2/52

The Correlation of Triaxial Compression Test Data on Cohesionless Granular Media

Corrélation des valeurs numériques d'essais de compression triaxiale sur les milieux granulaires sans cohésion

A. THURAIRAJAH, B.SC., PH.D., *The Engineering Laboratories, Cambridge University, Cambridge, Great Britain*
K. H. ROSCOE, M.C., M.A., A.M.I.MECH.E., *The Engineering Laboratories, Cambridge University, Cambridge, Great Britain*

SUMMARY

The results of an extensive series of triaxial compression tests on sand and glass beads, under varied drainage conditions, are correlated by the application of various corrections. Where necessary all data are first corrected for penetration of the rubber sleeve and an allowance is also made for the consequent volume change of the soil skeleton in undrained tests. By applying an elastic, as well as a boundary, energy correction to all the undrained data a unique surface is obtained in (p, q_w, e) space, where p is the mean normal effective stress, q_w is the corrected deviator stress, and e is the voids ratio. The interesting feature of this surface is its constant slope with respect to the e-p plane. When the same corrections are applied to the state paths for fully drained tests the peak corrected deviator stresses correspond to points on the unique surface for undrained test results.

SOMMAIRE

L'application de différentes corrections permet d'établir une corrélation entre les résultats d'une large série d'essais en compression triaxiale sur des grains de sable et de verre dans des conditions de drainage variées. Lorsque nécessaire, on effectue d'abord des corrections sur toutes les valeurs numériques pour la pénétration de la membrane de caoutchouc et on établit également une tolérance pour le changement de volume correspondant du squelette du sol dans les essais non drainés. En appliquant une correction d'énergie élastique, aussi bien que limite, à toutes les valeurs numériques non drainées, on obtient une surface unique dans l'espace (p, q_w, e), où p est la contrainte normale effective moyenne, où q_w est la contrainte déviatorique corrigée et où e est l'indice de vide. Une propriété intéressante de cette surface est sa pente constante par rapport au plan (e, p). Quand on applique les mêmes corrections aux chemins d'état pour les essais complétement drainés, les contraintes déviatoriques ayant subies la correction de pointe correspondent à des points sur la surface unique pour les résultats d'essais non drainés.

ROSCOE, SCHOFIELD, AND THURAIRAJAH (1963a) suggested that if a triaxial sample of soil, initially in equilibrium under a compressive stress system with an observed isotropic component p and deviatoric component q, undergoes a volumetric strain δv and a shear strain $\delta\epsilon$ due to stress increments of δp and δq, then in unit volume of the soil the total energy supplied $\delta E'$, the recoverable elastic energy stored δU, and the energy dissipated δW, are related by

$$\delta E' = \delta U + \delta W = q \cdot \delta\epsilon + p \cdot \delta v \tag{1}$$

Furthermore if $dW/d\epsilon$ is defined as q_w, then

$$q_w = q + p(dv/d\epsilon) - (dU/d\epsilon) \tag{2}$$

where $p(dv/d\epsilon)$ and $-dU/d\epsilon$ are called the "boundary" and the "elastic" energy corrections respectively. They showed that data from drained and undrained triaxial compression tests on normally consolidated samples of kaolin lay on a unique surface, which was a plane containing the voids ratio (e) axis, when plotted in (p, q_w, e) space. Some scatter in these results has been discussed by Roscoe and Thurairajah (1964).

In this paper it will be demonstrated that similar unique surfaces, namely planes containing the e-axis, can be obtained for cohesionless media. The media considered are Leighton Buzzard sand (passing 14 but retained on 25 B.S. sieve), glass beads (av. diam 0.78 mm), and Braehead silt (Penman, 1953). The test procedure and determination of sleeve penetration has been described by Roscoe, Schofield, and Thurairajah (1963b).

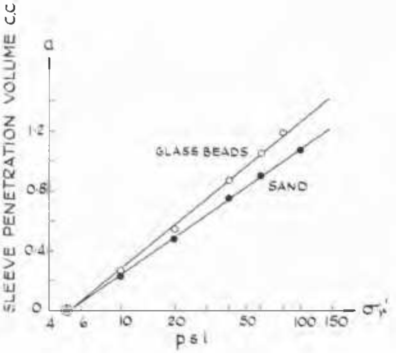


FIG. 1. Variation of sleeve penetration volume with effective radial stress for triaxial samples of sand and glass beads.

SLEEVE PENETRATION

The relationship between sleeve penetration volume (a cu.cm.) and effective radial stress (σ'_r psi) is shown in Fig. 1 in which it is assumed that $a = 0$ when $\sigma'_r \leqslant 5$ psi. The equation of the straight line for the sand is

$$a = 0.237 + 0.364 \log_e (\sigma'_r/10) \qquad (3)$$

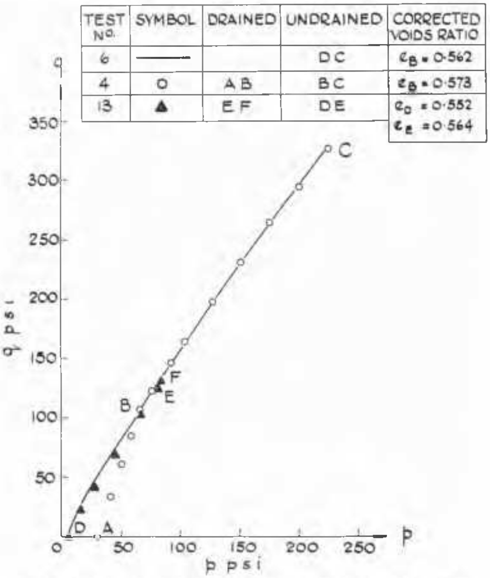
while for the glass beads it is $a = 0.278 + 0.242 \log_e (\sigma'_r/10)$. There is no sleeve penetration of the silt samples.

p, *q*, *e* DATA FOR DENSE SAND AND SILT

Before discussing the correlation of drained and undrained test data in (p, q_w, e) space, it will be shown, by considering tests on dense sand and silt samples in which conditions were suddenly changed from drained to undrained and *vice versa*, that a unique surface cannot be obtained in (p, q, e) space.

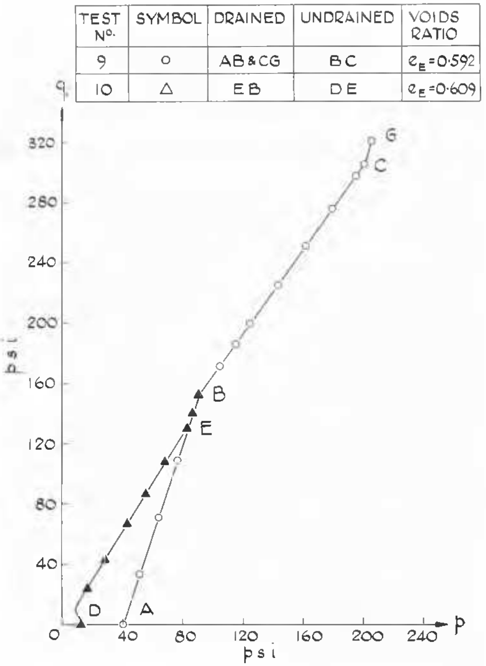
Sand

The (p, q) stress paths followed by three dense samples are shown in Fig. 2. In test 6 conventional undrained conditions prevailed throughout and the stress path is shown by the full line DC (but not the points). In test 4 the circular points A to B refer to drained conditions with peak drained deviator stress q_{max} at B. Thereafter, conditions were undrained as represented by the circular points B to C. In test 13 the triangular points D to E refer to undrained and E to F to fully drained conditions with drained q_{max} at F. The above results appear to show fair agreement between drained and undrained yield surfaces for triaxial samples of sand in (p, q, e) space, but no firm conclusion can be drawn since the voids ratio changed due to sleeve penetration in the so-called undrained tests.



TEST No.	SYMBOL	DRAINED	UNDRAINED	CORRECTED VOIDS RATIO
6	———		DC	$e_B = 0.562$
4	○	AB	BC	$e_B = 0.573$
13	▲	EF	DE	$e_D = 0.552$ $e_E = 0.564$

FIG. 2. Stress paths during triaxial compression tests on sand.



TEST No.	SYMBOL	DRAINED	UNDRAINED	VOIDS RATIO
9	○	AB & CG	BC	$e_E = 0.592$
10	△	EB	DE	$e_E = 0.609$

FIG. 3. Stress paths during triaxial compression tests on silt.

Silt

The two latter sand tests (4 and 13) were repeated on samples of silt in which constant voids ratio is maintained during undrained tests since the sleeve penetration is negligible. In Fig. 3 the silt test 9 (compare sand test 4) is shown by AB and CG for drained conditions and BC for undrained. Points B and G correspond to peak drained deviator stresses. In silt test 10 (compare sand test 13) the undrained stress path is DE and the subsequent drained path is EB with q_{max} at B. When the two samples were in the state represented by point E the voids ratio was 0.592 in test 9 and 0.609 in test 10. Since these are approximately the same, both samples, when at stress point E, are at virtually the same state point in (p, q, e) space. If a unique surface exists for drained and undrained tests in this space, then point B should coincide with point E and the straight line BC should be a continuation of the straight line portion of DE. This suggests that some form of correction should be applied to q if a unique surface is to be obtained. The application of the two corrections discussed in the introduction will now be considered.

CORRECTIONS

Boundary Energy Correction

For conventional *undrained* tests on the sand samples, of average bulk volume $V = 90.7$ cu cm, the *boundary energy correction for sleeve penetration*, as derived from Equation 3, is $-0.00409\ (p/\sigma'_r)(d\sigma'_r/d\epsilon)$. The same correction applies to the glass bead samples except the 0.00409 becomes 0.0047.

Elastic Energy Correction

Fatt (1957) verified experimentally for neoprene and steel spheres the following equation of Brandt relating V and p for a pack of uniform spheres.

$$V = b - cp^{2/3} \tag{4}$$

where b and c are constants. He showed that the isotropic compression of the packs was purely elastic, as found by Thurairajah (1961) for triaxial samples of the sand, glass beads, and silt which were also observed to have equal axial and radial strains. The measured elastic volume change of a sand sample ($V = 90.7$ cu cm) was 0.258 cu cm as p changed from 5 to 100 psi giving a value of $c = 0.0139$.

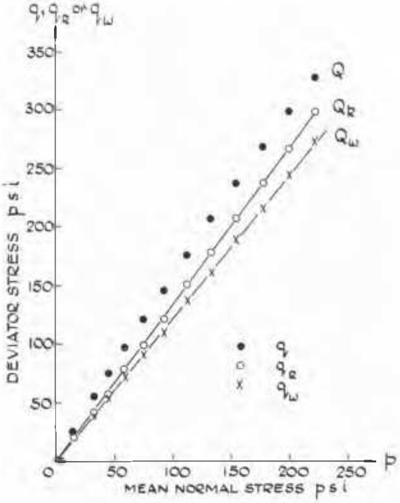


FIG. 4. Variation of q, q_R, and q_w with p during an undrained test on sand.

Hence from Equation 4 the volumetric strain consequent upon an isotropic stress increment δp is $\delta v = -\delta V/V = -\frac{2}{3}(-c/V)(\delta p/p^{1/3}) = -0.000102\ \delta p/p^{1/3}$. If it is assumed that a change in deviator stress δq has negligible effect upon the elasticity energy U, then under a general stress increment $(\delta p, \delta q)$ the elastic energy correction for a sand sample is

$$-\frac{dU}{d\epsilon} = -p\frac{dv}{d\epsilon} = -0.000102\, p^{\frac{1}{3}} \cdot \frac{dp}{d\epsilon} \tag{5}$$

Thurairajah found that the elastic volume change for the glass bead samples was 0.256 cu cm compared to the 0.258 cu cm for the sand, hence Equation 5 was also taken as the elastic energy correction for the beads.

CORRELATION OF TRIAXIAL DATA ON SAND

In Fig. 4 the curves OQ, OQ$_R$ and OQ$_w$ refer respectively to the (p, q), (p, q_R), and (p, q_w) stress paths for an undrained test on a dense sand sample. q_R is obtained from q by applying the boundary, but not the elastic, energy correction. It is apparent that both corrections are substantial for dense tests. The (p, q) stress paths for five undrained tests on sand samples, of initial voids ratio ranging from $e = 0.552$ to $e = 0.641$, are shown in Fig. 5. The paths for dense samples lie above those for loose and the slope of each path decreases with increase of p. This tendency is still apparent for the corresponding (p, q_R) stress paths in Fig. 6 but, as shown in Fig. 7, the (p, q_w) paths are straight and all lie on a plane containing the e-axis in (p, q_w, e) space. The line OQ$_w$ shown in Figs. 8 and 9 is the mean line obtained from the undrained tests in Fig. 7. The peak values of the observed deviator stresses q_{max} for six drained tests at varying initial voids ratios are represented by the

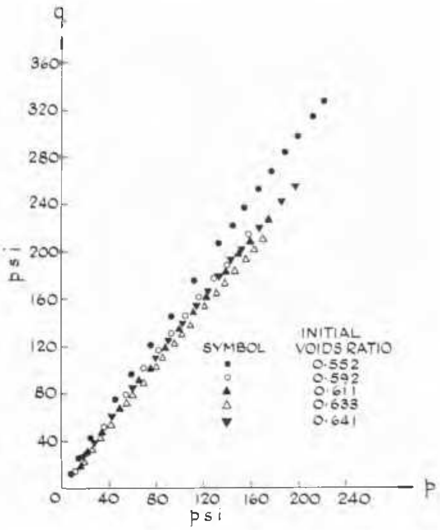


FIG. 5. Variation of q with p during five undrained tests on sand.

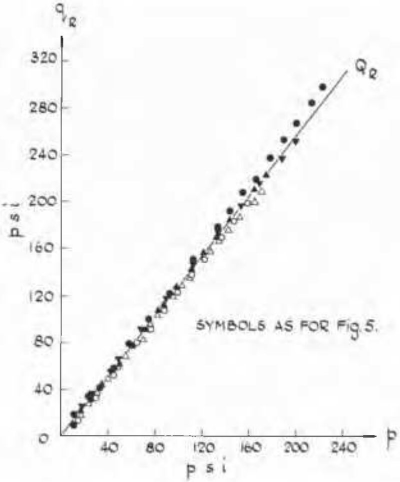


FIG. 6. Variation of q_R with p during the same tests as shown in Fig. 5.

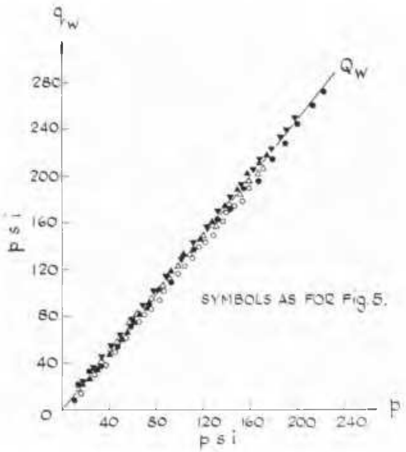


FIG. 7. Variation of q_w with p during the same tests as shown in Fig. 5.

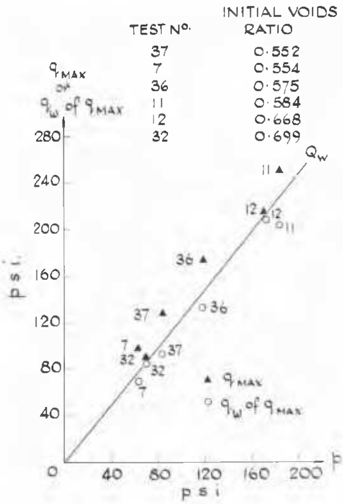


FIG. 8. Peak points of q during drained tests on sand. The line OQ_w is for undrained tests from Fig. 7.

triangular points in Fig. 8 and all lie above the line OQ_w. The points (circular) obtained after applying both energy corrections to these peak (triangular) points are also shown in the figure. The corrected points for loose samples are near the line OQ_w while those for dense samples lie considerably below. It has been found that the peak value of the corrected deviator stress $q_{w_{max}}$ occurs at a greater strain than q_{max}; the denser the sample, the greater the difference in strains corresponding to $q_{w_{max}}$ and q_{max}. The points plotted in Fig. 9 are $q_{w_{max}}$ for drained tests and they all lie close to the line OQ_w obtained for undrained tests. It should be noted that the elastic energy correction for a drained test in the regions of strain where the sample attains q_{max} and thereafter are negligible compared to the measured stress; the boundary energy correction is, however, of appreciable magnitude.

CORRELATION OF TRIAXIAL TEST DATA ON GLASS BEADS

Similar results to those obtained for the sand were observed for the glass beads as shown in Figs. 10, 11, and 12. All the data in these diagrams refer to undrained tests

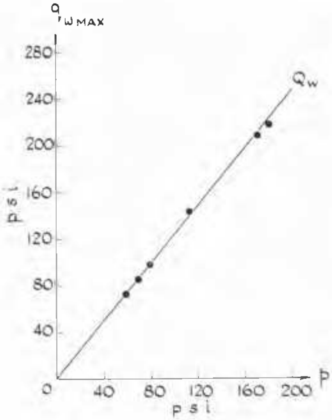


FIG. 9. Peak points of q_w during drained tests on sand. The line OQ_w is for undrained tests from Fig. 7.

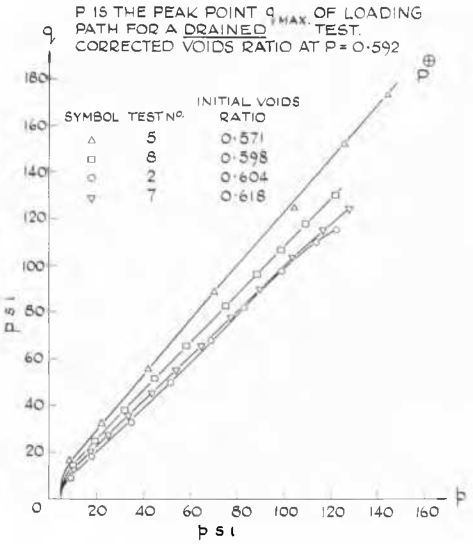


FIG. 10. Variation of q with p during four undrained tests on glass beads. Point P is q_{max} in a drained test. (Voids ratio at P = 0.592).

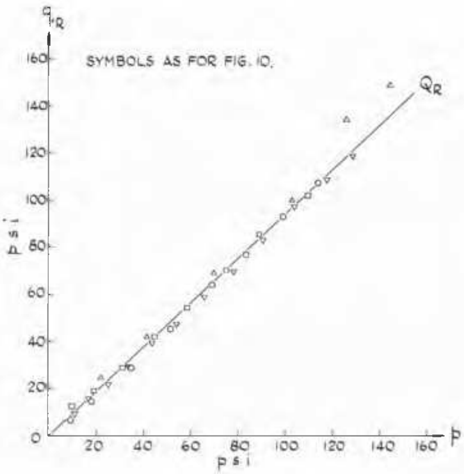


FIG. 11. Variation of q_R with p for same tests as shown in FIG. 10.

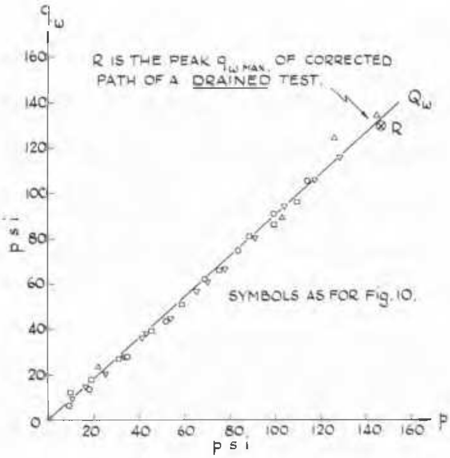


FIG. 12. Variation of q_w with p for same tests as shown in Fig. 11. Point R is $q_{w\max}$ in a drained test.

except point P in Fig. 10 which refers to $q_{\max}$ and point R in Fig. 12 denoting $q_{w_{\max}}$ for drained tests.

CONCLUSIONS

The experimental results on sand and glass beads suggest that correlation of the undrained and drained yield surfaces can be obtained if boundary and elastic energy corrections are applied to the observed deviator stresses. The corrected state paths, after the samples begin to yield, of undrained and drained tests lie on a plane containing the e-axis in (p, q_w, e) space. This behaviour is similar to that of normally consolidated clays as reported by Roscoe, Schofield, and Thurairajah (1963a).

REFERENCES

FATT, I. (1957). Compressibility of a sphere pack—comparison of theory and experiment. *Jour. Applied Mechanics*, Vol. 24, pp. 148–9.

PENMAN, A. D. M. (1953). Shear characteristics of a saturated silt measured in triaxial compression. *Géotechnique*, Vol. 3, pp. 312–28.

ROSCOE, K. H., A. N. SCHOFIELD, and A. THURAIRAJAH (1963a). Yielding of clays in states wetter than critical. *Géotechnique*, Vol. 13, pp. 211–40.

——— (1963b). A critical appreciation of test data for selecting a yield criterion for soils. *Symposium Laboratory Shear Testing of Soils* (Ottawa).

ROSCOE, K. H., and A. THURAIRAJAH (1964). On the uniqueness of yield surfaces for wet clays. *Symposium Rheology and Soil Mechanics, International Union of Theoretical and Applied Mechanics* (Grenoble).

THURAIRAJAH, A. (1961). Some shear properties of kaolin and of sand. Ph.D. thesis, University of Cambridge.