Effect of Measuring System on Pore Water Pressures in the Consolidation Test

L'Influence de la méthode de mesure sur les pressions interstitielles durant un essai de consolidation

W. H. PERLOFF, *Assistant Professor of Civil Engineering, Ohio State University, Columbus, Ohio, U.S.A.*
K. NAIR, *Assistant Professor of Civil Engineering, Indian Institute of Technology, Kanpur, India*
J. G. SMITH, *Graduate Student, Department of Civil Engineering, Ohio State University, Columbus, Ohio, U.S.A.*

SUMMARY

An analytical solution describing the effect of flexibility of the pore-water-pressure measuring system on the distribution of pore water pressure in a specimen during a consolidation test is presented. A method for fitting experimental and theoretical pore-pressure curves is suggested.

SOMMAIRE

Une solution analytique est présentée qui décrit l'effet de la flexibilité du système de mesure de la pression de l'eau interstitielle sur la distribution de la pression interstitielle dans un spécimen durant un essai de consolidation. Une methode pour corriger les courbes expérimentales et théoriques des pressions interstitielles est présentée.

IN THE CLASSICAL CONSOLIDATION THEORY it is assumed that upon the application of a stress $\Delta\sigma$ to a saturated soil sample the pore water pressure is instantaneously increased by an amount u_0 equal to $\Delta\sigma$ and that at any subsequent time the following equation holds:

$$\Delta\sigma' = \Delta\sigma - u \tag{1}$$

where $\Delta\sigma'$ is the effective stress, and u is the pore water pressure. Bishop and Eldin (1950), Lambe and Whitman (1959), and Skempton (1961) have shown that Equation 1 is not strictly correct, but that when considering volume change of "non-expansive" cohesive soils at "moderate" pressures the equation will be essentially valid. However, pore water pressures measured at the base of specimens during one-dimensional consolidation tests do not appear to verify the Terzaghi hypothesis. Generally, the measured pore water pressure achieves its maximum value after some finite time. Furthermore, this maximum value is usually less than $\Delta\sigma$. These effects have been shown to be related to the relationship between volumetric compliance of the pore-water-pressure measuring system and the volume compressibility of the soil skeleton (Whitman, *et al.*, 1961; Gibson, 1963). Whitman, *et al.* (1961) also presented an approximate solution to the equations governing the effect of volumetric flexibility on the measurement of pore pressure at the base during a single-drained consolidation test.

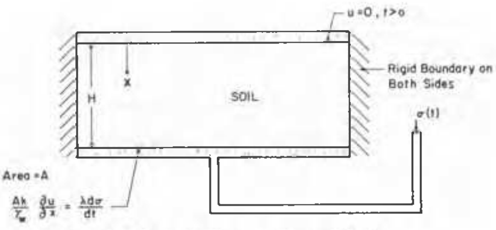

FIG. 1. Schematic diagram of consolidation test.

In this paper a closed form analytic solution is presented to the above problem, thus permitting a detailed examination of the influence of the system on the distribution of pore pressures in the sample during a consolidation test.

THEORETICAL ANALYSIS OF PORE PRESSURES DURING CONSOLIDATION

Fig. 1 is a schematic diagram of a "single-drained" consolidation test with provision for measurement of pore water pressure at the base. The pore water pressure in the soil sample is governed by the differential equation

$$c_v \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}, \text{ for } 0 \leqslant x \leqslant H, t > 0 \tag{2}$$

where c_v is the coefficient of consolidation, H is the sample thickness, and t is time. The condition of drainage at the surface implies

$$\text{at} \quad x = 0, u = 0 \text{ when } t > 0. \tag{3}$$

Because of the compliance of the pore-water-pressure measuring system, some water must flow into the system before it will observe a pressure change. This quantity of flow per unit time, q, is

$$q = \lambda \, d\sigma/dt \tag{4}$$

where λ is the volumetric compliance of the measuring system, i.e., the system volume change per unit pressure change, $d\sigma/dt$ is the rate of pressure change in the system. The flow out of the soil specimen at the base (utilizing Darcy's law) will be

$$q = \frac{Ak}{\gamma_w}\left(\frac{\partial u}{\partial x}\right)_{x=H} \tag{5}$$

where A is the area of the sample, k is the coefficient of permeability, and γ_w is the unit weight of water. Since the flow out the base of the soil must equal the flow into the measuring system, the boundary condition at the base becomes

$$\frac{Ak}{\gamma_w}\frac{\partial u}{\partial x} = \lambda \frac{d\sigma}{dt} \text{ for } x = H. \tag{6}$$

Equation 6 is determined in the same manner presented by Gibson (1963).

The pore-water-pressure distribution throughout the consolidation test specimen as a function of time is then given by

$$u = 2u_0 \sum_{n=1}^{\infty} \frac{(A_n + \eta^2/A_n)\sin A_n - \eta}{(A_n^2 + \eta^2 + \eta)\sin A_n} \sin\left(\frac{A_n x}{H}\right) e^{-A_n^2 T} \tag{7}$$

where u_0 is the initial uniform pore water pressure for $0 < x < H$

$\eta = AHm_v/\lambda$, the stiffness of the measuring system relative to that of the soil skeleton. This quantity was used by Gibson (1963) and is the inverse of "B" used by Whitman, *et al.* (1961).

$m_v = k/(c_v\gamma_w)$, the coefficient of volume compressibility of the soil skeleton

A_n, positive roots of the equation $A_n \tan A_n = \eta$

e, base of natural logarithms

$T = c_v t/H^2$, dimensionless time factor.

Equation 7 was determined by analogy from superposition of results given by Carslaw and Jaeger (1959) for problems of heat conduction through a flat plate in contact with a "well-stirred" fluid. As η becomes very large, Equation 7 must approach the Terzaghi equation. This may be readily verified. The measured pore water pressure at the base at any time will be

$$u_m = 2u_0 \sum_{n=1}^{\infty} \frac{(A_n + \eta^2/A_n)\sin A_n - \eta}{A_n^2 + \eta^2 + \eta} e^{-A_n^2 T}. \tag{8}$$

DISCUSSION

An approximate solution to the above problem for the measured pore pressure at the base was presented by Whitman, *et al.* (1961). They solved the differential equation by means of an electrical analogy. Their results were presented in the form of graphs showing the relationship between the ratio of the measured pore pressure to the applied load against the average degree of consolidation. Since a closed form solution was not presented, it is presumed that the average degree of consolidation in their figure is that obtained from the Terzaghi consolidation theory, and is therefore not really applicable to the problem considered.

The availability of a solution in closed form (Eq 7) permits a detailed examination of the influence of the pressure measuring system on the pore water pressures throughout the entire specimen. Figs. 2 through 5 show the isochrones of pore water pressure for a specimen drained at the top surface as determined from Equation 7 for various values of the stiffness factor, η. The dashed lines shown for comparison are the Terzaghi solution for a specimen undrained at the base ($\eta = \infty$). From physical considerations, and also from an examination of the figures, it can be seen that upon application of a load the pore pressure instantaneously goes to zero at both top and bottom of the sample, because the measuring system has a compressibility

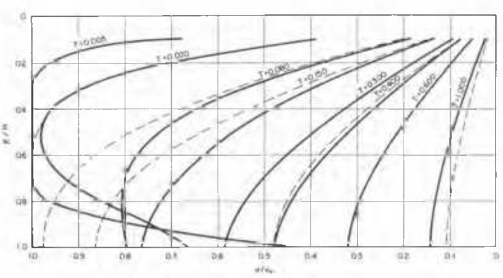

FIG. 2. Isochrones of pore water pressure in a consolidation specimen for $\eta = 10$.

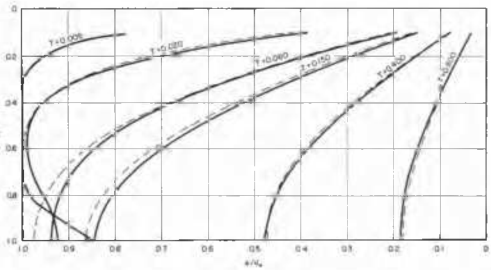

FIG. 3. Isochrones of pore water pressure in a consolidation specimen for $\eta = 50$.

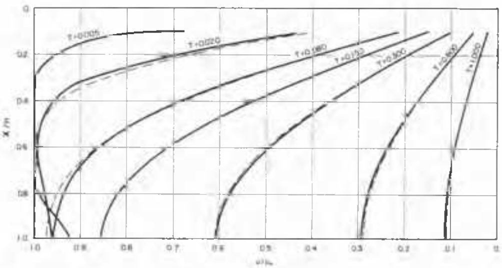

FIG. 4. Isochrones of pore water pressure in a consolidation specimen for $\eta = 100$.

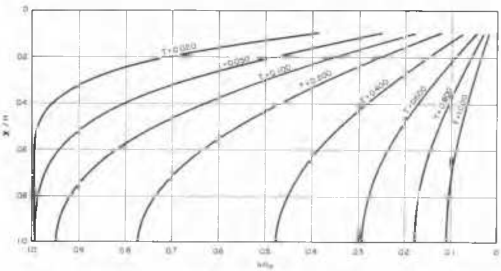

FIG. 5. Isochrones of pore water pressure in a consolidation specimen for $\eta = 1000$.

greater than zero causing water to flow across the face $x = H$. Hence, the soil at the base is immediately consolidated. After this initial compression, the pore water pressure at the base gradually increases to a peak value and then dissipates. A portion of the sample near the base swells during the time that the pore water pressure is increasing. Gibson (1963) reached similar conclusions on the basis of his solution for the problem of a spherical piezometer inserted into an infinite mass of soil.

It is possible to estimate the portion of the sample that swells from an examination of Figs. 2 through 5. It can be seen that at certain values of T the isochrones have a point of inflection. Thus there is more water flowing into than out of the portion of the sample below this point. From Fig. 2 ($\eta = 10$) it is seen that swelling begins at a particular time after the load application. Note that for $T = 0.005$ there is no inflection point, whereas for $T = 0.02$, there is an inflection point at approximately $x/H = 0.85$.

Flow across the face $x = H$ is evident from the fact that the pore pressure isochrones are not perpendicular to the surface $x = H$. It was assumed in the derivation that the coefficient of consolidation (c_v in Eq 2) is constant. Since swelling occurs as a consequence of the imposed boundary conditions it is necessary for the coefficient of consolidation to be the same in compression, rebound, and recompression for the solution presented in Equation 7 to be strictly valid. The above assumption is more nearly satisfied for overconsolidated than normally consolidated clays. It does, however, lead to a conservative estimate of the

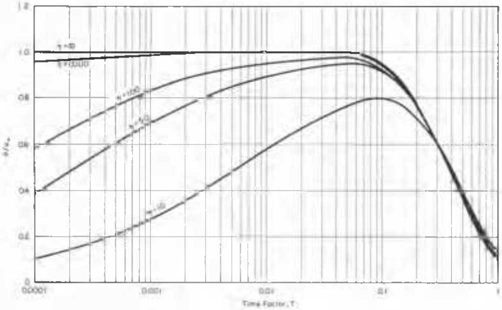

FIG. 6. Pore water pressure at the base as a function of time factor for various values of η.

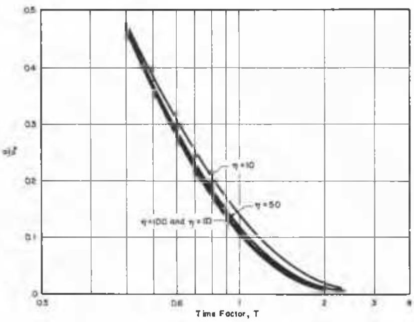

FIG. 7. Pore water pressure at the base as a function of time factor for various values of η.

time lag in normally consolidated clays. Whitman, *et al.* (1961) and Gibson (1963) reached similar conclusions.

Figs. 6 and 7 indicate the effect of system flexibility on the measured pore pressure at the base as a function of the time factor. The time factor in these figures (and in Eqs 7 and 8) is expressed in terms of the specimen thickness, *not* the distance from a drainage surface to the plane for which $\partial u/\partial x = 0$. The values for $\eta = \infty$ are those of the Terzaghi solution for no drainage at the base. At small time factors, as shown in Fig. 6, the measured pore water pressures are smaller than those predicted by the single drained case. At large time factors, as shown in Fig. 7, the measured values should be larger than those predicted by the Terzaghi solution. This result is not predicted by Whitman, *et al.* (1961), but follows directly from substitution into Equation 8. For time factors of approximately 0.4, the curves merge and cross. The influence of η on the precision of pore water pressure measurement can be seen in Fig. 8. This figure

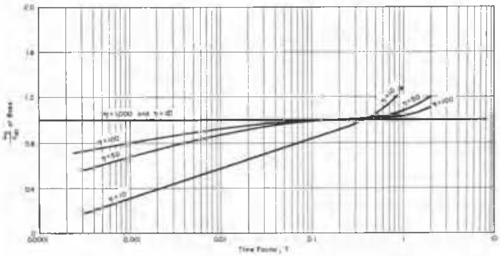

FIG. 8. Ratio of pore pressure at the base for various values of η to pore pressure at the base for $\eta = \infty$ as a function of time factor.

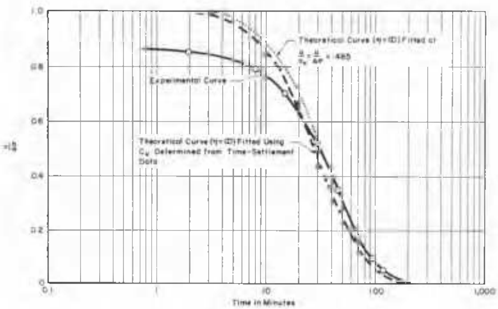

FIG. 9. Comparison of fitting methods (data from Girault, 1960).

shows the ratio of the predicted measured pore water pressure to the pore water pressure for $\eta = \infty$ (truly single drained). Although the pressure difference may be small at large time factors, it should be noted that the ratio u/u_∞ may be substantially larger than one. This may be of particular concern when very sensitive systems, with small values of η, are used to measure long-term pore water pressures.

Equation 8 also provides insight into another problem arising from attempts to measure pore water pressures at the base of a consolidating specimen. In order to compare experimental and theoretical pore-water-pressure curves it is necessary to fit the curves together at some point. This is done to obtain a relationship between the actual time, t, and the time factor, T. From Fig. 8 it is possible to determine the point at which the curves should be fitted in order to eliminate the effect of system flexibility from the determination of c_v with pore-water-pressure data. For values of η between 10 and 100 the measured pressure curves merge with the Terzaghi solution between $T = 0.34$ and $T = 0.39$, which, from Fig. 6, correspond to u/u_0 of 55 per cent and 48.5 per cent respectively. Note, however, that all curves coincide at $T = 0.39$ ($u/u_0 = 48.5$ per cent). Therefore, if the theoretical and experimental pore-water-pressure curves are fitted at this point, the determination of c_v will not be influenced by system flexibility. This is illustrated in Fig. 9, which shows the results of pore-water-pressure measurements during one pressure increment at the base of a single drained consolidation specimen of a brown Crosby B silty clay. These data, and a detailed description of the experimental procedure, are given by Girault (1960). Two theoretical curves ($\eta = \infty$) are shown, one determined by using c_v from the time-settlement data, the other determined by fitting the pore water pressure data at $u/\Delta\sigma = 0.485$. Clearly the values of c_v determined by these two methods are different. Although, for these data, one might intuitively fit theoretical and experimental results at u/u_0 approximately 0.5, Equation 8 (see Fig. 8) provides theoretical justification for doing so.

CONCLUSIONS

Examination of an analytical solution for pore water pressures set up in a specimen during a single drained consolidation test with pore-water-pressure measurement at the base, leads to the following conclusions:

1. The pore water pressure at the base goes to zero instantaneously upon application of the load, and then increases to a peak value before dissipating.
2. Approximately the lower 15 per cent of the specimen is subject to swelling during a part of the time the pore pressure at the base is building up to its maximum.
3. The measured pore pressure at the base is smaller than the Terzaghi theoretical values ($\eta = \infty$) during the initial stages of the test and greater during the later stages.
4. It is predicted that measured pore water pressures should coincide with the Terzaghi theoretical ($\eta = \infty$) values at $T = 0.39$, for all values of η considered. This suggests a means for eliminating the effect of measuring system flexibility when comparing theoretical and experimental results.

REFERENCES

BISHOP, A. W., and G. ELDIN (1950). Undrained triaxial tests on saturated sands and their significance in the general theory of shear strength. *Géotechnique*, Vol. 2, pp. 13–32.

CARSLAW, H. S., and J. C. JAEGER (1959). *Conduction of heat in solids.* 2nd ed. Oxford, Clarendon Press.

GIBSON, R. E. (1963). An analysis of system flexibility and its effect on time lag in pore-water pressure measurements. *Géotechnique*, Vol. 8, pp. 1–9.

GIRAULT, P. (1960). A study on the consolidation of Mexico City clay. A thesis submitted to the faculty of Purdue University, in partial fulfilment of the requirements for the degree of doctor of philosophy.

LAMBE, T. W., and R. V. WHITMAN (1959). The role of effective stress in the behavior of expansive soil. *Quart. Colorado School of Mines*, Vol. 54, p. 34.

SKEMPTON, A. W. (1961). Effective stress in soils, concrete and rocks. In *Pore Pressure and Suction in Soils*, pp. 4–16. London, Butterworth.

WHITMAN, R. V., A. M. RICHARDSON, and K. A. HEALY (1961). Time lags in pore pressure measurements. *Proc. Fifth International Conference on Soil Mechanics and Foundation Engineering* (Paris), Vol. 1, pp. 407–11.