On the Stability of Strip Foundation Footings

Stabilité des semelles de fondations continues

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SUMMARY

This paper sets forth the results of theoretical investigations of the bearing capacity of strip foundation footings having rectangular and downward, wedge-shaped, tapering cross-sections in soils possessing internal friction and little cohesion.

Based on the law of mechanical similarity, the solution of the problem of the bearing capacity of a foundation with a non-symmetrical lateral load with respect to the vertical axis of the foundation is reduced to the solution of a symmetrical problem. For wedge-shaped foundations, tables of non-dimensional parameters are computed with the help of which it is possible to calculate rapidly the bearing capacity of the foundation taking into account the bulk forces.

A solution without taking into account the bulk forces is given in a closed form. The difference in the values of the bearing capacity, with and without taking into account the bulk forces, for a flat foundation does not exceed 10 per cent. Therefore, in a number of cases the bearing capacity of such a foundation can be determined without taking into account the bulk forces.

SOMMAIRE

Le présent mémoire expose les résultats de recherches théoriques pour le calcul de semelles continues à section rectangulaire, cunéiforme et trapézoïdale rétrécie vers le bas, sur des sols granulaires à faible cohésion.

En partant de la loi de la similitude, l'auteur réduit la solution du problème de la capacité portante de la fondation soumise à une charge inclinée asymétrique à celle d'une fondation chargée d'une façon symétrique. Dans le cas d'une semelle cunéiforme, l'auteur donne des paramètres permettant un calcul rapide de la portance de la fondation, compte tenu des forces volumétriques.

La différence obtenue entre ces deux solutions dans le cas d'une fondation plate ne dépasse pas 10 pour cent. Il est donc conclu que dans plusieurs cas la portance de telles fondations peut se calculer en négligeant les forces volumétriques.

THE PRINCIPAL PROBLEM in the theory of the stability of foundations is their bearing capacity. In this paper the results of investigations concerning the bearing capacity of strip foundations are given. These investigations are based on the solution for a foundation of wedge-shaped transverse profile. From this a solution for a foundation with a flat toe is obtained as a particular case. Foundations with a flat toe are a commonly accepted solution, whereas the wedge-shaped, trapeziform foundations are nowadays mainly employed in buildings located over excavations (Bureau of Technical Assistance, 1958).

The solutions as set forth below refer to the case when the foundation is rigidly connected with the building frame and cannot turn freely. Therefore, the moment when the foundation loses stability is preceded by the moment when the soil on both sides of the foundation reaches the limiting stress condition. The bending moment, which occurs here as a result of embedding the foundation in the superstructure, is balanced by the moment of the soil reaction pressure on the inclined faces of the foundation.

We shall make use of the idea of the reduced stress, which is the vector sum of the actual stress and the normal compressive stress,

$$H = K \cot \phi, \tag{1}$$

where K and ϕ are the coefficient of cohesion and the angle of the internal friction of soil respectively. The vector of the reduced stress p forms an angle ϕ with the normal to the area on which it acts. Its normal and tangential components are equal to

$$\left.\begin{aligned} \sigma_n + H &= p \cos \phi \\ \tau_{nt} &= p \sin \phi. \end{aligned}\right\} \tag{2}$$

From these the well-known condition of the limiting equilibrium of a loose medium is obtained (Sokolovsky, 1954):

$$\tau_{nt} = (\sigma_n + H) \tan\phi. \tag{3}$$

If we apply this reasoning to the contact plane, we obtain

$$\tau_{nt} = -(\sigma_n + H) \tan \delta_0, \tag{4}$$

where σ_n and τ_{nt} are the normal and tangential stresses on the surface of contact of the foundation with the soil, and δ_0 is the angle between the reduced stress and the normal to the contact line.

Sometimes the relation

$$\tau_{nt} = -\sigma_n \tan \delta \tag{5}$$

is taken as the condition of the interaction of the foundation with the soil in the contact plane. Here the designations are the same as before, with the exception of δ, which designates the angle of friction between the foundation and the soil. It should be noted that the acceptance of condition (4) will make the solution much simpler. For instance, in the case where the bulk forces (the weight of the soil itself) are not taken into account, the solution based on condition (4) is obtained in a closed form. The results of such a solution are given at the end of the paper.

However, it is impossible to make use of condition (4), since no experimental or theoretical data about the value of angle δ_0 are available. Therefore, the following method for obtaining the solution is suggested. First, condition (4) is taken as the condition of interaction of the foundation with the soil in the contact plane, angle δ_0 in condition (4) being

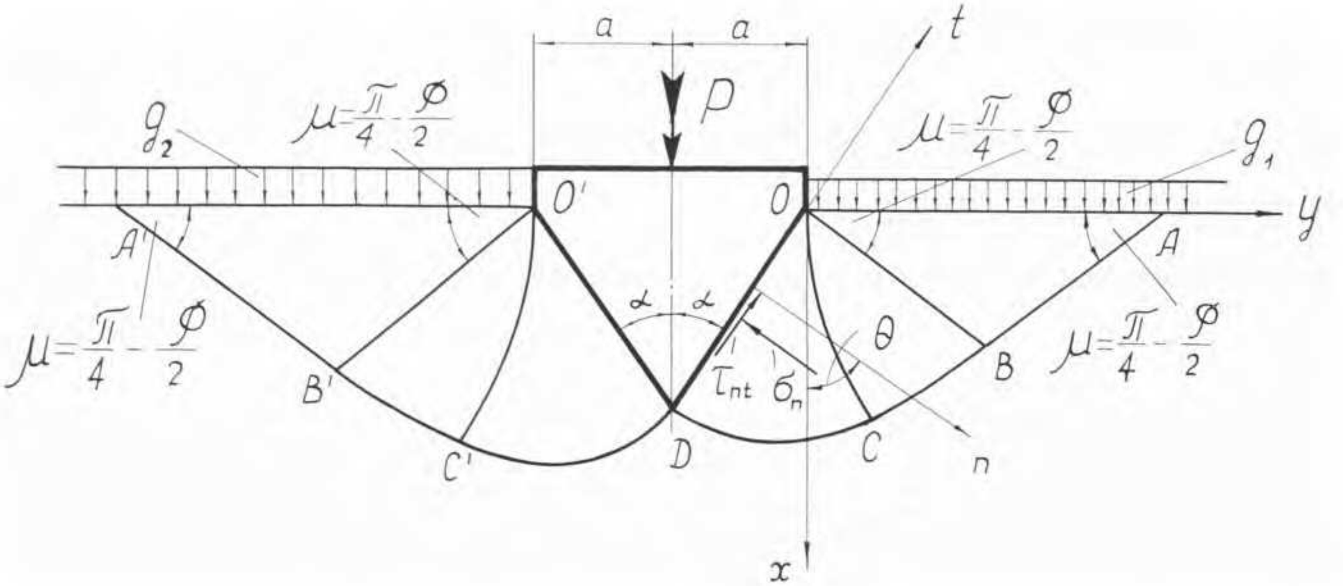

FIG. 1. Computation scheme for strip wedge-shaped foundation.

assumed to be equal to the angle δ of internal friction between the foundation and the soil:

$$\tau_{nt} = -(\sigma_n + H)\tan\delta. \tag{6}$$

Having determined the contact normal stresses σ_n we find the tangential stresses τ_{nt} by multiplying the normal stresses by the coefficient of friction, i.e. by $\tan\delta$. Further, in determining the bearing capacity of the foundation, we assume that stresses σ_n and τ_{nt} are applied to it. The condition $\delta \leqslant \phi$ is imposed on the solution, otherwise the shifting of soil over soil rather than the shifting of the foundation over soil will take place, in case the equilibrium is disturbed. This suggestion makes it possible to obtain a solution in which the numerical results differ from those of other solutions by approximately 5 per cent, which satisfies the accuracy of engineering calculations.

In the solution given below, the friction between the soil and the vertical faces of the foundation is neglected and the weight of the soil located above the wedge-shaped part of the foundation is considered as a uniformly distributed lateral load. The solution is given both with and without taking into account the bulk forces (the weight of the soil itself). First, we shall present the solution in which the bulk forces are taken into consideration. In the general case the bearing capacity of the foundation can be found by determining the passive soil pressure for the retaining wall.

Let us suppose there is a uniformly distributed lateral load of intensity g_1 on the right of the foundation and of intensity g_2 on the left of the foundation (Fig. 1). Then considering the inclined face of the foundation as a retaining wall acted upon by the passive pressure, we calculate the pressure for it twice—once taking the intensity of the lateral load g_1 and the second time taking the intensity g_2. By summing up the two solutions we shall obtain the solution of the problem under consideration.

The normal stresses on the rear face of the retaining wall can be represented (Sokolovsky, 1954) in the form

$$\sigma_n = \sigma_r - H, \tag{7}$$

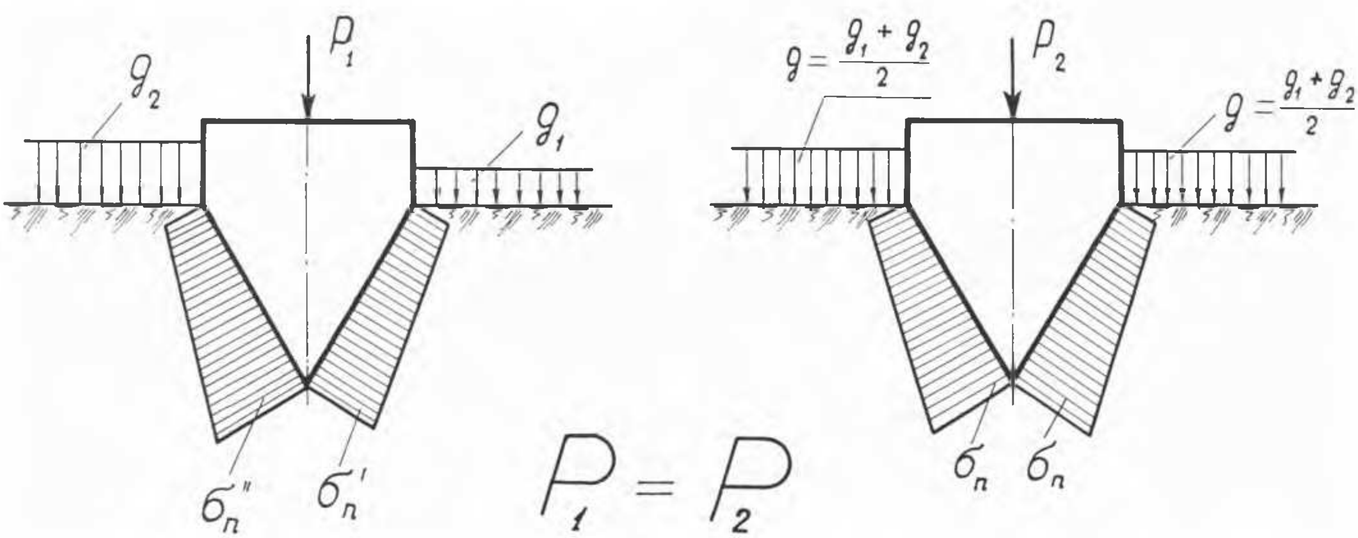

FIG. 2. Normal contact stresses with non-symmetrical and symmetrical lateral load.

in which

$$r = \cos\delta[\cos\delta + \sqrt{(\sin^2\phi - \sin^2\delta)}]. \qquad (8)$$

Here σ is the function to be determined. The tangential stresses on the contact surface are found according to Eq 5.

Two favourable conditions were established on the basis of the investigations carried out. The first favourable condition is that function σ varies linearly along the inclined face of the foundation. This made it possible to simplify the numerical solution of the loose medium statics (Sokolovsky, 1954) considerably and to find the value σ only at two points of the contact line, e.g. at points O and D (Fig. 1). In so doing, the usual calculations are cut down many times.

The second favourable condition is as follows. Suppose we have two foundations: one foundation with a non-symmetrical lateral load of intensity g_1, acting on the right side of the foundation and of intensity g_2 on the left side, and one foundation with a symmetrical load of intensity $g = (g_1 + g_2)/2$ (Fig. 2). It is clear that the reactive pressures of the earth on the inclined faces of the foundation with symmetrical and non-symmetrical lateral loads will be different, but as it happens, the bearing capacity of both foundations is the same. This condition makes it possible to shorten the solution considerably in the case of non-symmetrical lateral load, by changing it for the solution of a symmetrical problem.

The results obtained have made it possible to compute tables of non-dimensional parameters which help to calculate the bearing capacity of the wedge-shaped foundation in 15 to 20 minutes. The detailed method (Sokolovsky, 1954) used to take one calculator not less than eight days to carry out this task, but now, using the simplified method, one calculator will require only about 5 hours to complete the solution. The tables have been calculated on the grounds of a developed simplified method (Mintskovsky, 1960) on an electronic calculating machine (Ural) in the Calculating Centre of the Academy of Sciences of the Ukrainian Soviet Socialist Republic.

The bearing capacity of the foundation is determined with the help of the table according to the formulae

$$\left.\begin{aligned} P &= 2a\{[(g+H)/(1+H)]K\sigma_{\mathrm{av}}r - H\} \\ &\qquad \times (1 + \tan\delta.\cot\alpha), \\ r &= \cos\delta[\cos\delta + \sqrt{(\sin^2\phi - \sin^2\delta)}], \\ \sigma_{\mathrm{av}} &= [\gamma a(\sigma - \sigma_0)/2ky] + \sigma_0. \end{aligned}\right\} \qquad (9)$$

The designations used in these formulae are: P—the bearing capacity of the foundation, a—half-width of the foundation, α—half of apex angle of the foundation, γ—the bulk weight of the soil, g—the average value of the intensities of the lateral loads acting on the right and on the left of the foundation (Fig. 1), i.e. $g = (g_1 + g_2)/2$. The value H is determined according to Eq 1. The rest of the designations are as before. The values σ_0, σ, and y are given in the tables compiled by the author and included in his research work. The tables are computed for different values of the coefficient of cohesion of the soil, the internal angle of friction of the soil, the angle of slope of the foundation face to the vertical, and the coefficient of contact friction.

For foundations with obtuse angles (120° to 180°) at the apex the cited solution was inappropriate, since accuracy of calculation is lost during the computation. Therefore, for foundations with a flat toe the bearing capacity is determined with the help of detailed calculations on the grounds of the general solution (Sokolovsky, 1954), which, naturally, makes the calculations complicated. Also, investigations show that the value of the bearing capacity of a flat foundation calculated on the basis of the general solution (Sokolovsky, 1954), taking into account the bulk forces, differs from the corresponding value calculated on the basis of the solution cited below, without taking into account the bulk forces, by not more than 10 per cent. Therefore, in many cases it is expedient to compute the bearing capacity of a foundation with a flat toe on the basis of the solution, without taking the bulk forces into account.

In this paper, applying the suggested scheme of calculation, in which Eqs 5 and 6 are used, we obtain the solution of the problem in a closed form, which makes the calculations much simpler. The corresponding solution is presented below. Investigations have shown that when the foundation is pressed into the soil, the failure zone is of the form shown in Fig. 3.

We shall make use of the functions presented by Sokolovsky (1954).

$$\left.\begin{aligned} \xi &= \tfrac{1}{2}\cot\phi\ln\sigma + \rho, \\ \eta &= \tfrac{1}{2}\cot\phi\ln\sigma - \rho, \end{aligned}\right\} \qquad (10)$$

$$\sigma = (\sigma_1 + \sigma_2)/2 + H, \qquad (11)$$

in which σ_1 and σ_2 are the main normal stresses, σ_1 being accepted as $> \sigma_2$, and ρ is the angle between axis x and main axis 1.

The stress components in the co-ordinate system nt, which is turned by an angle θ (Fig. 1) with regard to the co-ordinate system xy are as given in

$$\left.\begin{aligned} \sigma_n &= \sigma[1 + \sin\phi\cos 2(\rho - \theta)] - H, \\ \sigma_t &= \sigma[1 - \sin\phi\cos 2(\rho - \theta)] - H, \\ \tau_{nt} &= \sigma\sin\phi\sin 2(\rho - \theta). \end{aligned}\right\} \qquad (12)$$

Now we proceed to the determination of the contact stresses. For purposes of reference we shall consider the right inclined face of the foundation OD and the horizonal border of the soil OA. First, we shall assume that the soil border OA is free from external load (Fig. 3). In the process of pressing in the foundation, the soil tends to bulge at the horizontal border OA. Therefore compressive stresses acting horizontally are applied to the soil element at this border. Bearing in mind that the compressive stresses (Sokolovsky, 1954) are given the sign +, we shall have $\sigma_y > \sigma_x$. Since, in addition $\tau_{xy} = 0$ and $\sigma_x = 0$ along the border OA, on the basis of equalities (12) along this border there should be

$$\sigma = H/(1 - \sin\phi), \quad \rho = \pi/2 + m\pi, \qquad (13)$$

in which m is a whole number. In order to meet the condition $\sigma_y > \sigma_x$, we should take $m = 0$. Therefore, substituting (13) in (10) we shall obtain

$$\xi = \tfrac{1}{2}\cot\phi\ln[H/(1 - \sin\phi)] + \pi/2. \qquad (14)$$

Further, as agreed above, we shall make use of Eq 6. Substituting the value τ_{nt} according to (6) and (12) and excluding first σ then ρ from the first and the third of these formulae we shall find

$$\left.\begin{aligned} \rho &= \theta - \frac{\delta}{2} - \frac{1}{2}\arcsin\frac{\sin\delta}{\sin\phi} + m\pi, \\ \sigma &= \frac{\sigma_n + H}{\cos^2\phi\cos\delta}(\cos\delta \pm \sqrt{\cos^2\delta - \cos^2\phi}). \end{aligned}\right\} \qquad (15)$$

For the problem under consideration we should take $m = 0$

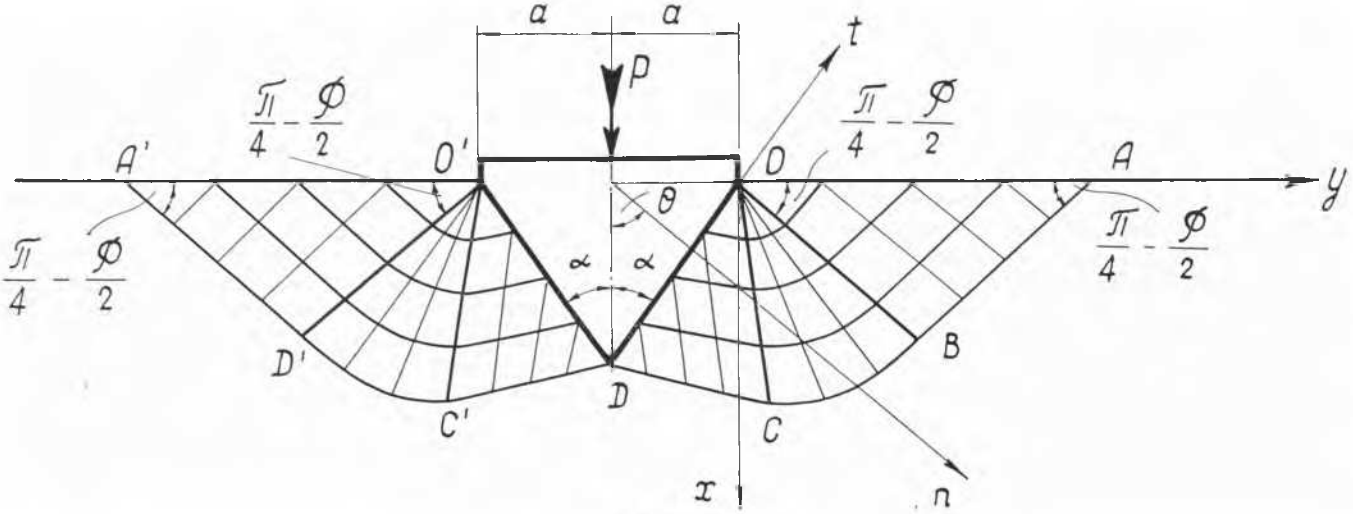

FIG. 3. Failure zones.

and assume the radical as having the sign minus. Then substituting the values ρ and σ in the first formula (10) we shall obtain the value ξ at the contact line

$$\xi = \frac{\cot\phi}{2}\ln\sigma + \theta - \frac{\delta}{2} - \frac{1}{2}\text{arc sin}\,\frac{\sin\delta}{\sin\phi}. \qquad (16)$$

Following Mintskovsky (1952) we shall assume that the bearing capacity of the soil is exhausted and that the value σ_n has reached the critical value along the entire length of the contact line, and is constant. Then, on the basis of Eqs 12 and 15 we conclude that the function σ also has a constant value along the contact line. Similarly, from Eq 13 we conclude that function σ has a constant value near the horizontal surface of the earth OA (Fig. 1) as well. On these grounds it follows from Eqs 14 and 16 that the functions ξ in the regions OAB and ODC have constant values.

To determine the contact stresses we shall make use of the condition that the functions σ and ρ should be continuous in the entire region ABCDO with the exception of point O, and for this it is necessary and sufficient that the functions ξ, which are constant in the regions OAB and ODC are equal to one another.* Therefore comparing Eqs 14 and 16 and substituting $\theta = \pi/2 - \alpha$, we obtain,

$$\sigma = \left[\frac{H}{1-\sin\phi}\right]\cdot e^{\left[2\alpha + \delta + \text{arc sin}\,\frac{\sin\delta}{\sin\phi}\right]\tan\phi} \qquad (17)$$

If the symmetrical lateral load of intensity g is taken into consideration, the formula for σ takes the form

$$\sigma = \left[\frac{g+H}{1-\sin\phi}\right]\cdot e^{\left[2\alpha + \delta + \text{arc sin}\,\frac{\sin\delta}{\sin\phi}\right]\tan\phi} \qquad (18)$$

*At the point O the stress components suffer final breaches (Sokolovsky, 1954).

The normal stresses are found according to the first formula of Eq 12 in which ρ should be substituted, according to Eq 15 with $m = 0$. In this way we obtain

$$\sigma_n = \sigma\left[1 + \sin\phi\cos\left(\delta + \text{arc sin}\,\frac{\sin\delta}{\sin\phi}\right)\right] - H. \qquad (19)$$

Further, in accordance with the computation scheme presented at the beginning, the tangential stresses are found in the following way:

$$\tau = -\sigma_n\tan\delta. \qquad (20)$$

And finally, from the equilibrium condition we find the bearing capacity of the foundation

$$P = 2a\sigma_n(1 + \tan\delta.\cot\alpha), \qquad (21)$$

in which $2a$ is the width of the foundation, and α is half of the angle at the apex. The rest of the designations are the same as before. For a flat foundation take $\alpha = \pi/2$ in Eqs 17, 18, and 21.

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