Foundations Subject to Moment

Fondations soumises à des moments

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SUMMARY

A series of loading tests were carried out on model surface footings to investigate the moment-rotation relationship under conditions of constant eccentricity and constant centric load. The peak moment capacity was found to be slightly higher than the value derived from the Meyerhof concept of effective width. Predicted values ranged from 0.98 to 0.83 of recorded values for models of sizes 36 in. by 12 in. to 12 in. by 12 in. Contact pressure distributions along the short and long centre lines showed qualitative characteristics of a linear elastic supporting soil provided a plastic fringe adjustment was made. However, the quantitative comparison was not convincing. The effective width of all stages was more closely approximated by the subgrade modulus concept. Plastic strains were manifested at all stages including conditions of repeated loading. The final rotation was found to be dependent on the loading path. It was evident from these results that the simple linear elastic-plastic model was not fundamentally acceptable.

SOMMAIRE

Une série d'essais de charge a été effectuée sur des modèles d'empattements de surface afin d'étudier les rapports entre les rotations dues aux moments dans les conditions d'excentricité constante et de charge concentrique constante. On a constaté que le moment maximal était légèrement supérieur à la valeur dérivée du concept de largeur effective selon Meyerhof. Les valeurs anticipées s'étendaient de 0.98 à 0.83 des valeurs enregistrées pour des modèles variant de 36 par 12 pouces à 12 par 12 pouces. Les distributions des pressions de contact le long des lignes centrales courtes et longues ont démontré des caractéristiques qualitatives d'un sol d'appui aux propriétés linéaires élastiques, à condition qu'on introduise une correction pour la zone plastique périphérique. Cependant, la comparaison quantitative n'était pas convaincante. La largeur effective de tous les étages se trouvait approchée de plus près par le concept du module de réaction. Des déformations plastiques se manifestaient à tout étage, y compris les conditions de chargement répété. On a constaté que la rotation finale dépendait de la succession dans laquelle on faisait appliquer les charges. En raison de ces résultats il est devenu évident que le simple modèle élastique-plastique linéaire n'est pas acceptable en principe.

THE NEED FOR TREATING THE STRUCTURE and supporting soil as a single compatible unit in both the elastic and plastic methods of design and analysis has been recognized for a considerable period. Existing analytical methods can be applied to the structure provided the load-deformation and the moment-rotation relationships of the foundation units are established. However, the derivation of a theoretical relationship is dependent on a knowledge of the fundamental stress-strain law for the soil. At the present time such a law has not been satisfactorily established although recent investigations have made significant contributions (Rowe, 1962; Roscoe, *et al.*, 1963).

The engineering necessity for predicting soil deformations and stress distributions has forced the use of existing mathematical models, notably the Hookean model, the visco-elastic model, and the elastic-plastic model. Although the elastic model is supported by principal stress measurements within a loaded soil mass, plastic strains are generally manifested and the fact that the total strains cannot be satisfactorily predicted by the theory indicates it is not a fundamentally acceptable concept. Empirical adjustments can be made on the basis of experimental work and the usefulness of the concept depends on the degree of approximation introduced.

The investigations described in this paper were aimed at establishing the moment-rotation relationships of a rigid footing on densely packed sand. This allowed a comparison to be made with the elastic theory on the basis of contact stress distribution, rotation produced by a specified force-moment combination, effective width, and stress history. As the tests were continued until failure was evident it was also possible to compare predicted and measured peak load or moment values.

EXPERIMENTAL APPARATUS

A mechanism was designed to apply moment and centric vertical force independently to the model footings (Lee, 1961). Controlled deformations and rotations were applied, the resulting force system being recorded by a centric load cell and vibrating wire dynamometers within the tension and compression members of the mechanism. Friction within the linkage system was reduced by the use of knife edges. Calibration tests showed that the difference between the tensile and compressive forces did not exceed 50 lb.

The mechanism was assembled on the model footing before transferring the model to the prepared supporting mass of soil. By initially mounting the model on three load cells centricity of the central load was tested and adjusted, and the magnitude and direction of the applied moment was measured. These investigations showed that it was unnecessary to calibrate the unit before each test and assembly was usually carried out with the aid of two theodolites using a suitable reference line on the rigid base of the mechanism.

Effects of geometric changes on the loading system during tests were fully investigated and calibration charts prepared to enable the immediate calculation of the force system for any defined distortion. Two types of pressure cells were used to record the contact normal stress—the vibrating wire boundary type (Lee, 1960) and the electrical resistance (Trollope and Lee, 1961). In some tests the latter cells were also placed within the soil at a depth of one in. from the surface.

In all tests the supporting soil was a dry dense sand (Trollope and Lee, 1961) contained in a fabricated box of plan dimensions 6 ft by 6 ft with a total depth of 5 ft. After repeated trials it was established that a mean density to within $\pm$ 1 per cent could be developed by placing the sand in 4-in. to 6-in. layers and vibrating for a fixed period with a plate vibrator. Vibration was carried out on a grid system. The final surface was levelled by scraping then revibrating.

The results presented in the paper were obtained with 1⅛-in.-thick steel models of plan dimensions 13 in. by 36 in., 12 in. by 36 in., 12 in. by 24 in., and 12 in. by 12 in. Relative deformations of the models were small compared with surface displacement of the soil. Further tests on footings of greater flexibility are not considered in this paper.

EXPERIMENTAL RESULTS

Peak Moment Capacity

Meyerhof (1953), recognized that the conventional centric load solution for the rigid-plastic soil model could be applied to the case of a non-centrically loaded surface footing if the width was reduced by twice the resultant eccentricity. For a cohesionless soil the ratio of the peak moment, M, for any specified centric load, P, to the maximum possible peak moment the supporting soil can withstand, M_m, is given by the expression

$$M/M_m = 27/4 \cdot P/P_c[1 - \sqrt{(P/P_c)}] \qquad (1)$$

where P_c is the peak centric load when there is no moment applied. Fig. 1 is a dimensionless plot of M/M_m *versus* P/P_c showing the Meyerhof relationship compared with the experimental values. There was no significant difference between the peak moment capacity established under constant eccentricity conditions and under constant centric load conditions.

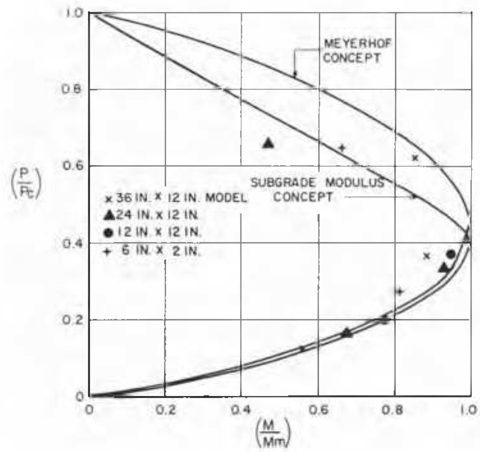


FIG. 1. Plot of P/P_c *versus* M/M_m for model footings.

Use of the subgrade modulus concept based on a limiting stress condition leads to the relationships:

For $1 \geqslant P/P_c \geqslant \frac{1}{2}$ or $1 \geqslant e/B \geqslant \frac{1}{6}$,

$$M/M_m = 16/9(1 - P/P_c). \qquad (2a)$$

For $0 \leqslant P/P_c \leqslant \frac{1}{2}$ or $\frac{1}{6} \leqslant e/B \leqslant 0$

$$M/M_m = 16/3 \cdot P/P_c(1 - 4/3 \cdot P/P_c). \qquad (2b)$$

where e denotes the eccentricity.

An average P_c was established by centric load tests. Recorded peak values corresponded to ϕ' of 43° to 44° compared with triaxial values of 42° to 44°. Although it was not possible to make final conclusions from this correlation due to the sensitivity of the bearing capacity factor to ϕ' within the range considered and to the differences in stress state, it appeared that the effect of side restraint imposed on the supporting soil was of secondary importance. It is evident that any edge effects were less significant with moment tests since the effective width was reduced.

The experimental values agreed quite closely with predicted values up to the maximum moment ratio. The absolute values of peak moment were underestimated by Meyerhof (1953), Fig. 2. It will be noted that, as the theoretical plot of Fig. 2 is based on the experimental value of P_c, any side effects during these tests would elevate the theoretical moment values, so that the moment capacity predicted by Meyerhof could be more conservative than that suggested by the results plotted in this figure. The ratio of predicted to measured maximum moment was 0.98 for the 36-in. model, 0.89 for the 24-in. model, and 0.83 for the 12-in. model.

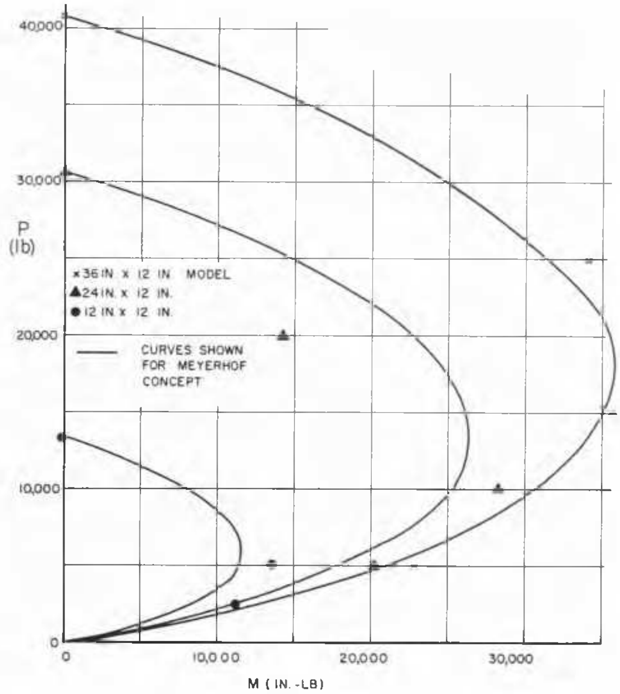


FIG. 2. Plot of P *versus* M.

The experimental evidence was too limited to finally evaluate the validity of the concepts for the P/P_c range in excess of one-half but it appeared that the Meyerhof concept led to an overestimate of the moment ratio. A failure surface became evident on the compressive side of the model at a rotation of the order of 3°. For a typical test (5,000 lb centric load) the first surface was at a distance of 5 in. from the edge of the model. When the footing was rotated further, an extra failure surface was developed on either side at a distance of approximately 10 in. from the edge.

Distribution of Normal Contact Stress

Typical contact stress distributions along the short centre line are shown in Fig. 3. A comparison between the distributions developed by a constant centric load sequence and a constant eccentricity load sequence showed that there were no significant differences for the same load-moment combination. Such a comparison is shown in Figs. 3(a), 3(b), and 3(c).

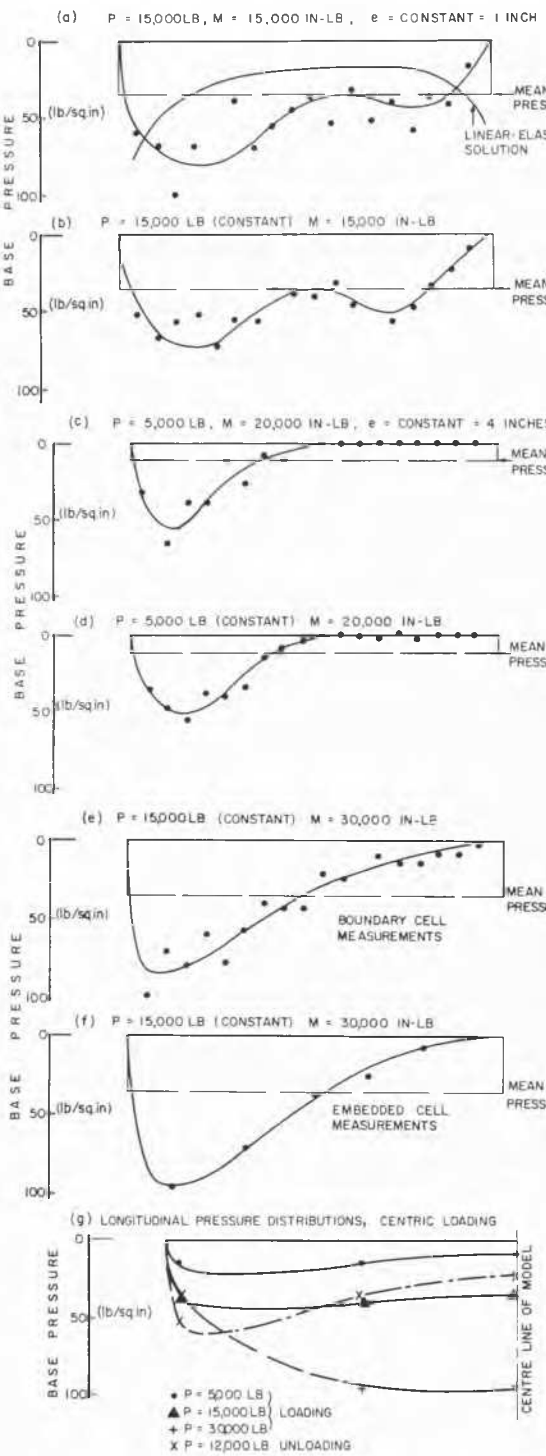


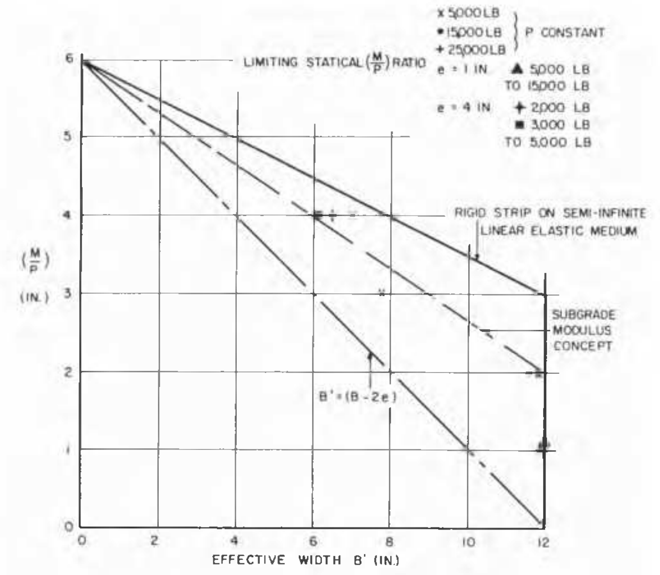
FIG. 3. Base pressure distributions for constant centric force and constant eccentricity.

The typical scatter of individual boundary pressure cell readings was increased in the present test series due to the difficulty of establishing a prepared soil surface sufficiently level to ensure that the base of the model and all cell diaphragms were initially in uniform contact with the soil. The relative scatter decreased with increasing moment or load. When the cell readings of two equivalent tests were averaged the scatter was within the range of ± 5 lb/sq.in. Readings plotted in Fig. 3 were not averaged.

Experience had shown that the scatter of cell readings was less with the embedded type of cell (Trollope, Lee, and Morris, 1962). This was confirmed by the present test results, as shown, for example, in Fig. 3(f), which is a plot of the readings recorded by a series of cells embedded one inch below the contact surface. These values are compared with the boundary cell readings.

A comparison of the pressure diagrams under repeated loading sequences investigated showed that a specific force-moment combination produced base pressure distributions essentially equivalent to the virgin distributions. As this observation suggests elastic behaviour of the supporting soil it is relevant to investigate whether the elastic model is supported by further experimental evidence.

The elastic normal stress distribution is plotted in Fig. 3(a). It has been well established by experiment that the edge stresses are very small for a granular supporting soil but even if a plastic failure "correction" (Schultze, 1958) is applied the measured and theoretical ordinates are not comparable, although the general distribution is similar in some respects. Even for a L/B ratio of 3 the average pressure across the short centre line (Fig. 4) was considerably



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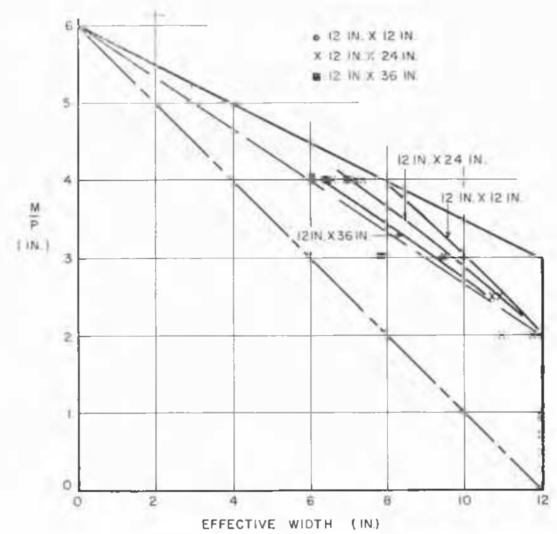

B

FIG. 4. Effective width plots.

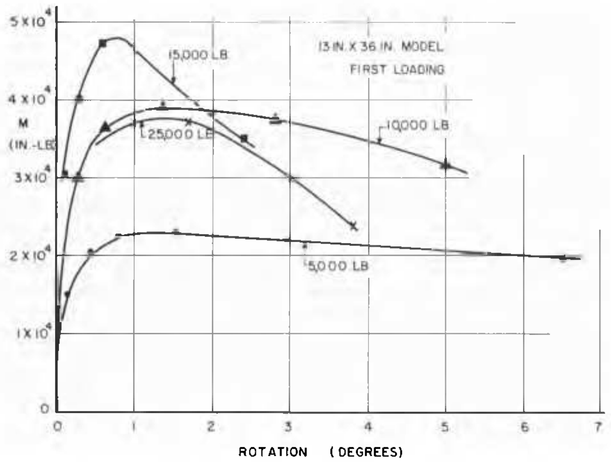

FIG. 5. Moment rotation relationship for 13 in. by 36 in. model. Constant centric load tests.

greater than the over-all average, a feature not predicted by the elastic concept.

Stress distributions along the longitudinal centre line are shown in Fig. 3(g). These are similar to the distributions across the width under centric loading and exhibit a very uniform distribution or slightly void type of distribution except at high P/P_c ratios. The greater concentration of stress towards the central region is assisted by the model deformation despite the fact that differential deflection is only of the order of 0.06 in. over a distance of 18 in. Such small differential deflections were particularly significant on unloading as shown in Fig. 3(g) where the loading and unloading longitudinal distributions are compared.

Measurements of the contact stress distributions across the width of the model at sections away from the centre line indicated that the distributions were geometrically similar. By accepting this observation it was then possible to evaluate the total volume of the pressure diagram and the position of the centroid thus providing a statical check on the experimental results.

The centroid was within $\pm$ 0.2 in. of the theoretical position of M/P from the centre line. The total volume of the diagram varied between 1.0 and 1.10 of the recorded load. The over-registration could have been due to the lack of precision in the pressure ordinates near the edges of the models where the stress gradients were relatively steep.

Figs. 4A and 4B show the variation of effective width with load eccentricity. Although the effective width was essentially independent of the loading history there was a significant difference between predicted and measured values. Complete contact was maintained up to a M/P ratio of 2 instead of approximately 3 and at all stages the effective width was less than the linear-elastic value. Fig. 4B is a summary of the results for three width-length ratios.

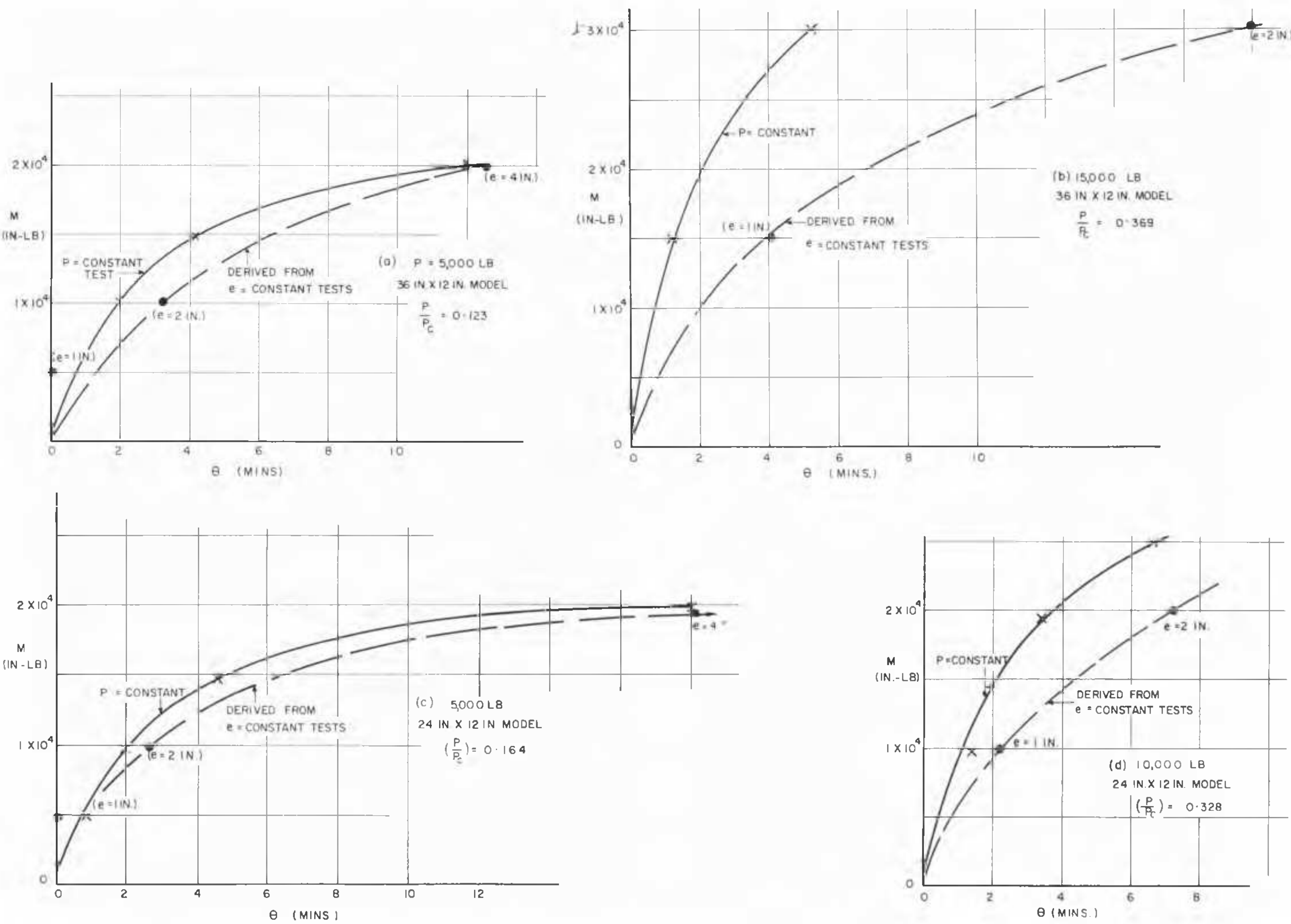

FIG. 6. Moment rotation relationships for fifth loading. Constant centric load tests and constant eccentricity tests.

Moment-Rotation Relationships

Typical experimental results are shown in Fig. 5 for the 13 in. by 36 in. model. When a constant centric load was applied there was a significant plateau at low P/P_c ratios, but, at higher ratios an unstable type of moment-rotation curve was manifested. Peak moment capacity was developed at rotations between 0.75 and 1.5°. In all tests the models were prevented from slipping laterally. Without such restraint failure would occur at much lower moments than it does with full side restraint.

Elastic strain behaviour was not exhibited in any test. At all load levels there was an irrecoverable strain component. A comparison of the rotations developed by different loading paths showed that the final rotation was significantly affected by the loading history. The greatest absolute differences obviously occurred under first loading but as shown in Fig. 6 linear elastic strain behaviour was not developed under repeated loading although the plastic strain component was progressively diminished with increasing load repetitions unless an incremental failure occurred. The results plotted in Fig. 6 corresponded to the fifth increase of moment in the constant P tests, and to the fifth increase of the equivalent eccentric load in the constant eccentricity tests. Individual experimental points obtained from constant eccentricity tests of 1 in., 2 in., and 4 in. were superimposed on the moment-rotation relationship for constant centric loads.

It was evident from these results that the rotation produced by a specific moment was influenced by the mean principal stress. When the mean principal stress during the moment application was smaller, as with the constant eccentricity tests compared with the constant centric load tests, the rotation per unit moment was increased. This typical non-linear characteristic is in contradiction to the linear theory.

To establish the degree of approximation of the linear elastic theory it is possible to compare the calculated unloading modulus (Trollope, Lee, and Morris, 1962) with unloading triaxial test modulus values. A typical group of results is plotted in Fig. 7 based on the equation (Lee, 1962)

$$E/(1 - \nu^2) = P/BL \cdot e/B \cdot I_s/\tan\theta \qquad (3)$$

for $e/B \leqslant \frac{1}{4}$ and similar equation can be derived for greater eccentricities. The influence factor I_s was considered to be equal to the rigid footing value for the particular L/B ratio.

Whereas tests on centric loaded footings established a rebound modulus which was virtually independent of the stress level, the moment tests showed that the rebound and reload moduli both decreased with centric load intensity. There was no correlation between the calculated values and the triaxial rebound values which were found to be 60,000 lb/sq.in. to 80,000 lb/sq.in.

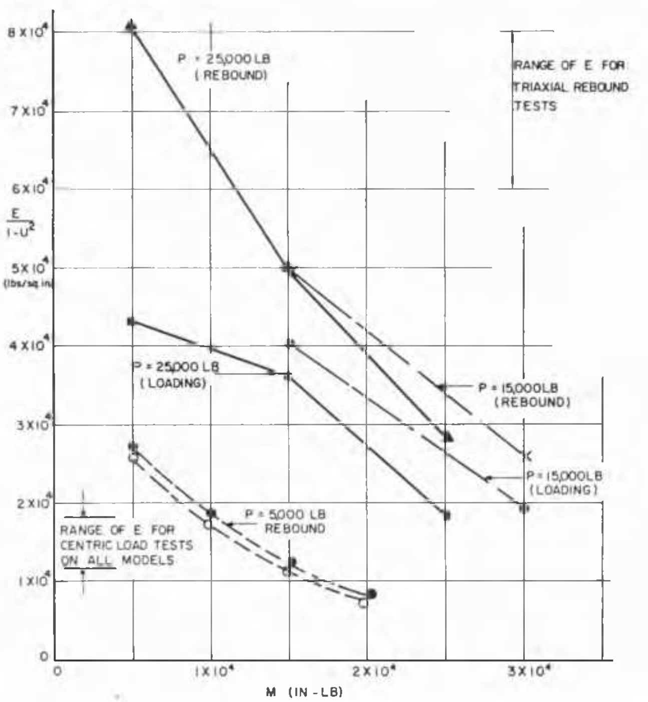

FIG. 7. Rebound and reloading moduli for moment increase and decrease under constant centric load.

CONCLUSIONS

The experimental evidence in this paper does not support the linear elastic model for surface footings on a dense sand except on the basis that the contact stress distribution is effectively independent of the loading history. Non-linear strain behaviour was manifested and at all stress levels there was a plastic component of strain even after several loading cycles.

The Meyerhof effective width concept leads to a reasonable but conservative prediction of the ultimate moment capacity despite the fact that the effective width and failure surfaces are not those assumed by Meyerhof.

It was evident that the simple mathematical models do not apply to granular soils and that a solution of this problem, in common with any stability problem, can only be achieved from a knowledge of the fundamental stress strain law.

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