removal of σ_1 and σ_3 is shown in Fig. 3b. Since the total stress is zero, and the pore water stress, $u_0 - \Delta u$, acts with equal magnitude in all directions, it follows that the effective stress acting on the sample is an isotropic stress system, herein called σ'_0, and that,

$$\sigma'_0 = \Delta u - u_0. \quad (3)$$

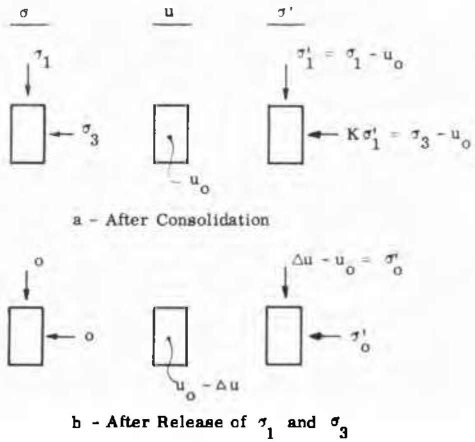

FIG. 3. Effect of removal of anisotropic stresses.

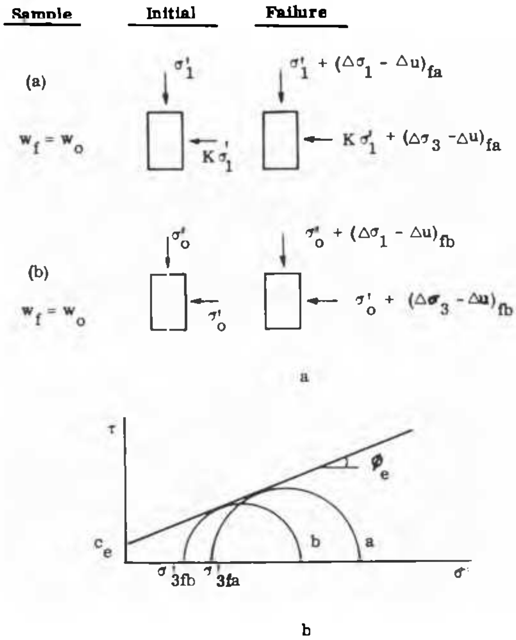

FIG. 4. Initial and failure effective stresses for samples a and b.

In order to evaluate the magnitude of the effective stress, σ'_0, the change in pore pressure, Δu, will first be calculated,

$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]. \quad (4)$$

For saturated clays, $B = 1$, and $\bar{A} = BA$ is the pore-pressure coefficient for the release of the deviator stress, herein called $\bar{A}_0$. Thus, $\Delta u = \sigma_3 + \bar{A}_0(\sigma_1 - \sigma_3)$. Substituting for $\sigma_1 = \sigma'_1 + u_0$ and $\sigma_3 = K\sigma'_1 + u_0$, equation (4) yields

$$\Delta u = \sigma'_1[K + \bar{A}_0(1 - K)] + u_0.$$

Substituting this value of Δu in equation (3), the residual isotropic effective stress acting on the sample can be calculated:

$$\sigma'_0 = \sigma'_1[K + \bar{A}_0(1 - K)]. \quad (5)$$

The coefficient $\bar{A}_0$ in this equation, which represents the change in pore water pressure due to decrease of the deviator stress, is in general considerably smaller than the coefficient $\bar{A}$ for a corresponding increase in deviator stress. For example, for normally consolidated samples of a soft silty clay from San Francisco Bay, numerous tests gave an average value of $\bar{A}_0 = 0.20$ (Noorany, 1963; Seed, Noorany, and Smith, 1964). For Boston Blue clay, $\bar{A}_0 = 0.11$, and for a group of clays from Kawasaki, Japan, $\bar{A}_0 \approx 0.17$ (Ladd and Lambe, 1963).

The stress transfer from the anisotropic condition to the isotropic stress, σ'_0, brought about by the release of σ_1 and σ_3, involves no net volume change, and thus, the sample water content remains the same. Therefore, if a pair of identical samples are first consolidated anisotropically, and then, under undrained conditions, σ_1 and σ_3 of one sample are removed, there will exist two samples at identical void ratios, but under different effective stress systems. Consequently, if both samples are loaded to failure, their undrained strengths would be expected to be different. From a comparison between the effective stresses at failure for these two samples, the shear strength parameters, c_e and ϕ_e, may be deduced.

Fig. 4a illustrates the effective stress conditions for a pair of anisotropically consolidated saturated samples, herein called samples a and b. Subsequent to the anisotropic consolidation, under undrained conditions, the stresses, σ_1 and σ_3 of sample b have been removed. Then, both samples have been loaded to failure. Although the failure water content is the same for the two samples, both the initial and the failure effective stress conditions are quite different. Fig. 4b illustrates the Mohr circles for the failure effective stresses shown in Fig. 4a. Since the void ratios of the two samples a and b at failure are equal, the component c_e of the shear strength (Eq 1) will be the same for both samples and the difference between the strengths of these two samples will be due only to unequal normal effective stresses on the planes of failure at failure, σ'_{ff}. Thus the straight line tangent to the two Mohr circles representing the effective stresses at failure for the samples a and b (Fig. 4b), will make an angle, ϕ_e, with the horizontal; the intercept of this line on the shear stress axis will be the cohesion parameter c_e for the two samples.

This concept provides the basis for the proposed experimental method for the determination of the strength parameters, c_e and ϕ_e, using pairs of anisotropically consolidated specimens.

TESTING PROCEDURE

To illustrate the details of the method, the results of a series of tests conducted on undisturbed samples of San Francisco Bay mud* will be explained. Three pairs of identical undisturbed samples, trimmed side by side, were consolidated anisotropically, using $K = \sigma_{3c}/\sigma_{1c} = 0.5$, to minor principal stresses of 0.4, 0.6, and 0.8 kg/sq.cm. A back pressure of 1.0 kg/sq.cm. was used during consolidation to insure complete saturation. Keeping $K = 0.5$ constant throughout the consolidation stage, the lateral and axial stresses were built up in several increments of $\Delta\sigma_3 = 0.10$ kg/sq.cm. and $\Delta\sigma_1 = 0.20$ kg/sq.cm., the minimum time interval between the load increments being 24 hours. This procedure proved successful, and no bulging or bending of the samples was observed.

After the completion of consolidation, the drainage valves were closed and the deviator stress on one sample of each pair was increased to failure. Having closed the drainage valve, the stresses, σ_1 and σ_3 on the second sample of each pair were simultaneously removed and the resulting decrease in pore water pressure was measured. Because of an initial back pressure of 1.0 kg/sq.cm., the tension developed subsequent to the release of σ_1 and σ_3 did not exceed -1.0 kg/sq.cm. The samples were then loaded to failure. The variation in pore water pressure during the undrained shear was measured using Statham pressure transducers installed in the base plates of the triaxial cells.

A typical test result is shown in Fig. 5. The variations of

*The soft silty clay in San Francisco area is a marine deposit composed primarily of illite and chlorite with some montmorillonite, vermiculite, and kaolinite. $w_L = 88$ and $w_P = 43$. The soil has a sensitivity of about 10, and at natural water content (about 90 per cent), has a shear strength of approximately 400 psf.

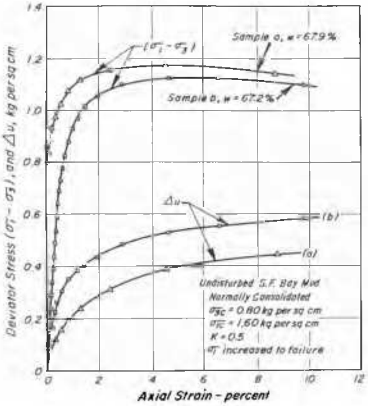

FIG. 5. Variations of the deviator stress and pore water pressure with axial strain for samples a and b.

Reference to Figs. 5 and 6 and Table I will show that the water contents of the two samples tested in each pair were not exactly the same, even though the samples in each pair were trimmed side by side and consolidated at the same time and in the same manner. Since, for an accurate measurement of c_e and ϕ_e, it is essential that both samples

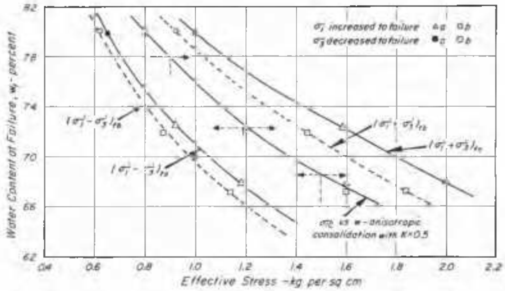

FIG. 7. Effective stress *versus* water content relationships at failure for samples a and b.

TABLE I. SUMMARY OF TRIAXIAL TEST RESULTS

Sample No.	σ_{1c} (kg/sq.cm.)	σ_{3c} (kg/sq.cm.)	w_f (per cent)	σ'_0 (kg/sq.cm.)	$(\sigma'_1-\sigma'_3)_f$ (kg/sq.cm.)	Δu_f (kg/sq.cm.)	σ'_{3f} (kg/sq.cm.)	σ'_{1f} (kg/sq.cm.)	$(\sigma'_1-\sigma'_3)_f$ (kg/sq.cm.)
1–a	1.60	0.80	67.9	—	1.18	0.39	0.41	1.59	2.00
1–b	1.60	0.80	67.2	0.90	1.13	0.55	0.35	1.48	1.83
2–a	1.20	0.60	72.5	—	0.92	0.27	0.33	1.25	1.58
2–b	1.20	0.60	72.0	0.67	0.87	0.38	0.29	1.16	1.45
3–a	0.80	0.40	79.9	—	0.65	−0.05	0.20	0.85	1.05
3–b	0.80	0.40	80.1	0.42	0.62	−0.35	0.15	0.77	0.92

NOTE: In pairs 1 and 2, failure was induced by increasing σ_1; pair 3 was tested with decreasing σ_3. A back pressure of 1.0 kg/sq.cm. was used in all tests.

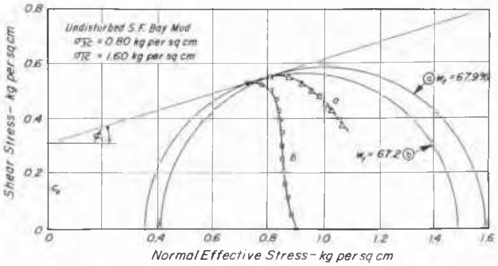

FIG. 6. Mohr circles at failure and vector curves for samples a and b.

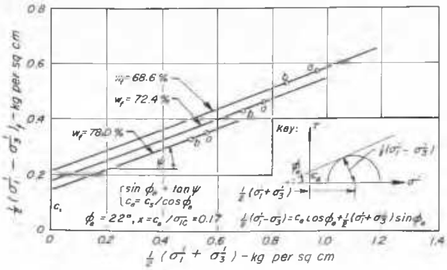

FIG. 8. Relationship between ½$(\sigma'_1-\sigma'_3)_f$ *versus* ½$(\sigma'_1+\sigma'_3)_f$ for samples a and b.

deviator stress and pore water pressure with axial strain have been plotted for two samples, a and b, initially consolidated under $\sigma_{1c} = 1.60$ kg/sq.cm. and $K = 0.5$. Similar data obtained on two other pairs of samples are summarized in Table I. In Fig. 6, the two Mohr circles marked a and b represent the state of effective stresses at failure for the two samples a and b shown in Fig. 5. The envelope to the two circles gives $\phi_e = 18°$, and $c_e = 0.31$ kg/sq.cm. Also plotted in Fig. 6 are the vector curves for samples a and b, showing the variation of the normal effective stress on the plane oriented at an angle $45 + \phi_e/2$ with the horizontal. It is readily apparent that the stress paths for the two samples a and b are quite different.

a and b have an identical water content, the values of c_e and ϕ_e directly deduced from each pair (Fig. 6) will be slightly in error. A more accurate result may readily be obtained by compilation of the data for all three pairs in the manner shown in Fig. 7.

The relationship between the major principal stress, σ_{1c}, and the water content at the end of consolidation is first plotted.* Also shown in this figure are values of $(\sigma'_1-\sigma'_3)_f$ and $(\sigma'_1+\sigma'_3)_f$ for samples a and b at different water con-

*The choice of σ_{1c} for this plot was for the purpose of presenting a clear figure. No eminent influence of σ_{1c} in controlling the water content is implied.

tents. From these curves, the quantities $(\sigma'_1 - \sigma'_3)_{fa}$, $(\sigma'_1 + \sigma'_3)_{fa}$, $(\sigma'_1 - \sigma'_3)_{fb}$, and $(\sigma'_1 + \sigma'_3)_{fb}$ (for samples *a* and *b*) may be determined at constant water contents. The values selected at water contents of 78 per cent, 72.4 per cent, and 68.6 per cent (corresponding to $\sigma_{1c} = 0.90$, 1.20, and 1.50 kg/sq.cm.) have been plotted in Fig. 8. From this plot, the angle of internal friction, ϕ_e, and the cohesion, c_e, for each water content may be determined. From the data in Fig. 8, $\phi_e = 22°$ and appears to be constant. The parameter, c_e, increases with decreasing water content, but the value of $x = c_e/\sigma_{1c} = 0.17$ (or $X = c_e/⅓(\sigma_{1c} + 2\sigma_{3c}) = 0.25$) remains constant.

CONCLUSIONS

The proposed experimental method for the determination of the Hvorslev shear-strength parameters, c_e and ϕ_e, in clays requires that several (preferably three or more) pairs of identical specimens be consolidated anisotropically at different confining pressures. In each pair, one specimen is sheared undrained and the pore water pressure is measured. On the second specimen in each pair, the anisotropic stress condition is released without change in water content and the sample is then subjected to unconsolidated-undrained shear with pore water pressure measurements.

The test results are first plotted to show one-half of the sum and the difference of the effective major and minor principal stresses at failure as a function of the water contents of the sample as shown in Fig. 7. From this plot values of these terms may be read off at the different values of water content and plotted as shown in Fig. 8, providing two points for each value of water content. By drawing straight lines through these pairs of points, the cohesion, c_e, and the angle of friction, ϕ_e, may be determined as shown in Fig. 8.

A disadvantage of this method is that the two Mohr circles representing the effective stress conditions at failure for samples a and b (on the two points in $½(\sigma'_1 - \sigma'_3)$ *versus* $½(\sigma'_1 + \sigma'_3)$ plot), may be very close to each other for insensitive clays. However for sensitive soils they are sufficiently far apart to permit the desired interpretation of the test data and the method has the advantage that test data for samples at the same water content are readily obtained without the necessity of performing tests in overconsolidated samples.

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