Investigation of the Tensile Deformability of Soils Using Hollow Cylinders

Étude de la déformabilité des sols sous tension effectuée au moyen de cylindres creux

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SUMMARY

The authors suggest the investigation of the tensile deformability of soils with hollow cylinders subjected to plane state of stress. The corresponding solutions for stresses and strains have been deduced from the fundamental assumptions of the theory of elasticity, taking into consideration orthogonal axially symmetric deformation anisotropy. These solutions enable one to obtain the deformation constants in certain limited load intervals and in a given time after the load application. The tests made so far prove that deformation anisotropy has to be taken into account.

Analogous solutions for stresses and strains is given for the plane state of strain. The corresponding tests permit the investigation of the influence of the mean principal stress, if different from either the minimum or maximum principal stress. For this purpose the hollow cylinder deformability tests can also be applied usefully in cases where all principal stresses are compression stresses.

SOMMAIRE

Les auteurs proposent la recherche de la déformabilité sous tension des sols au moyen de cylindres creux soumis à un état plan de contraintes. Ils donnent les solutions correspondantes pour les contraintes et les déformations en se basant sur les hypothèses fondamentales de la théorie de l'élasticité et en considérant une anisotropie de déformation orthogonale symétrique par rapport à l'axe du cylindre. En utilisant ces solutions, on peut trouver les constantes de déformabilité correspondant à certains intervalles de charge limités et à un moment donné après l'application de la charge. Les essais effectués jusqu'à présent prouvent la nécessité de prendre en considération l'anisotropie de déformation.

Une solution analogue pour les contraintes et les déformations est présentée pour un état plan de déformation. Les essais correspondants permettent une étude de l'influence de la contrainte principale moyenne lorsque celle-ci diffère de la contrainte principale, maximum ou minimum. Dans ce but, les essais de déformabilité faits sur des échantillons en form de cylindres creux triaxiaux annulaires peuvent être intéressants aussi pour des états de contraintes de compression.

IN THE SOIL MECHANICS LABORATORY of the University of Ljubljana the first tensile deformability tests were made using specimens of rectangular profile with expanded heads; in the horizontal position the specimens were exposed to uniaxial tension.* The results of similar extensive tests of eightfold specimens with egg-shaped expanded heads were reported by Hasegawa and Ikeuti (1964). Hollow cylinder triaxial tests may have the following advantages: (*a*) the stress state is better defined without uncontrolled stress concentration; (*b*) tests can be made at various stress states; (*c*) deformation anisotropy can be taken into account; (*d*) long-term and drained tests can more easily be carried out.

Our interpretation of the hollow cylinder tensile tests† will be based on the assumption of orthogonal anisotropy in the directions of cylindrical co-ordinates whose one axis coincides with the axis of the specimen. In so far as the compression stresses are smaller than the preconsolidation pressure, the same order of magnitude of deformation in the tangential tensile and in the radial compression directions could be expected, provided that the structure of the specimen is isotropic in the planes perpendicular to the sample axis. By considering deformation anisotropy, the deformability differences due to the hysteresis effect can be determined within the same order of magnitude. If the radial or axial compression stresses surpass the preconsolidation pressure, different orders of magnitude of compression and tension moduli can be expected.

The solutions for stresses and strains which will be deduced in the following paragraphs by considering elastic anisotropy are valid for any plane state of strain or stress. Nevertheless, the use of hollow cylinders for tests with three compression principal stresses are recommended only if one is studying the dependence of the deformation parameters on a mean principal stress different from either the minimum or the maximum normal stress. For other stress states the conventional triaxial tests are preferred (Šuklje, 1963).

Soil being a non-linear viscous elastic medium, the investigations based on the assumptions of the theory of elasticity can represent only an approximation valid for certain limited stress intervals in a given time elapsed after loading or—when investigating the relaxation effects—after retention of the displacements. On the condition that stress-strain relationships can be expressed by linear rheological models, elastic solutions can be applied for any consolidation degree.

ELASTIC SOLUTIONS FOR STRESSES AND STRAINS IN HOLLOW CYLINDERS

Only the plane (axial symmetric) states of stress and strain will be considered, assuming orthogonal deformation

*I. Markič's thesis for the Bachelor's degree, done under the guidance of the senior author in 1961.

†Similar tests have previously been made by Wu, *et al.* (1963) in order to study the failure envelope of soils.

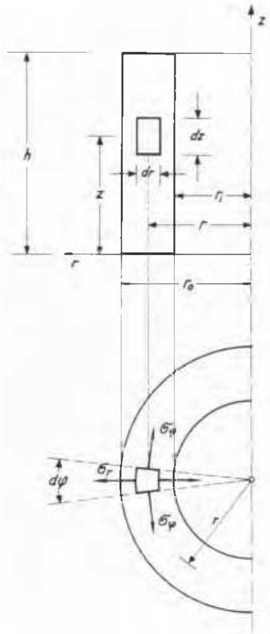

FIG. 1. Symbols.

anisotropy in the directions of the cylinder co-ordinates (r, φ, z). E_r, E_φ, and E_z represent the linear deformation moduli in the directions r, φ, and z, ν_{ik} (i, k = r, φ, z) being the Poisson ratio determining the influence of the stress σ_k on the strain ϵ_i. Symmetrical radial hydrostatic loading is denoted as p_i inside and p_o outside the specimen; the inside radius may be denoted by r_i and the outside radius by r_o. The total reaction in the direction z may be denoted P.

Plane State of Strain

The fundamental expressions for strains are:

$$\epsilon_r = \frac{\sigma_r}{E_r} - \nu_{r\varphi}\frac{\sigma_\varphi}{E_\varphi} - \nu_{rz}\frac{\sigma_z}{E_z} \tag{1}$$

$$\epsilon_\varphi = -\nu_{\varphi r}\frac{\sigma_r}{E_r} + \frac{\sigma_\varphi}{E_\varphi} - \nu_{\varphi z}\frac{\sigma_z}{E_z} \tag{2}$$

$$\epsilon_z = -\nu_{zr}\frac{\sigma_r}{E_r} - \nu_{z\varphi}\frac{\sigma_\varphi}{E_\varphi} + \frac{\sigma_z}{E_z} \tag{3}$$

$$\epsilon_z = 0. \tag{3a}$$

The assumption that the deformation work does not depend on the turn of applying loads in principal directions, yields

$$\nu_{ik} = \nu_{ki}\,(E_k/E_i). \tag{4}$$

By taking into account this relation one obtains from Eq 3a:

$$\sigma_z = \nu_{rz}\sigma_r + \nu_{\varphi z}\sigma_\varphi \tag{5}$$

and further, by inserting Eq 5 into Eqs 1 and 2:

$$\epsilon_r = k\sigma_r - l\sigma_\varphi \tag{6}$$

$$\epsilon_\varphi = -l\sigma_r + m\sigma_\varphi \tag{7}$$

with the following meaning of the signs k, l, and m:

$$k = (1/E_r) - (\nu^2_{rz}/E_z) \tag{8}$$

$$l = (\nu_{\varphi r}/E_r) + [(\nu_{rz}\cdot\nu_{\varphi z})/E_z] \tag{9}$$

$$m = (1/E_\varphi) - (\nu^2_{\varphi z}/E_z). \tag{10}$$

Further, the well-known expressions

$$\epsilon_r = \partial u/\partial_r \tag{11}$$

$$\epsilon_\varphi = u/r \tag{12}$$

may be considered, u being the displacement in the radial direction, as well as the equilibrium condition

$$\partial\sigma_r/\partial r + (\sigma_r - \sigma_\varphi/r) = 0 \tag{13}$$

giving

$$\sigma_\varphi = r(\partial\sigma_r/\partial r) + \sigma_r. \tag{13a}$$

Now, Eqs 11, 12, and 13a should be inserted in Eqs 6 and 7, then the displacement u in Eq 7 differentiated with respect to r, and the resulting derivation equalized with Eq 6. In this way the following differential equation is found

$$mr^2(\partial^2\sigma_r/\partial r^2) + 3mr(\partial\sigma_r/\partial r + (m-k)\sigma_r = 0. \tag{14}$$

Its solution is:

$$\sigma_r = Ar^{x_1} + Br^{x_2} \tag{15}$$

with the following meaning of the parameters x_1, x_2

$$x_{1,2} = -1 \pm \sqrt{k/m}. \tag{15a}$$

The constants A and B can be deduced from the boundary conditions for r_i and r_o:

$$A = \frac{p_i r_o^{x_2} - p_o r_i^{x_2}}{r_i^{x_1} r_o^{x_2} - r_i^{x_2} r_o^{x_1}} \tag{16}$$

$$B = \frac{p_i r_o^{x_1} - p_o r_i^{x_1}}{r_i^{x_2} r_o^{x_1} - r_i^{x_1} r_o^{x_2}} \tag{17}$$

Inserting Eq 14 into 13a one gets

$$\sigma_\varphi = A(x_1+1)r^{x_1} + B(x_2+1)r^{x_2}. \tag{18}$$

Expressing then, in Eqs 6 and 7, σ_r by Eq 15 and σ_φ by Eq 18, we get

$$\epsilon_\varphi = Ar^{x_1}[m(x_1+1) - l] + Br^{x_2}[m(x_2+1) - l]. \tag{19}$$

The equilibrium condition in the direction z, applied to the cross-section z = h, yields

$$2\pi\int_{r=r_i}^{r_0} \sigma_z r d\zeta = P. \tag{20}$$

For σ_z Expression 5 may be inserted taking into account Eqs 15 and 18. By integrating, one gets

$$\nu_{rz}\left[\frac{A}{x_1+2}r^{x_1+2} + \frac{B}{x_2+2}r^{x_2+2}\right]_{r=r_i}^{r_0} \tag{21}$$

$$+ \nu_{\varphi z}\left[\frac{A(x_1+1)}{x_1+2}r^{x_1+2} + \frac{B(x_2+1)}{x_2+2}r^{x_2+2}\right]_{r=r_i}^{r_0} = \frac{P}{2\pi}.$$

When testing hollow cylinders the loads p_i, p_o, and P can directly be measured while the linear strains ϵ_φ in the tangential direction, corresponding to $r = r_i$ and $r = r_o$, can be found from the observed volume change of the liquid in the cell inside ($r = r_i$) and outside ($r = r_o$) the specimen. For certain values of the coefficient k/m the corresponding pairs of the coefficients m and l can be computed from Eq 19 set up for $r = r_i$ and $r = r_o$ at given values of p_i and p_o in the observed load interval. Also, the right values of the pair satisfy Eq 19 for $r = r_i$ at another combination of the pressures p_i' and p_o' within the observed load itnerval on condition $p_i'/p_o' \neq (p_i/p_o)$; thereby it is assumed that in the observed load interval the elastic constants corresponding to a certain time do not change and do not depend on the ratio p_i/p_o provided that all stresses are increasing in the same sense. The Poisson ratios ν_{rz} and $\nu_{\varphi z}$ can then be found from Eq 21 set with the obtained value of k/m for two combinations of p_i', p_o' and p_i, p_o in the observed loading interval. In the three equations (8, 9, and 10) four deformation parameters, E_r, E_φ, E_z, and $\nu_{\varphi r}$ remain unknown quantities. These parameters can be obtained only with a supplementary assumption, e.g., $E_\varphi = E_z$, which holds in preconsolidated samples with isotropic structure. The complete computation of the deformation parameters still takes much time.

Plane State of Stress

For plane state of stress Eqs 1, 2, and 3 with

$$\sigma_z = 0 \tag{22}$$

become simplified. In an analogous way as in the plane state of strain Expressions 15 and 18 result again for stresses σ_r and σ_φ with following different meaning of the parameters x_1 and x_2:

$$x_{1,2} = -1 \pm \sqrt{\alpha} \tag{23}$$

$$\alpha = E_\varphi/E_r. \tag{24}$$

With this signification for the parameters $x_{1,2}$ Eqs 16 and 17, expressing the constants A and B, remain unchanged, while the following expression can be deduced for the linear strain ϵ_φ in the tangential direction:

$$\epsilon_\varphi = \frac{1}{rE_\varphi}\left\{Ar^{\sqrt{\alpha}}(\sqrt{\alpha} - \nu_{r\varphi}) - Br^{-\sqrt{\alpha}}(\sqrt{\alpha} + \nu_{r\varphi})\right\}. \tag{25}$$

Knowing the inside and outside pressures p_i and p_o Eq 25 can be set for both limit linear strains $\epsilon_{\varphi i}$ and $\epsilon_{\varphi o}$ corresponding to the radii r_i and r_o and to the observed axial displacements Δh and volume changes inside (ΔV_i) and outside (ΔV_o) the specimen. These two equations yield for various assumed values $\alpha = E_\varphi/E_r$ the corresponding pairs of elastic parameters E_φ and $\nu_{r\varphi}$. The pair should be chosen to satisfy Eq 25 also at another combination of p_i' and p_i whereby the same assumptions and conditions, as stated in the foregoing paragraph, must be taken.

TEST EQUIPMENT

The test equipment has been arranged for tests corresponding to the plane state of stress. The inside diameter of the hollow cylinder specimens is $2r_i = 4$ cm, the outside diameter $2r_o = 6.4$ cm, and the height $h = 8$ cm. On both

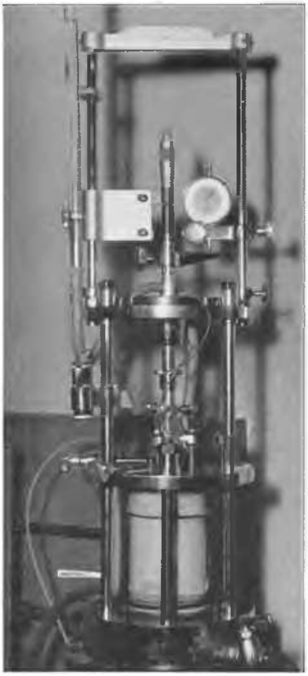

FIG. 2. Triaxial cell for hollow cylinder specimens subjected to plane state of stress.

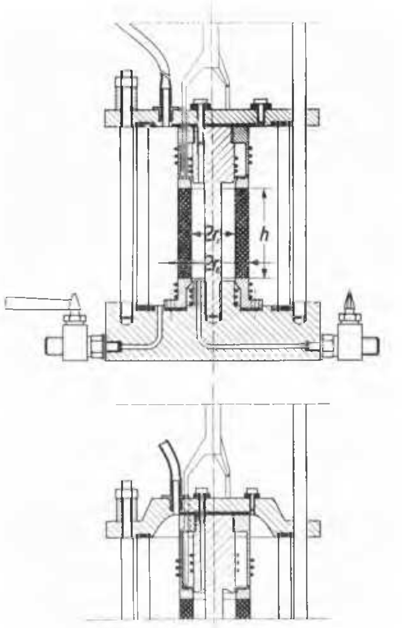

FIG. 3. Cross-section of the triaxial cell: top, executed; bottom, improved design.

lateral sides the specimens are protected by rubber membranes fixed to the base and to the top of the test cell in order to prevent the penetration of the liquid into the specimen and into the space above it. The inside liquid is connected with the pressure catch, with the mercury manometer, and with a volumeter of 0.1 cu. cm. accuracy, while the outside liquid of the equipment, used so far, has been connected only with a simple glass volumeter serving also for the measurement of the small hydrostatic pressure in the outside space of the cell. The displacements in the axial direction are registered by a comparator; the measuring axis is fastened to a micrometer screw and three vertical branches of the axis touch an aluminium ring lying on the upper surface of the specimen. Three light indicators connected in parallel facilitate maintenance of the contact between the measuring axis and the specimen without reaction pressure in the vertical direction. The details are shown in Fig. 3 (top) representing the cross-section of the test cell used in the tests made so far, and in the photograph (Fig. 2). Fig. 3 (bottom) represents an improved design of the top of the cell with better conditions for the elimination of the air caught in the liquid. The influence of the air bubbles and of the accuracy of the measuring system can be further reduced if the tests are made on larger specimens with diameters $2r_i \geqslant 7$ cm and $2r_o \geqslant 10$ cm and a height $h \geqslant 15$ cm. The volume change of the liquid in the outside space of the cell should be connected with a precise volumeter and the testing capacity of the device improved so that it could produce higher pressures in the outside liquid.

The equipment could be adopted for tests corresponding to the plane state of strain by designing a dynamometer measuring the vertical reactions; in this case the bearing capacity of the micrometer and of the contact vertical branches of the measuring axis should be adequately reinforced.

The equipment used so far did not permit either a free drainage or pore-pressure measurement. The corresponding improvement could be made without special difficulties.

EFFECTS OF ANISOTROPY AND OF CREEP ON THE DEFORMATION PARAMETERS

Fig. 4(a) represents the loading diagram $p_o = p_o\ (p_i)$ of two successive loading degrees of a specimen of the same tertiary clay with the water content $w_0 = 23$ per cent, Fig. 4(b) the corresponding graphs relating the volume changes ΔV_i and ΔV_o as well as the axial displacements Δh to time t, and Fig. 4(c) the solutions of Equation 25 resulting from plots (a) and (b) for various values of the coefficient $\alpha = E_\varphi/E_r$ and for various times t. In this test series the observed load interval was not divided in two steps with different p_o/p_i ratios in order to get the third equation of the type (25) enabling the final determination of the correct α value. Nevertheless, the plots presented in Fig. 4(c) allow the following conclusions: (1) The assumption of the isotropy does not lead to real values of the Poisson ratio. (2) The effect of the anisotropy on the value of the tensile deformation modulus is less important. (3) In the tsests presented the correct values of the coefficient α are between 2 and 4. (4) With increasing time the Poisson ratio decreases as well as the deformation modulus.

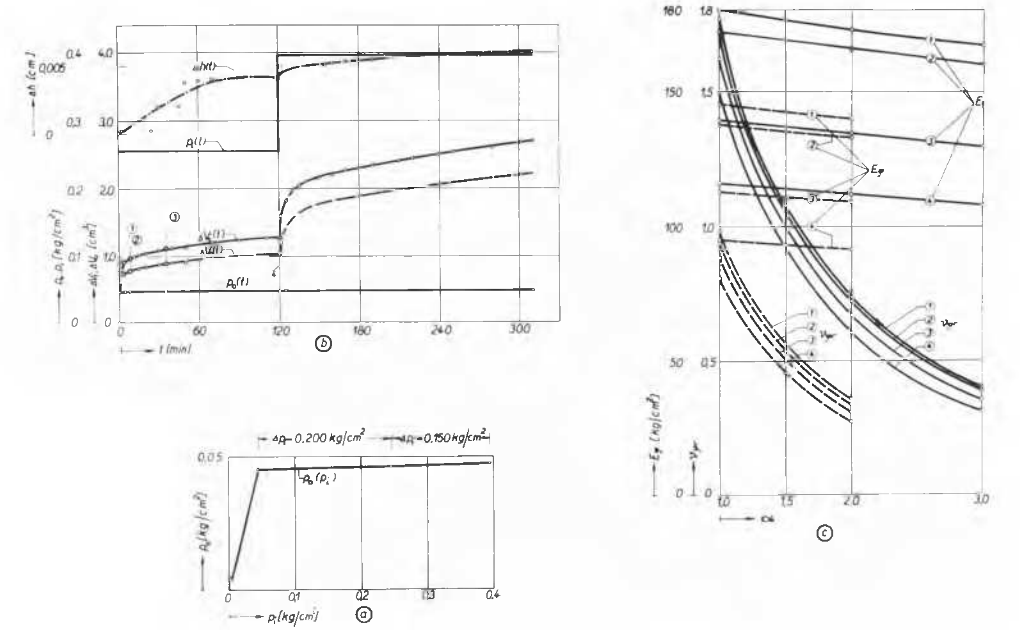

FIG. 4(a). The diagram $p_o = p_o(p_i)$ of a test executed with an undisturbed clay sample ($w_o = 23\%$). (b) Time dependent plots of p_o, p_i, ΔV_i, ΔV_o and Δh of the same test. (c) The resulting modulus E_φ and Poisson's ratio $\nu_{\varphi r}$ corresponding to various assumed modulus ratios $\alpha = E_\varphi/E_r$.

Owing to the insufficient accuracy of the volumeter connected with the outside space of the cell, the measured ΔV_0 changes are not reliable enough. The group of the dashed lines $\nu_{\varphi r} = \nu_{\varphi r}(\alpha)$ and $E_\varphi = E_\varphi(\alpha)$ corresponds to assumption $\Delta V_0 = \Delta V_i$ which is on the one limit of the probable accuracy interval. The rather important differences of the deformation parameters require the improvements of the equipment proposed in foregoing paragraphs.

In the observed load interval the deformation speed decreases and the strains tend to a final value. In the subsequent load interval the plot $\Delta V_i = \Delta V_i(t)$, which is also shown in Fig. 4(b), the hardening effect disappears; the deformations increase with constant rate. In the plot $\Delta V_i = \Delta V_i(p_i)$ this interval is already situated outside the yield point. Thus, the yield point could be taken as the limit value of the permissible stresses.

CONCLUSION

The use of the hollow cylinder specimens is suitable for deformability tests at stress states with one tensile principal stress. In limited stress intervals the deformation parameters can be computed on the basis of elastic solutions provided that the deformation anisotropy is taken into account. As the computed values are rather sensitive to the accuracy of the measurements, full attention should be paid to the elimination of the air bubbles in the cell liquid serving for producting pressures and volume change measurements in the inside and outside space of the cell. In order to reduce the relative influence of errors, the authors recommend tests with large samples, precise volumeters, the gauging of the effects of temperature and of deformability of single parts of the testing device as well as the use of organic testing liquid in the cell. The deformation parameters should be expressed as function of the time.

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