INTERNATIONAL SOCIETY FOR
SOIL MECHANICS AND
GEOTECHNICAL ENGINEERING

SIMSG ISSMGE

This paper was downloaded from the Online Library of the International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE). The library is available here:

https://www.issmge.org/publications/online-library

This is an open-access database that archives thousands of papers published under the Auspices of the ISSMGE and maintained by the Innovation and Development Committee of ISSMGE.

Shear Strength of Clays and Safety Factors as a Function of Time

La Résistance au cisaillement des argiles et les facteurs de sécurité en fonction du temps

W. E. SCHMID, *Associate Professor of Civil Engineering, Princeton University, Princeton, N.J., U.S.A.*
S. KITAGO, *Director, Soil Mechanics Laboratory, Hokkaido University, Sapporo, Japan*

SUMMARY

The concept of intrinsic stress is introduced and the shear strength of clays is assumed to be a function of this intrinsic stress. Test results show that the resulting shear-strength parameters are independent of the stress history and depend on strain rate or load duration. The practical application of the concept to computing the change of safety factors with time is briefly indicated.

SOMMAIRE

Le principe de tension intrinsèque est introduit et la résistance au cisaillement des argiles est supposée être fonction de cette tension. Les résultats des essais effectués montrent que les paramètres relatifs à la résistance au cisaillement sont indépendants des tensions du passé et dépendent seulement de la vitesse de déformation ou de la durée de la mise sous charge. Des applications pratiques du principe au calcul de la variation des coefficients de sécurité en fonction du temps sont brièvement indiquées.

THE CLASSICAL THEORIES of failure assume that stress conditions alone determine the state of failure of a material irrespective of the load duration and the stress history and independent also of the temperature. The Mohr-Coulomb theory, as a special case, also ignores this influence of time and energy level. Recently, however, these factors have received increased attention because their importance can no longer be overlooked (Murayama and Shibata, 1961; Vialov and Skibitsky, 1961; De Wet, 1961; Perloff and Osterberg, 1962; Mitchell, 1964).

That clay soils have time-dependent shear properties has been recognized for a long time (Vicat, 1833; Terzaghi, 1931), but the profession still frequently approaches problems of time-dependent stress-strain behaviour of clay soils as if they could be analysed completely on the basis of the theory of elasticity or plasticity without regard to the actual rheological properties. Time effects are often discussed, if at all, merely on a qualitative basis. One reason for this may be the complexity resulting when time is considered an additional variable in any problem. Another reason may be the difficulty of abandoning old and familiar concepts that usually are sufficient for most structural materials, and another the lack of familiarity of most soil engineers with rheological theory, since its application to problems of soil mechanics is still developing.

There appears to be general agreement, however, that saturated clay soils do behave like viscoelastic or viscoplastic materials. As a consequence, the classical failure theories (such as the Mohr-Coulomb theory and its effective stress modification) cannot and do not completely describe the material behaviour of clay soils. Either they have to be modified to permit a quantitative assessment of stress history, temperature, and rate of loading or they have to be replaced by theories that include these effects.

While in the classical theories one or two material parameters such as yield stress or Young's modulus and Poisson's ratio are sufficient to describe the behaviour of an isotropic material, a larger number of parameters may be required to express the behaviour of a viscoelastic material, depending on the rheological model chosen. If, in addition, changes in the material itself must be considered (because a clay may change its void ratio or its water content during the test or during the lifetime of a foundation), the problem is a rather complicated one.

THE EFFECTIVE STRENGTH PARAMETERS

On the basis of effective stresses the Mohr-Coulomb theory assumes that the shear strength is given by:

$$\tau_f = c' + (\sigma - u)\tan\phi' \qquad (1)$$

Recent improvements in the techniques for measuring pore pressures in triaxial tests have led to numerous investigations of these effective strength parameters and most shear-strength research on clays revolves about the parameters of Equation 1. We have argued elsewhere that perhaps this is not all to the good (Schmid, *et al.*, 1960; Schmid, 1962). Even with the improved techniques, pore-pressure measurements still require a careful and slow experimentation that is possible in a research project but not very suitable for routine application in construction or foundation jobs. Also, investigators have reported such significant variations of pore pressures within test samples during triaxial tests that the observed values of u have to be taken with a certain reservation (Blight, 1963; Crawford, 1963). However, the most serious shortcoming of the Mohr-Coulomb theory, in our opinion, is its failure to include the effect of stress history or the effect of the variation of void ratio or water content. Two groups of otherwise identical soil samples, when subjected to a different consolidation history, will yield different effective stress parameters and, as stated also by Hvorslev (1960, p. 256), a proper representation of the effective stress Mohr-Coulomb failure criterion consists of a shear-strength line for normally consolidated clays plus a *family of hysteresis curves* for over-consolidated clays.

THE CONCEPT OF INTRINSIC STRESS

We propose to extend the effective stress concept by recognizing that any soil particle skeleton arrangement has a certain number of point contacts with a resulting force exchange from particle to particle at these contact points. This force exchange or transmission is an intrinsic phenomenon and may only be secondarily related to any exterior applied stress. It depends mainly on the number and the area of the particle contacts. The sum total of all the forces taken over a given nominal area gives a stress which we define as the intrinsic stress p_i. It would be very difficult, if not impossible, to measure this intrinsic stress directly. However, we know that in the case of a normally consolidated clay held at the consolidation load the intrinsic stress must be equal to the consolidating pressure.

Conversely, we may assume that in overconsolidated clays that do not recover their original void ratio upon unloading, the intrinsic stress p_i that remains locked into the particle skeleton corresponds to the equivalent consolidation pressure on the virgin compression curve. For example, if a soil is consolidated along a virgin compression line o–a (Fig. 1) and is then unloaded (rebound branch a–b), the

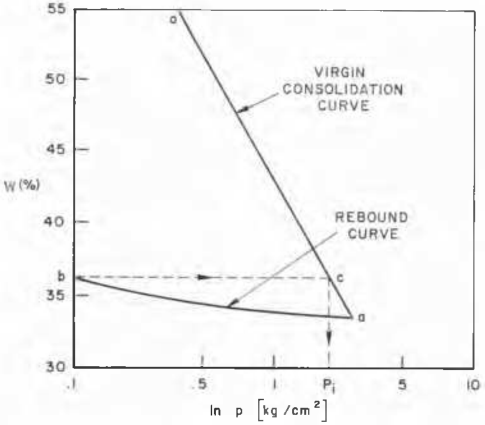

FIG. 1. Water content *vs.* consolidating pressure.

soil will have a water content (or void ratio) corresponding to point b. The equivalent consolidating pressure and, hence, the intrinsic stress at that void ratio would be given by point c on the virgin compression line. Accordingly, for normally consolidated clays the intrinsic stress is equal to the effective stress at zero pore pressure.

Since the most important effect of any stress history is usually a residual void ratio change, the concept of the intrinsic stress automatically takes into account this history.

SHEAR STRENGTH AND ITS TIME DEPENDENCE

To express the shear strength as a function of the intrinsic stress we note that, according to our definition, the intrinsic stress is equivalent to what in ordinary materials is considered the interior cohesion. It is therefore natural to express the shear strength as a function of this intrinsic stress:

$$\tau_f = \tfrac{1}{2}(\sigma_1 - \sigma_3) = p_i\mu^* = p_i \tan\phi^* \qquad (2)$$

where μ^* and ϕ^* would be analogous to a coefficient of friction and a friction angle on an intrinsic stress basis.

Since the virgin compression line expresses the relationship between water content and intrinsic stress, we may write:

$$p_i = a \cdot e^{-D(w-w_0)} \qquad (3)$$

and combined with Equation 2:

$$\tfrac{1}{2}(\sigma_1 - \sigma_3) = \tau_f = \mu^* \cdot a \cdot e^{-D(w-w_0)} = Ae^{-D(w-w_0)} \qquad (4)$$

where e denotes the base of the Napierian logarithm. Thus (4) predicts a linear relationship between the logarithm of the shear strength and the water content. Since void ratio and water content for saturated clays are directly related: $e = w \cdot G_s$, Equation 4 also predicts a linear relation between the logarithm of the shear strength and the void ratio. Experimentally this relationship does indeed exist and has been reported by a number of investigators, among them Rutledge (1947), Casagrande (1950), Calhoon (1954), Leonards (1955), Schmid (1960).

The two parameters of Equation 4 that thus describe the shear strength are the coefficients A and D. Coefficient A is a reference shear strength depending on the reference water content w_0. We suggest to choose the water content at the plastic limit for w_0. Since the remoulded shear strength at the plastic limit of a soil is constant, the resulting value of

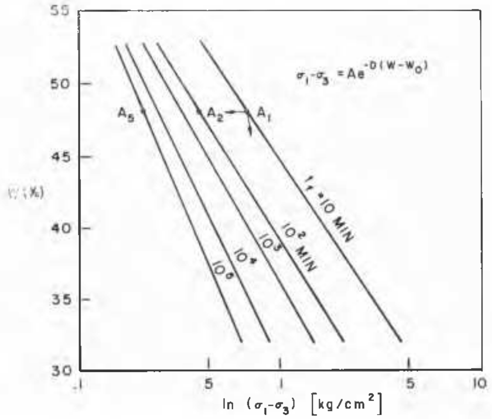

FIG. 2. Water content *vs.* shear strength for various times to failure.

A would be simultaneously a measure of the sensitivity to remoulding. Coefficient D is the slope of the failure line in the diagram showing water content *versus* the logarithm of the shear strength (Fig. 2).

It was pointed out earlier that if the shear strength of a clay soil is time-dependent, the parameters A and D should be functions of the load duration or the strain rate. The slower the strain rate or the longer the load duration, the smaller will be the shearing strength. Thus, the point A should move to the left as the load duration or the time to failure t_f increases (Fig. 2). The failure lines also may show a different slope.

This time dependence of A and D has indeed been found. Figs. 3 and 4 show the results of triaxial tests carried out by the junior author on load-controlled testing stands. Two clays, one predominantly kaolinite (Grantham) and one illite (Grundite), were tested. The test specimens were prepared in the laboratory from a common batch of saturated clay and were consolidated in the triaxial cells under different confining pressures. The specimens were then subjected to shear stresses while drainage was prevented. The stress was applied in uniform load increments that were held constant throughout the load interval. In order to make the total time to failure proportional to the load interval as well as for practical reasons, we chose stress increments that would cause failure after approximately ten load increments. To accomplish this, the ultimate strength was either estimated or found by a pilot test. The shear strength was taken to be the highest stress a specimen could withstand during an entire load interval. We attempted to increase the length of the load intervals always by one order of magnitude. However, for practical reasons, the following load intervals were chosen: 10 seconds, one minute, 10 minutes, 90 minutes. The results are shown in Figs. 3 and 4.

Even though there is some scatter, the data clearly show that parameters A and D are functions of the load interval or of the total time to failure. The scatter is more pronounced in the Grantham clay than in the Grundite mainly because the Grantham clay was tested first and we had less skill and experience in choosing the proper stress increments. Thus, the number of stress increments and load intervals varied widely from a low of 6 to a high of 18. For the Grundite clay, 90 per cent of all test specimens failed after 10 ± 1 load intervals.

Tests on Grundite clay at various constant strain rates by Perloff and Osterberg (1962) were evaluated on the same basis and also show a clear dependence of A on the strain

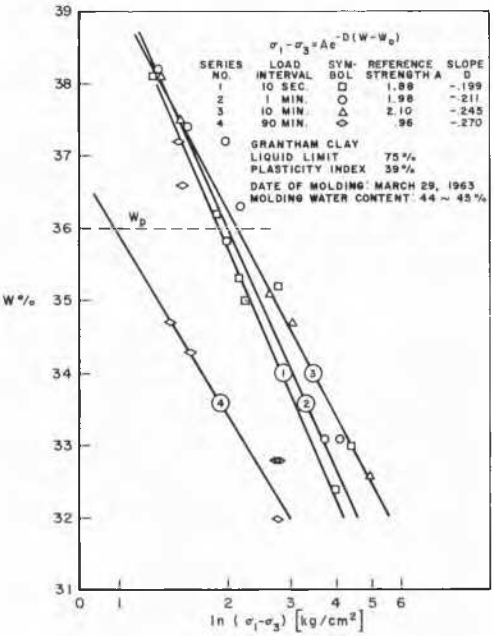

FIG. 3. Water content *vs.* shear strength for different load intervals on Grantham clay.

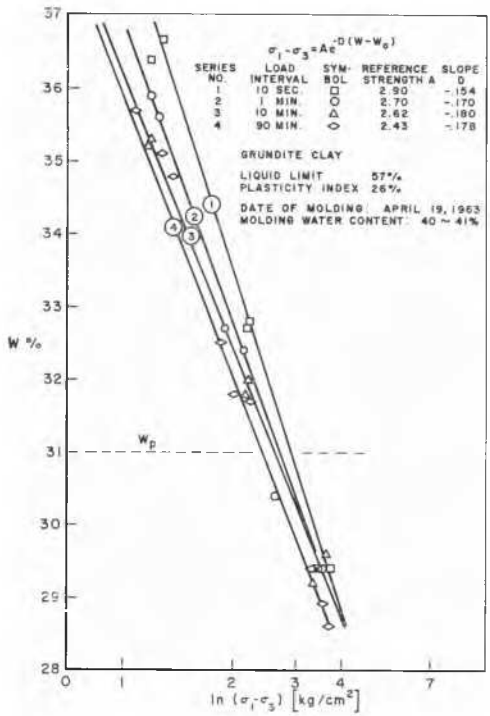

FIG. 4. Water content *vs.* shear strength for different load intervals of Grundite clay.

rate (Fig. 5). The data of Perloff and Osterberg seem to indicate that the value of D changes very little, if at all, and that perhaps the only time-dependent parameter is the value A. This may be so for certain soils. It certainly would make the problem much simpler. This will have to be investigated further, however. At any rate, the time dependence of the shear-strength parameters appears to be established and there are many interesting possible relationships that should be explored by further research. The important conclusions, however, that may be drawn from our tests are the following.

1. At any water content there exists a level of shear stress that can be safely sustained without excessive deformations or failure within a reasonable time span.

2. Beyond this safe level, however, there is not one unique shear strength but a range of values that represent shear strengths as a function of the time the stress must be carried. The longer a stress must be sustained, the lower it must be or, conversely, the higher the load, the sooner failure will occur. This time dependence, incidentally, is not unique to clays but has also been observed in other materials such as plastics, timber, and metals at elevated temperatures.

3. The load duration effect is normally not considered in conventional soil testing but further development and application of this method can lead to the quantitative analysis of the creep of slopes and of problems where failure occurs after the end of construction. So far a method for the quantitative analysis of such problems does not exist.

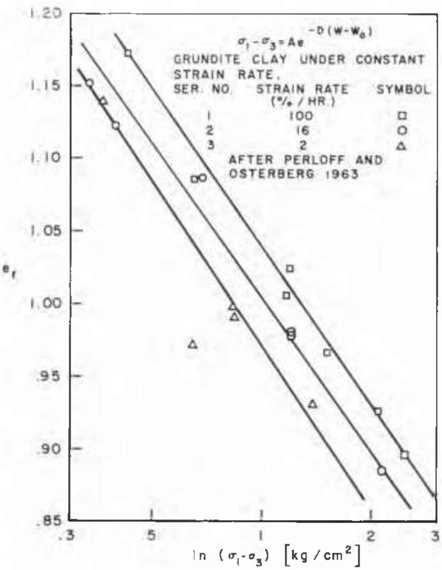

FIG. 5. Water content *vs.* shear strength for different strain rates on Grundite clay (after Perloff and Osterberg).

PRACTICAL APPLICATION

Frequently foundation failures occur after the end of construction and after the full load has been on a soil for some time.

According to the effective stress concept, the shear strength should be lowest immediately after the load application because then the pore pressure, u, is a maximum. As time proceeds, the pore pressure dissipates and the shearing strength increases. If these considerations reflect the actual situation, any failure should occur at the end of construction as soon as the full load is applied. Quite often, however, failure occurs a considerable time *after* the full load is applied. Thus, at least in some cases, there must be phenomena at work that cause the safety factor to *decrease* with time.

We can now compute the safety factor at any time. Consider for example the foundation problem shown in Fig. 6 where a trapezoidal load is applied at the surface of a clay layer. The instantaneous safety factor for the critical failure surface can be computed by conventional methods.

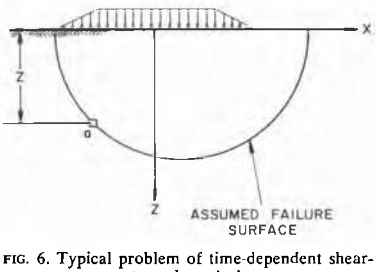

FIG. 6. Typical problem of time-dependent shear-strength analysis.

However, at any time $t > 0$ this safety factor may have changed. If we use the Mohr-Coulomb effective stress method, it can only have increased since for $t > 0$ pore pressures must be less and thus the shear strength must be higher. If, however, we use the method presented above, assuming shear-strength properties as they are shown in Fig. 2, the following reasoning is possible.

Any soil element along the assumed failure surface is subject to two influences; on the one hand the consolidation (or decay of pore water pressure) due to the normal stresses and, on the other hand, the decrease in shearing strength due to the load duration. Consolidation causes a decrease in the water content and thus an increase in strength. But consolidation requires time which, in turn, causes a decrease in strength. The question is whether the strength increase due to consolidation is balanced or overcompensated by the strength decrease due to the load duration.

If, in Fig. 2, we are initially at point A, and we consider consolidation only, we would move along the failure line down and towards the right. Considering the time effect alone (no change in w), we would move along a horizontal line towards the left. The vector sum of these movements determines whether the net result is an increase or a decrease of the shear strength. For a practical evaluation, one would have to consider a series of elements on a failure surface such as the one shown at depth z in Fig. 6 and compute for each of them, at given times t_1, t_2, t_3, etc., the degree of consolidation and the change in water content and compare the strength for that time from Fig. 2 with the applied stress. Thus, for a series of points, the safety factor may be found for any time t_i and can be integrated over the failure surface.

Admittedly, this requires a good deal of computation. With modern computers this is no longer a serious difficulty and programmes for some problems are in the process of development. Even so, the determination of the time variation of safety factors certainly will remain an intricate problem for some time and many facets still have to be investigated. The exact variation of A and D, for example, will have to be explored. Since very long time tests would be required to come into the time scale of field problems, possible methods of extrapolating short time tests to get long time variations of the parameters A and D will have to be found. Since the method proposed expresses shear strength in terms of water content or void ratio, greater emphasis is necessarily placed on the accurate determination of these properties and their variability.

CONCLUSIONS

The time dependence of the shear strength of clay soils is demonstrated and a relatively simple method is shown to assess this time dependence quantitatively. Actual tests show the variation of the shear-strength parameters A and D with the strain rate or with the load duration. However, these parameters promise to be independent of the stress history. The concepts developed make it possible to compute the variation of safety factors with time although it requires a great amount of computation. Extensive research will be required and may result in improved and shorter methods of testing.

ACKNOWLEDGMENT

The research reported here was sponsored by the Office of Naval Research as part of Contract Nonr-1858(34) with Princeton University. The tests were carried out by the junior author who was a visiting research associate in the Department of Civil Engineering.

REFERENCES

BLIGHT, G. E. (1963). The effect of nonuniform pore pressures on the laboratory measurement of the shear strength of soils. *Proc. NRC/ASTM Symposium on Laboratory Shear Testing of Soils.* In press.

CRAWFORD, C. B. (1963). Pore pressures within soil specimen in triaxial compression. *Proc. NRC/ASTM Symposium on Laboratory Shear Testing of Soils.* In press.

DE WET, J. A. (1961). The use of the energy concept in soil mechanics. *Proc. Fifth International Conference on Soil Mechanics and Foundation Engineering,* Vol. 1, pp. 403–6.

HVORSLEV, M. J. (1960). Physical components of the shear strength of saturated clays. *Proc. ASCE Research Conference on Shear Strength of Cohesive Soils,* pp. 169–273.

MITCHELL, J. K. (1964). Shearing resistance of soils as a rate process. *Jour. Soil Mechanics and Foundations Division ASCE,* Vol. 90/1, pp. 29–61.

MURAYAMA, S., and T. SHIBATA (1961). Rheological properties of clay. *Proc. Fifth International Conference on Soil Mechanics and Foundation Engineering,* Vol. 1, pp. 269–74.

PERLOFF, W. H., JR., and J. O. OSTERBERG (1963). The effect of strain rate on the undrained shear strength of cohesive soils. *Proc. Second Pan-American Conference on Soil Mechanics and Foundation Engineering,* Ib/2, pp. 103–28.

SCHMID, W. E., Y. KLAUSNER, and C. WHITMORE (1960). Rheological shear and consolidation behavior of clay soils. *Progress Report to the Office of Naval Research,* Princeton, December, 1960, 225 pp.

SCHMID, W. E. (1962). New concepts of shearing strength for clay soils, Part I & II. *Sols-Soils,* Vol. 1, pp. 31–42; Vol. 2, pp. 19–26.

TERZAGHI, K. (1931). The static rigidity of plastic clays. *Jour. Rheology,* Vol. 2, No. 2.

VIALOV, S. S., and A. M. SKIBITSKY (1961). Problems of the rheology of soils. *Proc. Fifth International Conference on Soil Mechanics and Foundation Engineering,* Vol. 1, pp. 387–91.

VICAT (1833). Recherches experimentales sur les phenomenes physiques qui precedent et accompagnent la rupture d'une certaine classe de solides. *Annales des Ponts et Chaussees,* pp. 201–68.