The Dynamic Response of Continuous Footings Supported on Cohesive Soils

Réponse dynamique de semelles continues supportées par des sols cohérents

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SUMMARY

This paper deals with the dynamic response of continuous footings supported on the surface of cohesive soils. The mode of deformation of the foundation is approximated by Fellenius' static rupture surface, and the resulting equation of motion is evaluated by a digital computer. The results are presented by diagrams expressing the influence of the transient load by means of a dynamic load factor for various natural frequencies of the foundation, various decay factors of the transient pulse, and various overload ratios. The results of this investigation serve as a useful guide for predicting the response of continuous footings subjected to exponentially decaying transient loads.

SOMMAIRE

Cet article présente la réponse dynamique des semelles continues reposant sur la surface de sols cohérents. Le mode de déformation est donné approximativement par la surface de rupture statique de Fellenius, et l'équation de mouvement qui en résulte est évaluée par une calculatrice digitale. Les résultats sont présentés sous forme de diagrammes exprimant l'influence de la charge transitoire au moyen d'un facteur de charge dynamique, de certains facteurs de décroissement de l'impulsion transitoire et de certains rapports de surcharge. Les résultats de cette recherche servent de guides utiles pour la prévision de semelles continues lorsqu'elles sont soumises à des charges transitoires qui peuvent être représentées approximativement par une impulsion décroissant exponentiellement.

THE STATIC BEARING CAPACITY of foundations has been extensively studied both analytically and experimentally. The dynamic bearing capacity problem is relatively new and most of the available technical literature (Carroll, 1963; Cunny and Sloan, 1961; Fisher, 1962; Johnson and Ireland, 1963; Landale, 1954; McKee and Shenkman, 1962; Triandafilidis, 1961; Wallace, 1961; White, 1964; Whitman, *et al.*, 1954) appeared during the last decade when the performance of foundations under transient loads became of concern to the engineering profession. The references cited are not intended to represent a complete bibliography on the subject of dynamic bearing capacity but only to provide an adequate background.

All analytical approaches attempted in the past are based on the assumption that soil rupture under transient loads occurs along a static rupture surface. Terzaghi's static bearing capacity approach (Terzaghi and Peck, 1948) postulates a symmetrical failure pattern similar to that suggested by Prandtl with modifications to account for footing roughness and soil weight. The failure patterns observed both in the laboratory and in the field are frequently one-sided. This does not prove the inadequacy of Terzaghi's approach but indicates that field and/or laboratory conditions fail to meet the requirements of the theory. Natural soil deposits are erratic, and homogeneous soil media cannot consistently be reproduced in the laboratory. Furthermore, small eccentricities of the applied loads are inevitable. These factors usually create one-sided shear failures.

Fellenius' method (Taylor, 1948) assumes that a continuous footing, without adjacent surcharge, fails along a surface that consists of a circular arc with its centre above the inner edge of the foundation. This method has been adopted for the formulation of the dynamic response problem since it offers a convenient simplification for transient loading conditions. The ultimate load carrying capacity along such a surface is in close agreement with both Terzaghi's approach for a rough footing and Prandtl's shear pattern (Fig. 1a).

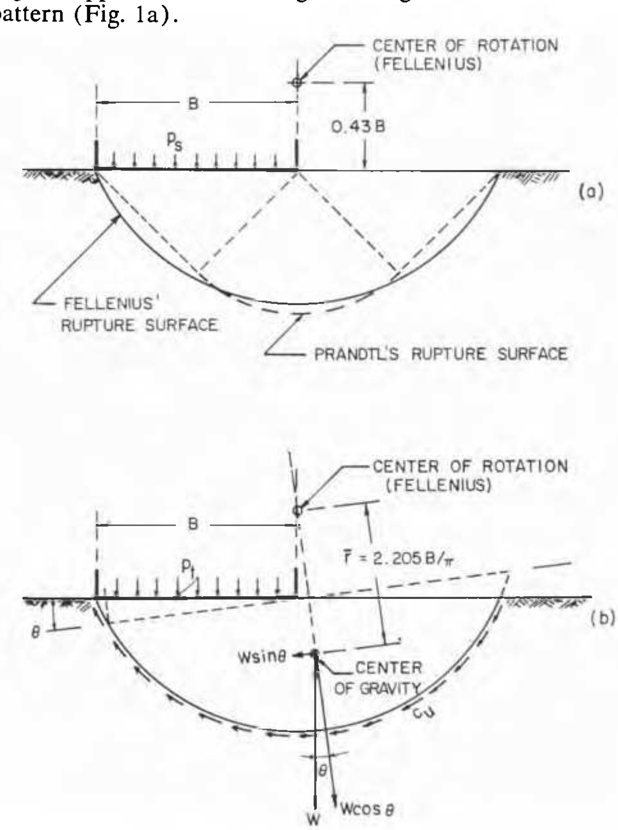

FIG. 1. Illustrations of mode of failure and of dynamic equilibrium of moving soil mass.

ASSUMPTIONS AND APPROXIMATIONS

The approach presented in this study is valid only for continuous footings supported on the surface of soft saturated clays. The soil mass participating with the foundation in the motion is considered to be a rigid body exhibiting rigid plastic stress-strain characteristics. The forcing function is assumed to be an exponentially decaying pulse, thus providing continuity for the entire event. The influence of strain rate on the shear strength of the soil and the dead weight of the foundation have not been introduced in the formulation of the dynamic response.

FORMULATION OF APPROACH

Based on the above assumptions and approximations the formulation of the dynamic response consists of equating around a fixed axis the moments of the forces that tend to disturb the equilibrium of the foundation to those that restore it. The only force which tends to disturb the equilibrium of the foundation is the externally applied dynamic pulse. The restoring forces consist of the shearing resistance along the rupture surface, the inertia of the soil mass participating in the motion and the resistance caused by the displacement of the centre of gravity of the soil mass (Fig. 1b).

Dynamic Pulse

The moment of the externally applied dynamic pulse M_{dp} is,

$$M_{dp} = \tfrac{1}{2} p_t B^2 \qquad (1)$$

in which p_t is the externally applied time-dependent pulse and B is the width of the foundation.

Soil Resistance

The static bearing capacity p_s of a continuous footing along Fellenius' rupture surface is given by

$$p_s = 5.54\, c_u \qquad (2)$$

where c_u is the undrained shear strength of the soil. The resisting moment due to shear strength M_{rs} around the centre of rotation is

$$M_{rs} = \tfrac{1}{2} p_s B^2. \qquad (3)$$

Soil Inertia

The applied pulse imparts to the soil mass an acceleration in the direction of motion. The resisting moment due to rigid body motion M_{ri} around a fixed axis is expressed as

$$M_{ri} = I_0 \ddot{\theta} \qquad (4)$$

in which I_0 is the polar mass moment of inertia of a circular segment and $\ddot{\theta}$ is the angular acceleration. The above equation could also be written in the form

$$M_{ri} = \frac{WB^2}{1.36g}\ddot{\theta} \qquad (5)$$

where W is the weight of the soil participating in the motion and is equal to $0.31\pi\gamma B^2$ per linear foot of foundation, g is the acceleration of gravity, and γ is the bulk unit weight of the soil.

Resistance due to Displacement of Centre of Gravity

The displaced position of the soil mass generates a restoring moment M_{rw} which can be expressed as

$$M_{rw} = W\bar{r}\sin\theta \qquad (6)$$

where $\bar{r}$ for this circular segment is equal to $2.205B/\pi$ (Fig. 1b).

For small angular rotations the argument of the angle can be accurately substituted by the angle itself, thus simplifying the resulting differential equation.

EQUATION OF MOTION

By equating the moments of the driving forces to those of the restoring forces the following equation of motion is obtained,

$$M_{dp} + M_{rs} + M_{ri} + M_{rw} = 0. \qquad (7)$$

Substituting, expanding, and rearranging terms,

$$\ddot{\theta} + (3g/\pi B)\theta = 0.68g/W(p_t - p_s). \qquad (8)$$

Substituting the term $3g/\pi B$ by the coefficient k^2, in Eq 8,

$$\ddot{\theta} + k^2\theta = 0.68g/W(p_t - p_s). \qquad (9)$$

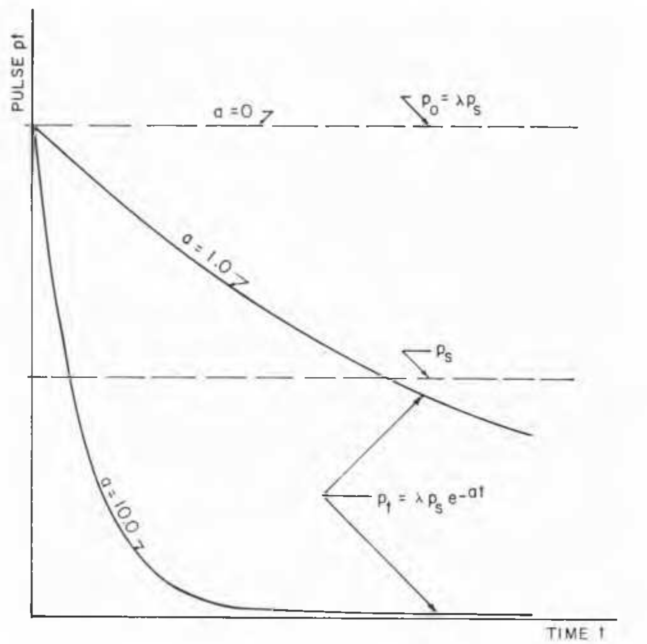

FIG. 2. Illustration of exponentially decaying stress pulses.

In Eq 9 the time-dependent pulse p_t is replaced by the exponential function (Fig. 2).

$$p_t = p_0 e^{-\alpha t}, \qquad (10)$$

where p_0 is the instantaneous peak intensity, t is the time, and α is a coefficient indicating the decay rate of the pulse. Furthermore, when the peak intensity p_0 is replaced in terms of an overload ratio λ and the static resistance p_s, the dynamic pulse can be expressed as

$$p_t = \lambda p_s e^{-\alpha t}, \qquad (11)$$

Substituting Eq 11 in Eq 9 we obtain,

$$\ddot{\theta} + k^2\theta = (0.68g/W)p_s(\lambda e^{-\alpha t} - 1). \qquad (12)$$

Substituting in Eq 12 the constants, $A = 0.68gp_s\lambda/W$ and $B = 0.68gp_s/W$ we obtain

$$\ddot{\theta} + k^2\theta = Ae^{-\alpha t} - B. \qquad (13)$$

In the above differential equation the constant coefficient k represents the angular velocity of the foundation in radians per second. The natural period of the foundation T is expressed as,

$$T = 2\pi\sqrt{(\pi B/3g)}. \qquad (14)$$

SOLUTION OF DIFFERENTIAL EQUATION OF MOTION

The solution of Eq 13 yields

$$\theta = C_1 \cos kt + C_2 \sin kt + \frac{A}{k^2+\alpha^2} e^{-\alpha t} - \frac{B}{k^2}. \qquad (15)$$

The constants C_1 and C_2 are evaluated from the initial conditions,

$$C_1 = \frac{B}{k^2} - \frac{A}{k^2+\alpha^2} \text{ and } C_2 = \frac{\alpha A}{k(k^2+\alpha^2)}.$$

Realizing that $k = 2\pi/T$ and substituting the constant coefficients A, B, C_1, and C_2 in Eq 15 the following equation is obtained

$$\frac{W}{0.68gp_s}[\theta] = \frac{T^2}{4\pi^2+\alpha^2T^2}\left[\left(1 - \lambda + \frac{\alpha^2T^2}{4\pi^2}\right)\cos 2\pi\frac{t}{T} + \frac{\alpha\lambda T}{2\pi}\sin 2\pi\frac{t}{T} + \lambda e^{-\alpha t} - \frac{\alpha^2T^2}{4\pi^2} - 1\right]. \qquad (16)$$

The above equation can be utilized to trace the history of motion. To obtain the peak angular rotation, Eq 16 is differentiated,

$$\frac{W}{0.68gp_s}[\dot{\theta}] = \frac{2\pi T}{4\pi^2+\alpha^2T^2}\left[\left(\lambda - 1 - \frac{\alpha^2T^2}{4\pi^2}\right)\sin 2\pi\frac{t}{T} + \frac{\alpha\lambda T}{2\pi}\cos 2\pi\frac{t}{T} - \frac{\alpha\lambda T}{2\pi}e^{-\alpha t}\right]. \qquad (17)$$

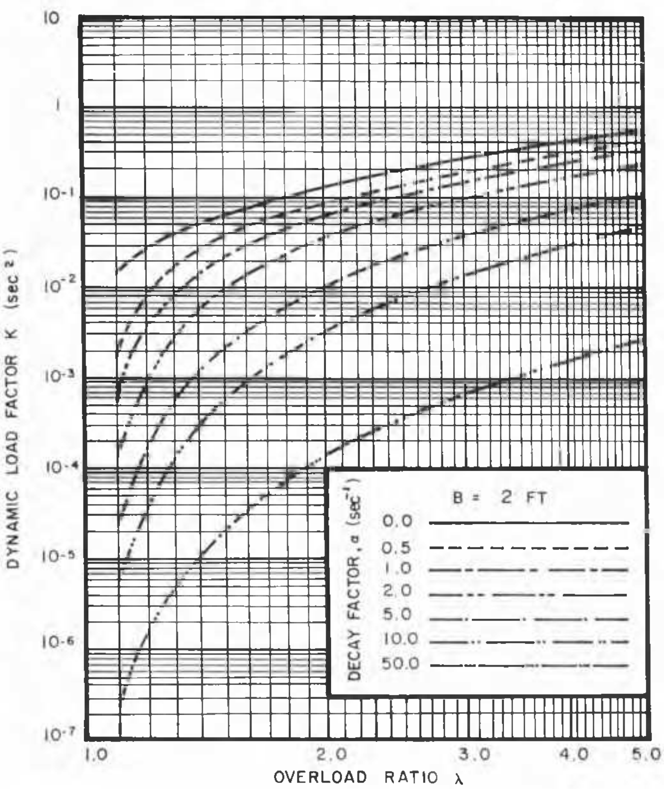

FIG. 3. Relationship between overload ratio and dynamic load factor for continuous footings 2 ft wide.

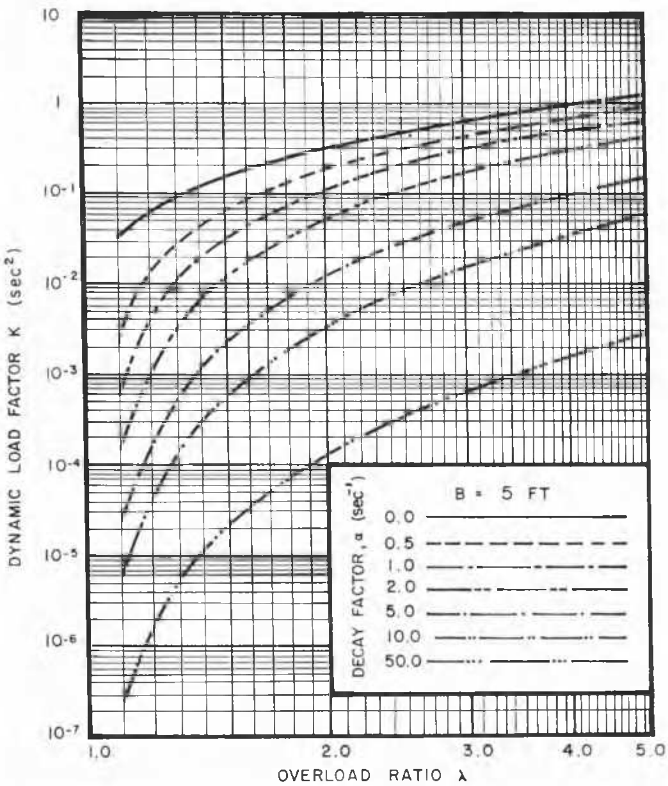

FIG. 4. Relationship between overload ratio and dynamic load factor for continuous footings 5 ft wide.

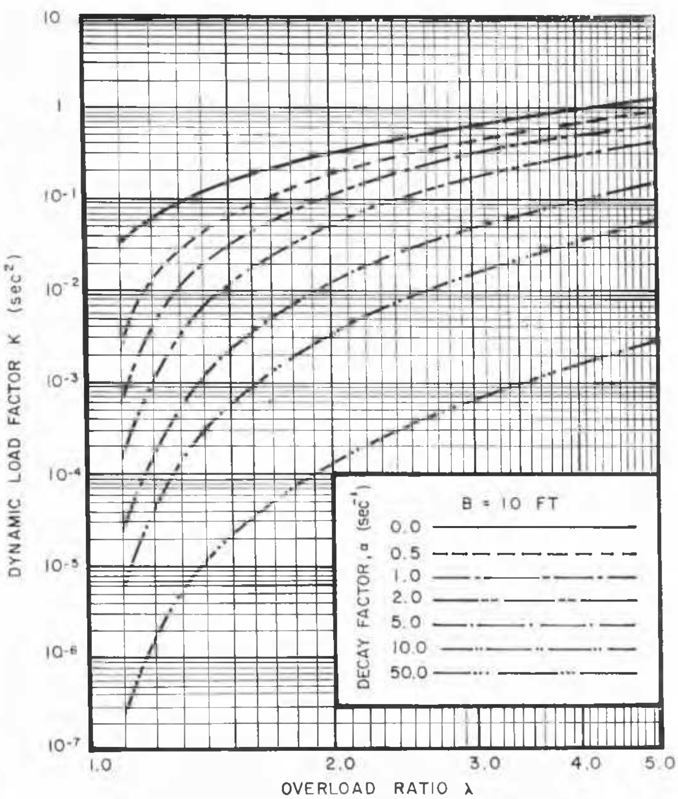

FIG. 5. Relationship between overload ratio and dynamic load factor for continuous footings 10 ft wide.

Substituting small increments of t in the above expression, a critical time t_c is obtained at which the righthand side of Eq 17 approaches zero. This critical time is subsequently substituted back in Eq 16, to obtain the maximum angular rotation θ_{max} in terms of a dynamic load factor K as follows,

$$W/0.68gp_s[\theta_{max}] = K(\sec^2). \quad (18)$$

This iteration procedure is used to determine dynamic load factors by a digital computer introducing various values of T, λ, and α.

Values of 1.60, 2.52, and 3.58 seconds are utilized for the natural period T. These correspond to foundation widths of 2, 5, and 10 ft, respectively. Values equal to 0.0, 0.5, 1.0, 2.0, 5.0, 10.0, and 50.0 $\sec^{-1}$ are introduced for the decay factor α. Similarly the influence of the overload ratio is evaluated for different values of λ equal to 1.10, 1.25, 1.50, 1.75, 2.0, 3.0, 4.0, and 5.0. The results are graphically presented in Figs. 3, 4, and 5.

DISCUSSION AND LIMITATIONS OF APPROACH

This study is valid only for cohesive soils. Such soils offer a better opportunity of formulating a simplified approach for transient loads since the shearing resistance under undrained conditions is independent of the normal stresses on the rupture surface. When the shear strength is a function of the normal stress, it is no longer possible to ignore the time dependency of the shearing resistance.

It is recognized that the assumption of rigid body rotation around a fixed axis is at variance with soil behaviour but provides a conservatively simplified mechanical model. It has been assumed that the soil exhibits rigid-plastic stress-strain characteristics without consideration of the response of the foundation prior to the mobilization of the full shear strength of the soil. Such an approach is not unreasonable provided that the deformations prior to the mobilization of the full shearing resistance are small compared to those within the plastic range.

The results of this investigation should not be applied for large rotations exceeding about 15 to 20 deg, since a substantial portion of the shearing resistance along the rupture plane vanishes when it bypasses the ground surface. In the formulation of the problem such a reduction in shearing resistance has not been introduced.

ACKNOWLEDGMENTS

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