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The Measured Horizontal Coefficient of Consolidation of Laminated, Layered or Varved Clays

Mesure du coefficient de consolidation horizontale d'argiles feuilletées, stratifiées ou varvées

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SUMMARY

The rate of drainage of a clay containing artificial layers of higher permeability has been studied by means of a series of radial drainage oedometers of new design. The measured horizontal coefficients of consolidation were found to vary with the ratio of the thickness of the clay layers to the diameter of the model in the manner predicted by theory.

SOMMAIRE

On a étudié la vitesse de drainage d'une argile qui contient des couches artificielles d'une perméabilité plus grande au moyen de nouveaux oedomètres à drainage radial. Après avoir mesuré les coefficients de consolidation horizontaux on a découvert que l'épaisseur des couches d'argiles divisée par le diamètre du modèle variaient de manière prédite par la théorie.

THE RATE OF CONSOLIDATION of foundation clays under large embankments where vertical sand drains have been installed is sometimes much faster than the predicted rate based on conventional laboratory tests. If the clay shows fine laminations when cut and split, is occasionally layered with silt, or is varved, consolidation flow can occur in the field by a process of radial flow in each layer combined with vertical flow from the less permeable to the more permeable layers. In this event the over-all coefficient of consolidation in the horizontal direction, $\bar{c}_h$, which results from fitting the average measured degree of consolidation to the radial flow solution for an isotropic soil, will exceed the horizontal coefficient c_{hc} for the clay layers alone. The ratio of these coefficients has been shown experimentally to increase with length of drainage or size of sample for a finely laminated clay (Rowe, 1959).

Recently Horne (1964) and Rowe (1964) have obtained a theoretical solution for the average consolidation of alternate layers of unlike compressibility, permeability and thickness. The ratio of the above coefficients of consolidation for the case of radial flow from a boundary diameter D to a well diameter D/n was found to be given by the solution of the following two equations:

$$\frac{\bar{c}_h}{c_{hc}} = \frac{\frac{b}{a}\cdot\frac{F_c}{F_s}+1}{\frac{1}{a}\,\frac{F_c}{F_s}+1} \tag{1}$$

and

$$\frac{H_c}{D}\sqrt{N_c} = \frac{\theta_c}{n\alpha_N}\cdot\sqrt{\frac{\frac{1}{b}+\frac{a}{b}\,\frac{F_s}{F_c}}{1-\frac{1}{b}}}\,. \tag{2}$$

With reference to Fig. 1, a = the ratio $(m_{vc}/m_{vs})(H_c/H_s)$, b = the ratio c_{hs}/c_{hc}, where m_v, c_h, and H are the compressibility due to vertical effective stress increase, the coefficient of consolidation in the horizontal direction, and the layer thickness, respectively. Suffix c refers to the less permeable or clay layer, and suffix s refers to the more permeable or silt layer. The ratio $N_c = c_{hc}/c_{vc}$, the ratio of coefficients for the clay in the horizontal and vertical direction. The factor $F_c = \theta_c/\tan\theta_c$, where θ_c is any chosen number between 0 and $\frac{1}{2}\pi$. The factor $F_s = \theta_s/\tanh\theta_s$, where θ_s satisfies the equation

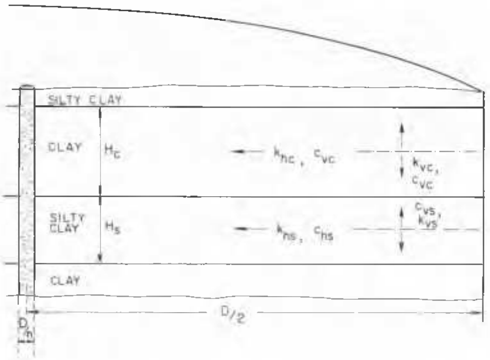

FIG. 1. The geometry and notation for a system of alternating layers.

$$\theta_s^2/F_s = (\theta_c^2/F_c)(H_s/H_c)(k_{vc}/k_{vs}) \tag{3}$$

and k refers to the permeability.

α_N is the first solution of the Bessel function:

$$\begin{aligned} U_1(n\alpha_N) &= 0 \\ &= Y_1(n\alpha_N)J_0(\alpha_N) - J_1(n\alpha_N)Y_0(\alpha_N). \end{aligned}$$

The object of the present paper is to give the results of observations on the rate of radial consolidation of an artificially layered clay in which discs of ceramic representing the more permeable layers were embedded between clay layers. In so far as the observations support the above equations for this special case they are believed to contribute to an understanding of the influence of the stratification and size of the sample on the measured over-all horizontal coefficient of consolidation.

PROPERTIES OF THE CLAY AND CERAMIC

All coefficients of consolidation were measured on remoulded normally consolidated clay subjected to an applied stress increment from 2 to 4 tons/sq.ft. The measured coefficient in the vertical direction, c_{vc}, was 5.1 sq.ft./yr and the associated permeability k_{vc} was 2.5×10^{-9} cm/sec. In the horizontal direction the coefficient c_{hc} was found to depend on the horizontal length of drainage, because the anisotropically consolidated clay showed laminations. The coefficients were measured in a new type of oedometer (Shields and Rowe, 1964) whereby drainage takes place radially to a thin central sand drain of diameter 1/20 by oedometer diameter. By this direct method, tests on the clay alone without the insertion of discs gave the results shown in Table I. The permeability, k_{vs}, of the ceramic was measured directly across its least dimension to be 3×10^{-6} cm/sec. It has been assumed that the value of k_{hs} was the same. The compressibility of the ceramic, m_{vs}, was very small compared to that of the clay, m_{vc}, so that the ratio $1/a \simeq 0$ for the range of layer thickness ratios studied. Likewise, c_{hs} for the ceramic was very large and $b \to \infty$.

TABLE I. MEASURED VALUES OF c_{hc} AND N_c

Diameter of oedometer D (inches)	c_{hc} (sq.ft./yr)	N_c
10	13.8	2.70
6	8.3	1.63
3	7.9	1.55

GEOMETRY OF THE LAYERING AND OF THE MODEL

All the radial drainage tests were made with an oedometer diameter/sand drain diameter ratio $n = 20$. Table II summarizes the sample geometry and mean consolidation times to 90 per cent, based on average settlement, each time being the mean of two tests. Column 5 in Table II is completed using the standard solution for radial flow at t_{90} for $n = 20$, namely $\overline{c_h} = (0.69D^2/t_{90})\text{in}^2/\text{min} = 2520(D^2/t_{90})\text{sq.ft/yr}$ where D is expressed in inches and t_{90} in minutes. Column 6 is completed using Table I. In Column 8 the ratio

$$\frac{b}{a} = \frac{k_{hs}}{k_{hc}}\cdot\frac{H_s}{H_c} = \frac{3 \times 10^{-6}}{2.5 \times 10^{-9}}\cdot\frac{1}{N_c}\cdot\frac{H_s}{H_c} = \frac{1200}{N_c}\cdot\frac{H_s}{H_c}.$$

THE THEORETICAL SOLUTION

For all the cases considered the ratio

$$\frac{H_s k_{vc}}{H_c k_{vs}} \leqslant \frac{0.14}{0.29} \times \frac{2.5 \times 10^{-9}}{3 \times 10^{-6}} = 4 \times 10^{-4}.$$

In this case, using Equation 3, $F_s = 1$. Inserting the special values of a and b into Equations 1 and 2

$$\overline{c_h}/c_{hc} = (b/a)\,F_c + 1 \tag{4}$$

and

$$\frac{H_c}{D}\sqrt{N_c} = \frac{\theta_c}{\sqrt{F_c}\,n\alpha_N}\left(\frac{b}{a}\right)^{-\frac{1}{2}}. \tag{5}$$

TABLE II. DETAILS OF TESTS AND REDUCTION OF RESULTS

Oedometer diameter D (in.)	Clay thickness H_c (in.)	Ceramic thickness H_s (in.)	t_{90} (mins)	$\overline{c_h}$ (sq.ft./yr)	$\overline{c_h}/c_{hc}$	$(H_c/D)\sqrt{N_c}$	b/a
10	8.60	0.60	7350	34	2.5	1.410	31
10	9.46	0.60	8700	29	2.1	1.550	28
10	3.96	0.14	2185	115	8.3	0.650	16
10	1.96	0.30	790	319	23.1	0.320	68
10	1.00	0.30	350	720	52.2	0.160	133
10	0.50	0.14	157	1600	116	0.082	124
10	0.29	0.14	148	1700	123	0.048	214
6	0.55	0.14	82	1105	133	0.117	187
6	1.95	0.28	572	158	19.1	0.415	106
3	2.00	0.26	438	52	6.5	0.830	101
3	1.00	0.16	141	161	20.3	0.415	124
3	0.49	0.16	47	483	61.0	0.204	254

TABLE III. CALCULATION PROCEDURE FOR THEORETICAL RELATION BETWEEN THE RATIOS $(\overline{c_h}/c_{hc})$ AND $(H_c/D)\sqrt{N_c}$

			$(H_c/D)\sqrt{N_c}$			$\overline{c_h}/c_{hc}$		
θ_c	$F_c = \frac{\theta_c}{\tan\theta_c}$	$\frac{\theta_c}{\sqrt{F_c}}$	$b/a = 30$	100	200	$b/a = 30$	100	200
0	1	0	0	0	0	31	101	201
0.2	0.9860	.202	.0396	.0217	.0153	30.6	99.6	198.2
0.4	0.9462	.411	.0806	.0443	.0314	29.4	95.6	190.2
0.6	0.8768	.641	.126	.0690	.0489	27.3	88.7	176.4
0.8	0.7769	.906	.178	.0976	.0691	24.3	78.7	156.4
1.0	0.6419	1.246	.244	.134	.0948	20.2	65.2	129.4
1.2	0.4664	1.760	.344	.189	.134	15.0	47.6	94.2
1.4	0.2413	2.850	.560	.307	.217	8.2	25.1	49.2
1.5	0.1063	4.600	.902	.495	.350	4.2	11.6	22.2
1.54	0.0482	7.02	1.38	.755	.535	2.4	5.8	10.6
1.555	0.0244	9.95	1.95	1.070	.757	1.7	3.4	5.8
1.565	0.0087	16.8	3.3	1.81	1.28	1.26	1.9	2.7

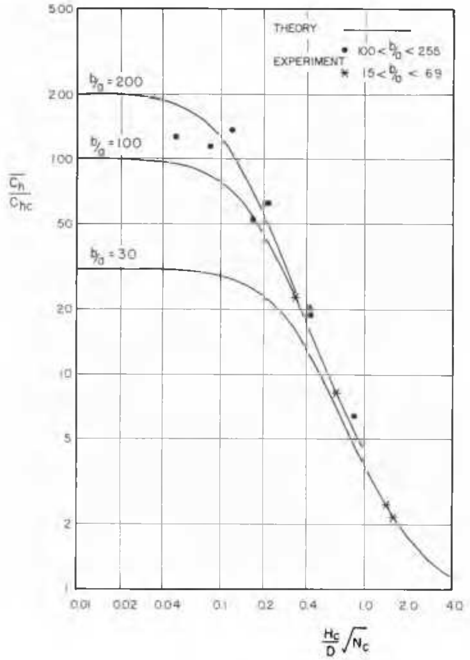

FIG. 2. Comparison between theory and experiment using ceramic discs embedded in normally consolidated clay.

For $n = 20$ the product $n\alpha_x = 0.929$. The solutions do not depend very critically on the b/a value except towards the maximum ratio of coefficients $\overline{c_h}/c_{hc}$, and solutions have been obtained for $b/a = 30$, 100, and 200. The steps in the solution of Equations 4 and 5 are illustrated in Table III.

The theoretical values, Table III, are compared with the experimental values, Table II, in Fig. 2. The thinnest ceramic disc, 0.14 inches, of 10-inch diameter was insufficient to bring the test observations on to the theoretical peak $\overline{c_h}/c_{hc}$ value. A similar comparison of results based on the measured pore water pressure gave equally good agreement for the chosen increment/pressure ratio of unity.

THE COEFFICIENT OF CONSOLIDATION IN THE FIELD

Further tests on alternate clay layers are in progress with the same objective, namely to check the theory against the measured component parameters. Meanwhile it may be noted that Fig. 2 shows, experimentally and theoretically, how the coefficient of consolidation of a layered deposit increases to a maximum with increase in the scale of the sample diameter tested. Although the permeability and thickness of the individual layers of a layered deposit cannot be measured, the sample scale may be varied so as to extrapolate to the maximum at field scale where D is large.

The over-all field value will also depend on the geometry of the stratification of the deposit as a whole. There may be occasional very permeable thin layers at spacings of many feet. Whereas the maximum $\overline{c_h}$ value deduced from tests on small samples may refer to the clay between these wider permeable layers, the over-all geometry and layer properties have to be used in conjunction with the theory to predict the influence of these occasional layers. It is highly desirable therefore that reports of dissipation rates of pore pressure in sand drain installations should include a detailed description of the stratification of the entire deposit as obtained by continuous sampling.

CONCLUSIONS

The predicted rate of consolidation of a layered deposit increases with the scale of the sample under analysis. Detailed observations on a clay deposit artificially layered with thin ceramic discs showed good agreement with the theory. A knowledge of all minute details of stratification over the entire depth of a clay deposit may allow an improved estimate of the rate of field consolidation where sand drains are installed.

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