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Consolidation of Thawing Soils

Consolidation des terrains en dégèlement

N. A. TSYTOVICH, D.SC., PROF., *Permafrost Soil Mechanics Laboratory, Research Institute of Bases and Underground Structures, Moscow, U.S.S.R.*
YU. K. ZARETSKY, M.SC., ENG., *Permafrost Soil Mechanics Laboratory, Research Institute of Bases and Underground Structures, Moscow, U.S.S.R.*
V. G. GRIGORYEVA, SC.COLLAB., *Permafrost Soil Mechanics Laboratory, Research Institute of Bases and Underground Structures, Moscow, U.S.S.R.*
Z. G. TER-MARTIROSYAN, ENG., *Permafrost Soil Mechanics Laboratory, Research Institute of Bases and Underground Structures, Moscow, U.S.S.R.*

SUMMARY

This paper states that the consolidation of thawing soils occurs with practically constant pore pressure and obeys the theory of filtration consolidation. A method is suggested for calculating settlement and pore pressure in thawing soils for one-dimensional problems of consolidation.

SOMMAIRE

Cette étude établit que le processus de la consolidation des terrains en dégèlement s'effectue pratiquement sous pression interstitielle constante et selon la théorie de la consolidation filtrante. On démontre une méthode de calcul de l'affaissement et de la pression interstitielle pour les terrains en dégèlement sous la consolidation unidimensionelle.

MAIN STAGES OF CONSOLIDATION OF THAWING SOILS

THE PROBLEM OF PREDICTING the magnitude of settlement and its variation with time is very important for estimating the capacity of foundation support on thawing permafrost soils by the method of limited strains (Tsytovich, 1960). The first experiments to determine the complete consolidation of thawing soils were carried out in 1934 (Tsytovich, 1934). The variations of the void ratio of thawing soils under the conditions of no side expansion were investigated also (Tsytovich, 1941) and the complete settlement of thawing soils was determined by the method of the test load (Lapkin, 1939, 1947).

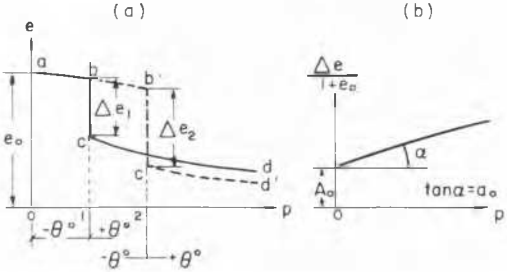

FIG. 1. Relationship between change of void ratio of frozen soils in the process of thawing and change of external pressure.

Fig. 1(a) illustrates the compression curve of the frozen soil during thawing: a-b corresponds to the frozen state, the convex part being directed upwards, which indicates the unconsolidated state of the frozen soil and increase in its compressibility in the process of increase of the load; b-c, where the curve is broken, corresponds to the subsidence of the soil as a result of thawing; and c-d corresponds to the consolidation of the soil after thawing. As shown experimentally (Tsytovich, 1957; Oushkalov, 1962) the maximum settlements occur in the process of thawing as a result of a jump-like change in the soil porosity. Fig. 1(b) illustrates the change in the void ratio of the thawing soil (Δe) as a function of the external pressure (p). With small changes in pressure (up to 3 to 5 kg/sq.cm. depending on the properties of the soil) (Shusherina, 1953) the respective section of the curve $\Delta e = f(p)$ can, with sufficient accuracy, be assumed to be straight. This assumption is used as the basis for the new method of calculation of the settlement of foundations on thawing soils (Tsytovich, 1941; Lapkin, 1939; Oushkalov, 1962).

The complete stabilized settlement of foundations on thawing soils (S) can be expressed as follows (Tsytovich, 1941):

$$S = \sum_{1}^{n} A_0 h_i + \sum_{1}^{n} h_i a_0 \sigma_{zi}, \qquad (1)$$

where h_i is the thickness of individual layers of thawed soils; A_0 is the so-called reduced coefficient of thawing; a_0 is the reduced coefficient of consolidation (corresponds to m_v); σ_{zi} is the average compressive stress in the soil layer under the effect of the external load and the weight of the soil itself.

As a result of the investigation of the time-settlement relationship of ice-saturated frozen soils (with the temperature close to that of melting) and thawing soils, it was concluded (Tsytovich, 1964) that a migration-viscous consolidation occurs at section a-b (with the constant negative temperature below freezing), that section b-c, with thawing of ice-saturated soils, is mainly characterized by filtration consolidation, while section c-d is characterized by the remaining filtration and secondary viscous consolidation.

EXPERIMENTAL INVESTIGATIONS OF COMPRESSIONAL CONSOLIDATION OF THAWING SOILS

The investigations were carried out with two types of artificially frozen heavily iced soils—montmorillonite ("kill" clay) and hydro-micaceous (surface loam-clay) composition—with the initial water content corresponding to the liquid limit. The thawing carried out was one-sided at a constant temperature and a constant load from 0.25 to 3.0 kg/sq.cm. The rate of thawing was regulated with the temperature of the settlement plate, corresponding to +2, +10, and +20 C. The conditions of linear thawing were preserved during all the experiments so that the movement of the thawing boundary was subjected to the known regularity: $h(t) = \alpha\sqrt{t}$. The tests were carried out in special non-heat-conducting oedometers and were accompanied by a change in the pore pressure during the process of thawing. Compression was carried out by one-sided filtration through the settlement plate. Drainage through the bottom surface of the specimen was prevented. The analyses of the results have shown that for the heavily iced soils, when the ice inclusions are uniformly spread over the whole volume of the specimen, the consolidation during thawing, irrespective of the type of soil, rate of thawing, and loads, is proportional to the square root of time (Fig. 2a). At the moment of complete thawing of the specimen a bend occurs in the consolidation curve, indicating the change in the stress regularity. During the whole period of thawing a direct proportionality between settlement and depth of thawing is observed, and the value of the relative settlement in time remains practically constant (Fig. 2b).

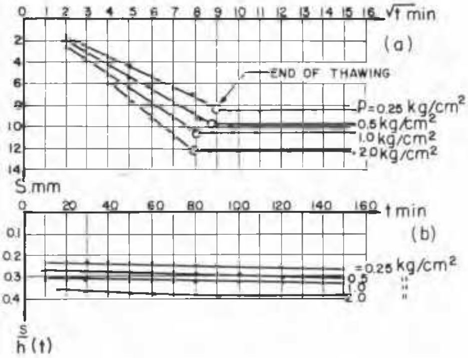

FIG. 2. Time settlement curves of thawing soils with different external loads: (a) in co-ordinates $S - \sqrt{t}$; (b) in co-ordinates $S(t)/h(t) - t$; soil-"kill" clay, $w_0 = 120$ per cent; settlement plate temperature $\theta = +20$ C.

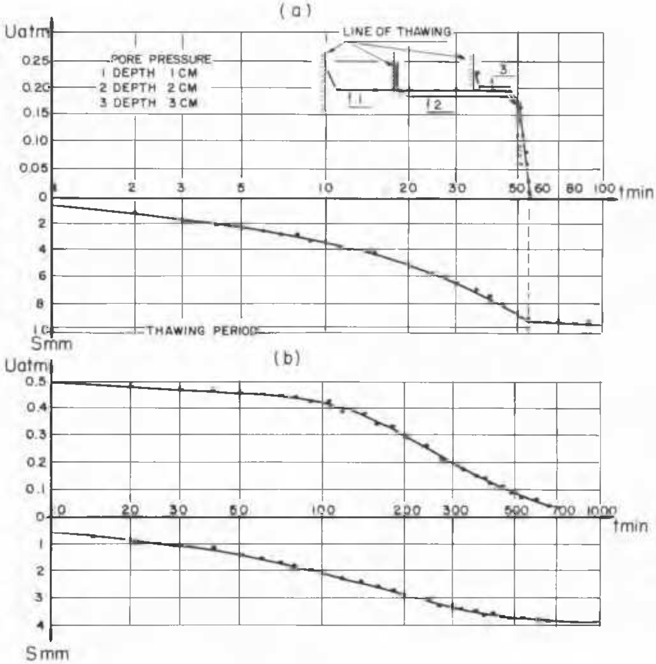

FIG. 3. Character of pore pressure dissipation in the process of consolidation and settlement with time (on semilogarithmic scale): (a) with thawing under load of 0.25 kg/sq.cm., piston temperature, $\theta = +40$ C.; (b) for unfrozen soil under load of 0.5 kg/sq.cm. "kill" clay, $w_0 = 120$ per cent.

It was shown experimentally that, with the thawing of the ice-saturated soils under load, the pore pressure in the thin layer at the thawing boundary attains the full value of the

TABLE I. PROPERTIES OF INVESTIGATED SOILS

Soil	Consolidation time, t (min)	External load, p (kg/sq. cm.)	Pore pressure, U (kg/sq. cm.)	Effective stress, $\bar{\sigma}$ (kg/sq. cm.)	$m = \bar{\sigma}/P$	Relative settlement, $S(t)/h(t)$	Void ratio, e_t	$\Delta e_t = e_0 - e_t$	$\Delta e_t/\bar{\sigma}$	Remarks
"Kill" clay	0	—	—	—	—	—	3.386			
$\theta = +40C$;	5	0.25	0.20	0.05	0.2	0.211	2.461	0.925	18.5	Thawing period
period of	25	0.25	0.20	0.05	0.2	0.219	2.426	0.960	19.2	
thawing,	50	0.25	0.20	0.05	0.2	0.238	2.340	1.016	20.9	
55 min;	55	0.25	0.20	0.05	0.2	0.245	2.311	1.075	21.5	
$w_0 = 117\%$	60	0.25	0.0	0.25	1.0	0.246	2.307	1.079	4.32	After thawing
	150	0.25	0.0	0.25	1.0	0.249	2.291	1.092	4.37	
"Kill" clay	0	—	—	—	—	—	3.425			
$\theta = +20C$;	15	0.5	0.35	0.15	0.3	0.210	2.496	0.929	6.19	Thawing period
period of	30	0.5	0.35	0.15	0.3	0.215	2.474	0.951	6.34	
thawing,	60	0.5	0.35	0.15	0.3	0.220	2.451	0.974	6.49	
100 min,	90	0.5	0.35	0.15	0.3	0.213	2.350	1.075	7.16	
$w_0 = 117\%$	100	0.5	0.35	0.15	0.3	0.241	2.345	1.080	7.20	
	110	0.5	0.0	0.5	1.0	0.249	2.323	1.102	2.20	After thawing
	1380	0.5	0.0	0.5	1.0	0.263	2.261	1.161	2.23	
Surface loam – clay	0	—	—	—	—	—	0.833			
$\theta = +20C$;	5	0.5	0.3	0.2	0.4	0.084	0.725	0.159	0.790	Thawing period
period of	15	0.5	0.3	0.2	0.4	0.089	0.715	0.168	0.840	
thawing,	30	0.5	0.3	0.2	0.4	0.099	0.697	0.186	0.930	
46 min;	45	0.5	0.3	0.2	0.4	0.103	0.689	0.195	0.970	
$w_0 = 31\%$	46	0.5	0.3	0.2	0.4	0.104	0.686	0.197	0.985	
	50	0.5	0.0	0.5	1.0	0.105	0.684	0.199	0.400	After thawing
	125	0.5	0.0	0.5	1.0	0.115	0.667	0.217	0.433	

θ, the temperature (deg. C) of the heater; $h(t)$, depth of thawing.

external load. Then, as the thawing boundary moves downwards a little further, the pore pressure in this layer quickly begins to decrease to a definite level, and during the whole period of thawing a practically constant pore pressure, a bit lower in value than the external load, is maintained in the whole melted zone (above the thawing boundary). Soil settlement in this period is intensive in spite of the low effective stress. At the time the specimen is completely thawed, the pore pressure quickly reduces to zero (Fig. 3a) and the strain develops the creep which is characteristic of the period of secondary consolidation. For the heavily iced soils the main settlement during thawing (92–98 per cent of the complete settlement) occurs while there are pore pressures, i.e., in the period of filtration consolidation.

The presence of a pore pressure in the melted zone of the thawing ice-saturated soils, the filtration coefficient of which is rather high, may be caused by the uninterrupted occurrence at the thawing boundary of a supermoistened layer with the maximum pore pressure, due to which the water head is maintained in the thawed layers of the soil. As is known, pore pressure dissipates slowly in unfrozen soils during the whole period of filtration consolidation (Fig. 3b), while in thawing soils, as shown above, filtration consolidation occurs practically with constant pore pressure which then dissipates quickly at the end of thawing.

In connection with the peculiarity of the stressed state of thawing ice-saturated soils, the process of their compressibility with time under the constant load has a somewhat unusual character, as is clear from Table I illustrating the main characteristics of consolidation of thawing specimens of "kill" clay and surface loam-clay.

ANALYSIS OF ONE-DIMENSIONAL CONSOLIDATION OF THAWING SOILS

In one-dimensional thawing under load of the ice-saturated soils, consolidation of the thawing layer occurs in compliance with the theory of filtration consolidation. The experiments described above have proved that at the boundary of thawing the super-moistened layer is maintained at all times, the mineral aggregates of this layer always being, as it were, in the suspended state, and that at the boundary of thawing ($Z = h(t), = \alpha\sqrt{t}$), the water receives all the external load. As a result of high filtration ability, the excess water is quickly removed and the pore pressure drops to some definite value under the present conditions. This value, as proved by experiments depends upon the type of the soil and constitutes a definite part of the external load. It can be assumed in calculations therefore (Fig. 4) that above the thawing boundary $z = h(t)$, where $\bar{\sigma} = 0$, some boundary $Z = L(t)$ exists, at which the effective pressures $R = R_0 + m\sigma_{z=L(t)}$ are equal.

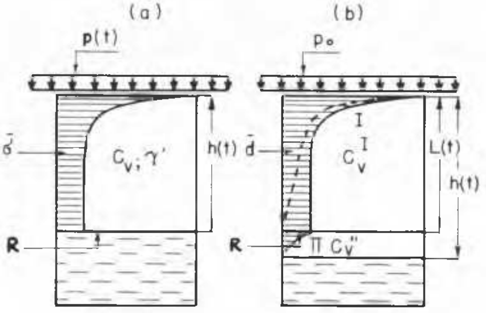

FIG. 4. Calculated diagrams of effective stresses in thawing soil: (a) without over-moistened layer; (b) with over-moistened layer.

Parameters R_0 and m are physical constants and can be determined experimentally. For instance, for the investigated soils $R_0 = 0$, and values of m are given in Table I. The layer of the thawed soils above boundary $Z = L(t)$ has a coefficient of compressibility* $\bar{a}_0$ and filtration k significantly lower than those which were characteristic for the soil immediately after thawing (in the region below boundary $L(t)$.

Two possible approaches can be considered in the analysis of one-dimensional consolidation of thawing soils. In the first the value of the region $[h(t) - L(t)]$ is neglected (Fig. 4a). The second approach presupposes the existence of two regions with significantly different coefficients of compressibility $\bar{a}_0$ and filtration k within the thawed layer (Fig. 4b).

First Approach

Let us assume the external load affecting the thawing bed to be increasing linearly with time:

$$P(t) = P_0 + nt. \tag{2}$$

Neglecting the value of the super-moistened layer and assuming $h(t) = L(t)$ we should assume that at the boundary $Z = h(t)$ the water does not receive the whole external pressure but a part of the complete stress σ_z. With $Z = h(t)$, the pressures affecting the skeleton in the general case equal

$$\sigma_{z\ h(t)} = R_0 + m(\gamma' h + P), \tag{3}$$

where R_0 and m are parameters, determined from the experiment. Taking into consideration that $\gamma_w \cdot u(Z, t) = \gamma' Z + P(t) - \bar{\sigma}$ the equation for determining the effective pressures in the thawing layer can be expressed as follows:

$$c_v(\partial^2\bar{\sigma}/\partial Z^2) = (\partial\bar{\sigma}/\partial t) \tag{4}$$

In this case the boundary conditions will be expressed as follows:

$$\left.\begin{array}{l} \text{with } Z = 0,\ \bar{\sigma} = P(t) = P_0 + nt \\ \text{with } Z = h(t),\ \bar{\sigma} = R = R_0 + m(\gamma' h + P_0 + nt). \end{array}\right\} \tag{5}$$

The initial condition is evident $h(t) = 0$ when $t = 0$.

The solution of Equation 4 with the boundary conditions (5) can be obtained from the equation below:

$$\bar{\sigma} = (P_0 + nt) - \frac{(1-m)P_0 - R_0}{\operatorname{erf}(\alpha/2\sqrt{c_v})} \operatorname{erf}(Z/2\sqrt{c_v t}) + m\gamma' Z + nZ^2/2c_v - \frac{(1 - m + \alpha^2/2c_v)nt}{N(\alpha/2\sqrt{c_v})} \times N(Z/2\sqrt{c_v t}), \tag{6}$$

where

$$N(Z/2\sqrt{c_v t}) = (Z/2\sqrt{c_v t})\exp(-Z^2/4c_v t) + \sqrt{\pi}(1/2 + Z^2/4c_v t)\operatorname{erf}(Z/2\sqrt{c_v t});$$

and $\quad \operatorname{erf}(x) = (2/\sqrt{\pi})\int_0^x \exp(-v^2)dv.$

In the particular case, when we neglect the influence of the weight of the soil and take $P = P_0 =$ const., the pore pressure will equal

$$u(Z, t) = (1/\gamma_w)[(1-m)P_0 - R_0] \times [\operatorname{erf}(Z/2\sqrt{c_v t})/\operatorname{erf}(\alpha/2\sqrt{c_v})] \tag{7}$$

Fig. 5, in compliance with formula 7, illustrates the curves characterizing the change in the pore pressure with movement of the thawing boundary depending on the values of the dimensionless parameter $\alpha/2\sqrt{c_v}$. The settlement of the bed thawing under the load can be easily expressed as follows:

$$S(t) = \bar{a}_0(D_1 P_0 \alpha\sqrt{t} + D_2 \alpha t + D_3 \alpha t^{3/2}), \tag{8}$$

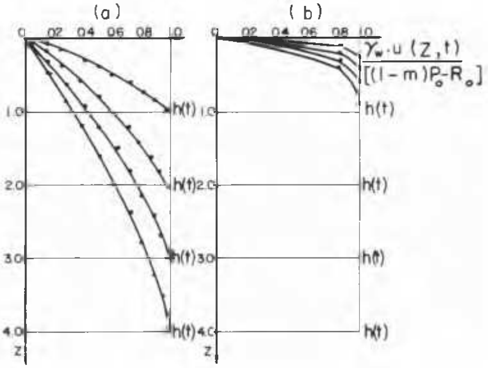

FIG. 5. Charts of reduced values of pore pressure $[\gamma_w/(1 - m)P_0 - R_0]u(Z,t)$ depending on dimensionless parameter $\alpha/2\sqrt{c_v}$; (a) with $\alpha/2\sqrt{c_v} = 1$; (b) with $\alpha/2\sqrt{c_v} = 10$.

where

$$D_1 = 1 - \frac{(1-m) - R_0/P_0}{\operatorname{erf}(\alpha/2\sqrt{c_v})}\left[\operatorname{erf}(\alpha/2\sqrt{c_v}) - \frac{1 - \exp(-\alpha^2/4c_v)}{\sqrt{\pi}(\alpha/2\sqrt{c_v})}\right],$$

$$D_2 = \gamma' m\alpha/2,$$

$$D_3 = 2n\left\{1/2 - [(1-m)/2]\left[1/3 + \frac{\sqrt{\pi}(\alpha/2\sqrt{c_v})\operatorname{erf}(\alpha/2\sqrt{c_v}) + \exp(-\alpha^2/4c_v) - 1}{3(\alpha/2\sqrt{c_v})N(\alpha/2\sqrt{c_v})}\right] - \frac{\sqrt{\pi}(\alpha^2/4c_v)\operatorname{erf}(\alpha/2\sqrt{c_v})}{3N(\alpha/2\sqrt{c_v})} - \frac{(\alpha/2\sqrt{c_v}\exp(-\alpha^2/4c_v) + (\alpha/2\sqrt{c_v})}{3N(\alpha/2\sqrt{c_v})}\right\}. \tag{9}$$

The result obtained shows that, neglecting the effect of the weight of the soils and with the constant external load, the relative settlement $S(t)/h(t)$ does not depend upon time. This conclusion agrees well with the experiment. The effect of the soil weight and the changes in time of the external load application greatly influence the character of occurrence of the settlement with time.

Second Approach

The problem in compliance with the second approach (Fig. 4b) can be easily considered under the stationary conditions at the boundaries $L(t)$ and $h(t)$, which equals the consideration of the problem by neglecting the weight of the soil and having a constant (in time) external load ($P = P_0$). The regularity of movement of the introduced

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* $\bar{a}_0$ is the compressibility coefficient of the soil in the process of its thawing under the load as compared with (a_0) used earlier which is the compressibility coefficient of the soil in a state of complete thaw.

boundary of division of the two regions $Z = L(t)$ should be assumed to be similar to the regularity of movement of the thawing boundary:

$$L(t) = \beta\sqrt{t} \tag{10}$$

Coefficient β will be determined later.

The determination of the effective pressures is reduced to the solution of the differential equation

in region I,

$$c_v(\partial^2\bar{\sigma}^{I}/\partial Z^2) = (\partial\bar{\sigma}^{I}/\partial t) \tag{11}$$

in region II,

$$c_v(\partial^2\bar{\sigma}^{II}/\partial Z^2) = (\partial\bar{\sigma}^{II}/\partial t). \tag{12}$$

The boundary conditions with $Z = L(t)$ and $Z = h(t)$ are established from the fact that the water appearing on the surface during thawing $Z = h(t)$ receives the whole external load, while at the boundary $Z = L(t)$ the water receives only a certain part of the external pressure. In other words:

$$\begin{aligned} &\text{with } Z = 0,\ \bar{\sigma}^{I} = P_0;\\ &\text{with } Z = L(t),\ \bar{\sigma}^{I} = R;\\ &\text{with } Z = h(t),\ \bar{\sigma}^{II} = 0. \end{aligned} \tag{13}$$

Solution of Eqs 11 and 12, taking into account the boundary conditions (Eq 13), gives the expressions for the effective stresses in both regions:

$$\left.\begin{aligned} &\text{within region I}\\ &\bar{\sigma}^{I} = P_0 - \frac{(1-m)P_0 - R_0}{\operatorname{erf}(\beta/2\sqrt{c_v^{I}})}\cdot\operatorname{erf}(Z/2\sqrt{c^{I}{}_{v}t})\\ &\text{within region II}\\ &\bar{\sigma}^{II} = (mP_0 + R_0)\cdot\frac{\operatorname{erf}(\alpha/2\sqrt{c_v^{II}}) - \operatorname{erf}(Z/2\sqrt{c_v^{II}t})}{\operatorname{erf}(\alpha/2\sqrt{c_v^{II}}) - \operatorname{erf}(\beta/2\sqrt{c_v^{II}})} \end{aligned}\right\}. \tag{14}$$

For the final solution of the problem it is necessary to determine the value of parameter β. This parameter can be found from the change in water content within region II. When $R_0 = 0$ and $m = 0$, i.e., when no consolidation occurs within region II and $\bar{\sigma}'' = 0$ (Fig. 4b, $\bar{\sigma}^{I}$ — dotted line), this condition will have the following expression:

$$(\beta/2\sqrt{c_v{}^{I}}) - (1 - w_{tot})\cdot(\alpha/2\sqrt{c_v{}^{I}}) = (1/\sqrt{\pi})\bar{a}_0{}^{I}P_0[\exp(-\beta^2/4c_v{}^{I})/\operatorname{erf}(\beta/2\sqrt{c_v{}^{I}})] \tag{15}$$

where $w_{tot} = w/(1 + w)$.

It can be said in conclusion that only the process of the filtration consolidation in the period of thawing is described here. It is necessary to add the value of the thermal settlement, $S(t) = 0.09\ w_{tot}i_0h(t)$ (where i_0 = relative ice content) occurring due to the reduction of the ice volume with the phase transitions.

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