Design of Deep Foundations

Calcul des fondations profondes

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SUMMARY

The present paper deals with the methods of footing design of shell and heavy sinking wells and caissons. The soils under such deep foundations may bear considerable pressure and be deformed when highly developed sliding zones exist. Methods of computing the ultimate load bearing capacity of footings are based upon: (*a*) a theoretical investigation of the development of sliding zones and the changes introduced by these zones when determining the foundation settlement in the case of plane strain; (*b*) the results of experimental work which enabled the correlation to be found between the value of ultimate settlement and the extent of the development of the sliding zone for plane and axial symmetrical problems. The limiting stress theory is used in determining the ultimate load bearing capacity. A comparison of theoretical and experimental data is given.

SOMMAIRE

Dans ce rapport sont exposées des méthodes de calcul de fondations pour puits-enveloppes, puits foncés et caissons. Le sol sous de telles fondations peut supporter des charges considérables et se déformer lorsque se développent des zones de glissement. Les méthodes de calcul de la charge ultime sur ce type de fondation s'appuient: (*a*) sur des recherches théoriques tenant compte de l'existence de zones de glissement, et de l'influence de ces dernières sur le calcul du tassement de la fondation dans le cas de la déformation plane; (*b*) sur des résultats expérimentaux qui ont permis de trouver une corrélation entre le tassement ultime et l'ampleur de la zone de glissement pour le problème dans un plan et dans les axes symétriques. La théorie des contraintes limitées est utilisée pour déterminer la capacité portante. Dans cet exposé sont comparés les résultats théoriques avec les données expérimentales.

WITH DEEP FOUNDATIONS (depth to width of foundation ratio exceeding 4), heavy loads of many kilograms per square centimeter can be transferred to soils such as sands and gravels with no risk of ground failure; the resistance of such footings depends greatly upon the soil's density, angle of internal friction, and cohesion, and upon the depth ratio of the foundation and the manner of its construction.

Soil deformations under deep foundations built with shell and heavy sinking wells and caissons differ considerably from those where foundations are built with large-diameter piles —ordinary piles or tubular piles with closed ends. In the first case, no significant displacement of the ground produced by the sinking part of the foundation is created, and no preliminary compaction of the soil takes place. In the second case, a pile of large cross-section which displaces the soil while sinking has well compacted ground under its end.

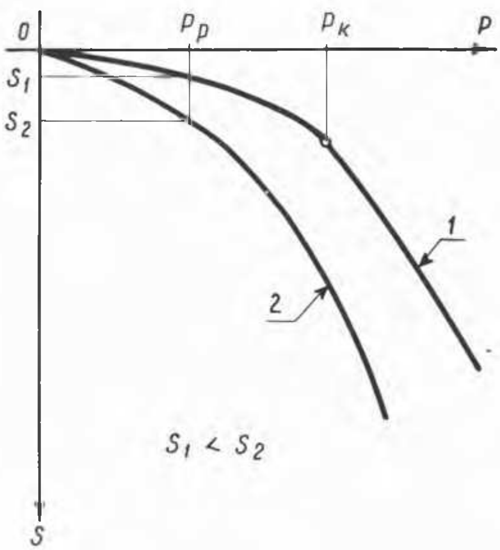

FIG. 1. Load-settlement curves: 1, for a tubular pile sunk with a closed end; 2, for a shell sinking well. (p_k, critical load; p_p, design load.)

The load-settlement curves for a shell sinking well, with excavation from within, and for a tubular pile driven with a closed end are shown in Fig. 1. From a comparison of the curves, it is evident that, with equal loads, the settlements of the foundation built up with a shell sinking well are much greater than those of a pile of the same size. Moreover, the load-settlement curve for the pile has a characteristic point corresponding to the beginning of a more intensive increase of settlement whereas the sinking well curve shows a smooth increase of settlement. Consequently, different criteria should be used in determining the ultimate load bearing capacity of soils when these types of deep foundation design are employed.

With single piles of large diameter it is necessary to compute the value of ultimate bearing capacity by the method formerly investigated (Berezantzev, 1960).

The ultimate load bearing capacity of shell and heavy sinking well and caisson footings, however, is defined solely by the value of ultimate settlement. With considerable pressures under the foundations, the soil is deformed when well-developed sliding zones exist. Therefore the settlement should be determined taking into consideration the presence of a combined stress state in the soil. The solution of such a complex boundary problem consists in determining stresses in the sliding zones and in the elastic half-space with the local change of boundaries in accordance with sliding zone contours, and in determining half-space deformations. It is necessary to assume certain boundaries for the sliding zones and make several trials to find a sufficiently correct solution. Thus, the problem should be solved by a numerical method requiring computing machines. However, the present state of theoretical research enables us to use a simplified device for an approximate assessment of the influence exerted by the sliding zones upon the change of settlement increase.

Conditions of plane strain in the case of a uniformly distributed load and a non-cohesive soil are considered below. Such conditions can be sufficiently well observed if the length of the foundations is more than 4 or 5 times its breadth; these foundations are often to be found in different structures (such as bridges) if they are built with caissons, heavy sinking wells, or shell sinking wells when the soil between the shells works together with them.

The weight of soil within the sliding zones is small in comparison to the pressure at the footing level, produced by the weight of the soil within the height equal to the depth of the foundation; therefore the sliding zone contours can be determined without taking into account the weight of soil in these zones. In this case, the locations at which local sliding planes commence will develop along a circular arc which passes through the boundaries of the loaded area and has tangents at those points (boundaries) which make an angle with the vertical equal to the angle of internal friction φ. The foundation pressure is accordingly calculated by the Puzyrevsky formula:

$$\sigma_H = \gamma D \frac{\cot\varphi + \varphi + \pi/2}{\cot\varphi + \varphi - \pi/2}. \qquad (1)$$

With the increase of σ ($\sigma > \sigma_H$) the sliding zones will develop (Fig. 2). The greatest influence upon the increase of settlement is produced by the displacements of the upper parts of the outer boundaries of these zones (AE and BF);

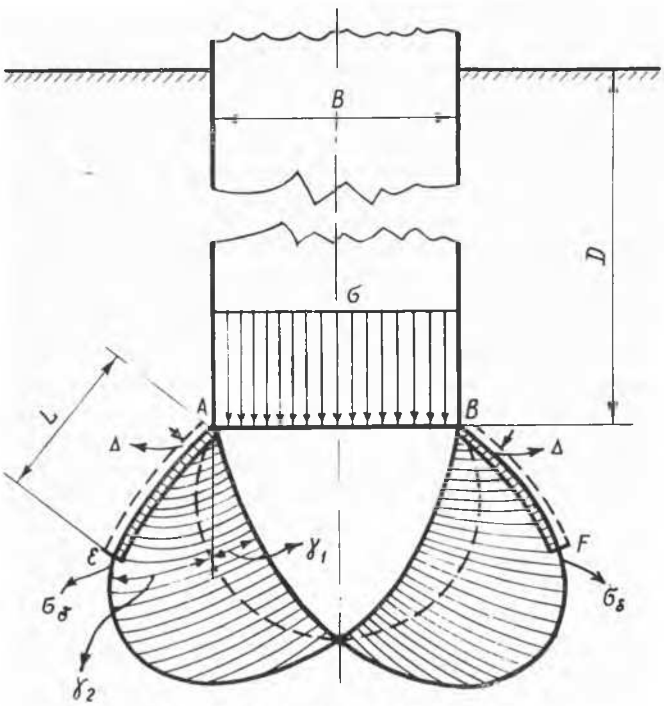

FIG. 2. Development of sliding zones.

these displacements are horizontal or upwardly inclined and occur because of compaction of the lateral zones of the ground. Within this degree of sliding zone development the foundation settlement can be approximately calculated as a sum of two values: settlement of the elastic half-space below the foundation, and additional settlement which is the result of the sliding zones displacements, Δ:

$$S = \frac{\omega PB}{E}(I - \mu^2) + \frac{2\Delta z_B}{B\cdot\cos\gamma_1 + z_B \sin|\gamma_1| + 2\Delta}. \qquad (2)$$

where
$$\Delta = \frac{\omega P_\delta L}{E}(1 - \mu^2);$$

$$z_B = L\cdot e^{-(\gamma_1 - \gamma_2)\tan\varphi}\cdot\cos\gamma_1;$$

$P = \sigma - \gamma D$; $P_\delta = \sigma_\delta - \xi\gamma D$; γ = unit weight of soil; E = modulus of linear deformation; μ = Poisson's ratio; $\xi = \mu/(1-\mu)$; the coefficient of lateral pressure

$$L = -B\frac{e^{(\pi/2)\tan\varphi}}{2\cos\left(\frac{\pi}{4} + \frac{\varphi}{2}\right)}\cdot\frac{\gamma_2 + \varphi}{\frac{\pi}{4} - \frac{\varphi}{2}}.$$

To calculate the values of the angles γ_1 and γ_2, determining the sizes of the upper parts of sliding zones, as well as the corresponding loads upon the footing σ and lateral normal pressure σ_δ, acting upon a part of contours L, it is necessary to use the solution of Fedorov (1958) for a mixed boundary problem for the semi-infinite load. The required formulae are:

$$\left\{\begin{aligned}
\gamma_1 &= -\frac{\frac{\pi}{4} + \frac{\varphi}{2}}{\frac{\pi}{4} - \frac{\varphi}{2}}\gamma_2 - \frac{\frac{\pi}{2}\varphi}{\frac{\pi}{4} - \frac{\varphi}{2}};\\
\sigma &= \gamma D\frac{e^{2(\gamma_1-\gamma_2)\tan\varphi}(1 + \cos 2\gamma_2)A_1}{(1 + \cos 2\gamma_1)A_2};\\
\sigma_\delta &= \sigma\frac{e^{2(\gamma_2-\gamma_1)\tan\varphi}(1 + \cos 2\gamma_1)\cos^2\varphi}{A_1},\\
&\text{where } A_1 = 1 + \cos 2\gamma_1 + \sin\varphi[(\pi - 2\gamma_1)\\
&\qquad\times\cos(2\gamma_1 + \varphi) + \sin(2\gamma_1 + \varphi) + \sin\varphi]\\
&\text{and } A_2 = 1 + \cos 2\gamma_2 + \sin\varphi[\sin(2\gamma_2 + \varphi)\\
&\qquad - (\pi + 2\gamma_2)\cos(2\gamma_2 + \varphi) + \sin\varphi].
\end{aligned}\right. \qquad (3)$$

Table I shows the results of computations based upon these formulae. In Fig. 3 the load-settlement curve is given for the foundation width B = 2m, the depth D = 16m. The soil is fine sand of medium density with γ = 1.7 tons/cu.m., φ = 30°, E = 400 kg/sq.cm., μ = 0.33.

TABLE I. COMPUTED SETTLEMENTS, S

γ_2	γ_1	$\gamma_1 - \gamma_2$	σ (kg/sq. cm.)	L (m.)	σ_δ (kg/sq. cm.)	S (cm.)
−30°	−30°	0°	15.2	0	—	14.4
−35°	−20°	15°	20.0	0.82	5.12	21.0
−40°	−10°	30°	25.1	1.65	5.00	29.5
−45°	0°	45°	30.6	2.47	4.80	39.4
−50°	10°	60°	36.6	3.30	4.58	51.1

The results obtained with the approximate solution shown above were compared to the results of laboratory tests* of foundation models with an 8 cm by 40 cm footing and depth

*The tests were carried out at the Laboratory of Soil Mechanics of the Leningrad Institute of Railway Engineers in 1961–62.

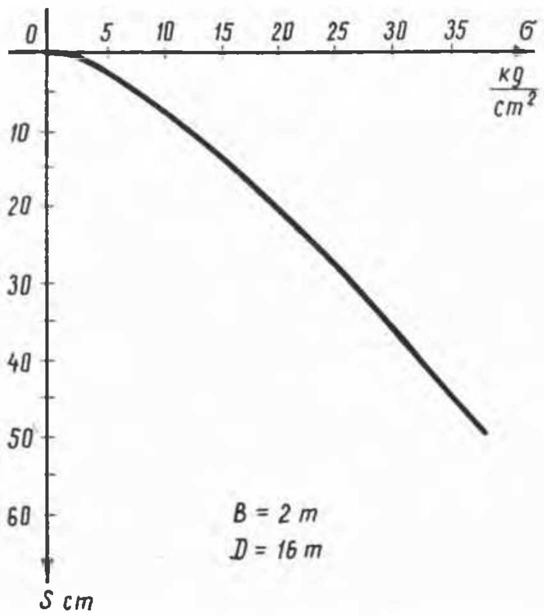

FIG. 3. Load-settlement curve of a foundation 2.0 m in breadth ($D/B = 8$).

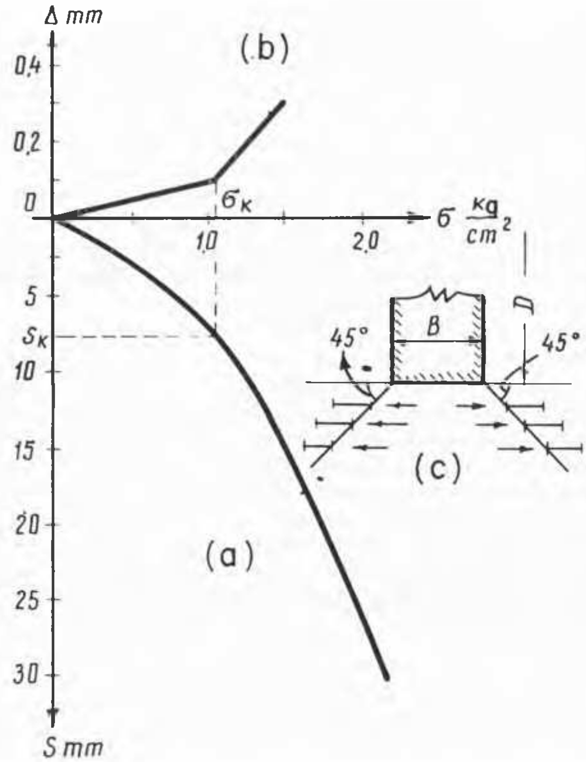

FIG. 4. Results of defining critical load by displacement of marks under foundation model: (*a*) load-settlement curve of foundation model; (*b*) curve of displacement of marks; (*c*) scheme of disposition of marks.

ratios of 8 and 12 in medium sand. Theoretical and experimental load-settlement curves closely coincided (Berezantzev, 1963).

Hence it is possible to state that there is a definite relationship between the relative settlement value (S/B) and the sliding zone size determined by angle γ_2. It is very difficult to define this relationship theoretically in general form. However, it is possible to find the correlation of the limiting relative settlement value, experimentally obtained in maintenance of different structures, to the lateral development of sliding zones. If such a correlation is available, the determination of the "critical" load value corresponding to the given limiting settlement value is quite simple.

To investigate this problem in cases of plane strain with the model tests referred to above, the relation of the pressure under the model to the horizontal ground displacements in planes, passing through the edges of the model and inclined to the horizon at the angle = 45°, was established. (The locations of the measuring marks are given in Fig. 4.) The same kind of experiments* with sands were made with circular cylindrical foundation models 6 to 11 cm in diameter and with circular cylindrical foundation 32 to 122 cm in diameter under field conditions (Berezantzev and Jaroshenko, 1962; and Berezantzev, *et al.*, 1963). Numerous experimental data accumulated showed that the relative settlement value does not exceed 0.15 to 0.25 (average 0.2) when the sliding zones reach boundary surfaces crossing the foundation edges and are inclined to the horizon at the angle of 45°. The moment when the sliding zones reached these boundary surfaces was fixed by a characteristic point in the curves showing the displacement of the marks sunk in the soil. Curves illustrating one of the tests are shown in Fig. 4. Experimental data and results are given in Table II.

For an approximate determination of the "critical" load corresponding to the relative settlement within the limits of 0.15 to 0.25, it is possible to make use of the design scheme in Fig. 5 based upon the following assumptions which make it differ from the scheme given in Fig. 2: (1) When approaching the straight lines, limiting the development of

*The experiments were carried out at the Leningrad and Moscow Institutes of Railway Engineers and at the Transport Construction Research Institute.

TABLE II. TEST RESULTS

No.	Model breadth, B (cm.)	Model depth, D (cm.)	$\frac{B}{D}$	Unit weight of soil, γ (tons cu.m.)	Angle of internal friction, φ (degrees)	Number of experiments	"Critical" unit load σ_K (kg/sq. cm.)	Relative settlement at σ_K, S_K/B	Theoretical value of $\sigma_K(\sigma_{K.K})$ (kg/sq. cm.)	Shape of Model footing
1	6.0	72	12	1.80	41	2	12.4	0.17	7.2	circular
2	8.0	96	12	1.80	41	2	4.7	0.21	5.3	rectangular (l = 40 cm.)
3	8.0	64	8	1.80	41	2	3.8	0.15	3.7	rectangular (l = 40 cm.)
4	8.0	32	4	1.77	39	5	8.8	0.16	2.8	circular
5	11.4	136.8	12	1.81	41	7	18.8	0.22	13.8	circular
6	11.4	68.4	6	1.17	45.5	4	17.5	0.15	8.4	circular
7	11.4	114	10	1.16	43.5	4	19.2	0.25	10.1	circular
8	32.5	520	16	1.80	30	1	39.1	0.16	14.7	circular
9	63.0	510	8.1	1.90	35	1	37.9	0.15	28.7	circular
10	122.0	1220	10	1.30	30	1	41.7	0.15	23.8	circular

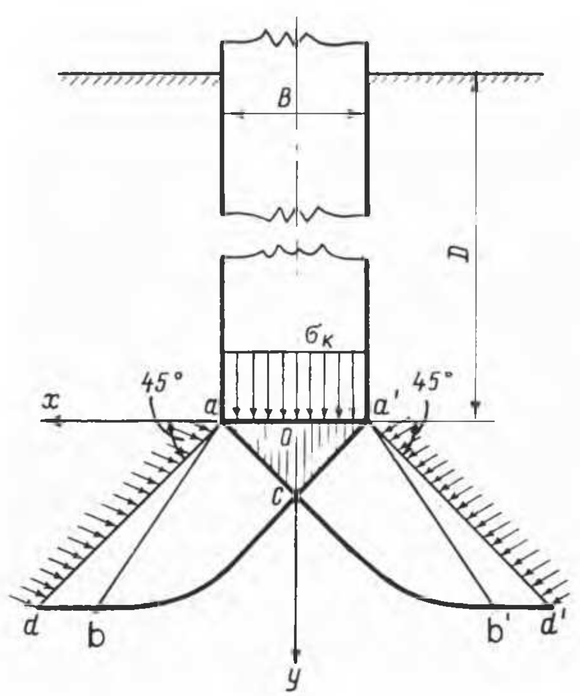

FIG. 5. Theoretical scheme for determining critical load in case of plane strain.

sliding zones, inclined to the horizon at an angle of 45°, the slip lines are horizontal. This condition determines the angle of inclination of the surcharge pressure upon the limiting straight lines from the normal. (2) The normal component of surcharge intensity q (Fig. 5) is equal to the weight of a vertical volume of soil. (3) Under the foundation there is a stiff compacted soil core with the shape of a rectangular triangle; the stresses on the surface of the core are inclined from the normal at angle φ. In case of a plane problem, the soil limiting equilibrium equation (Sokolovsky, 1960), under the assumed conditions, gives the following shape of sliding lines: in the a d b and a′ d′ b′ regions (Fig. 5) there are two families of parallel straight lines, the sliding lines of the first family incline from the x axis at: $\pi/2 - \varphi$; Ldab $= L$d′a′b′ $= \pi/4 - \varphi$. In regions b a c and b′ a′ c one family is a bundle of straight lines while the other family consists of logarithmic spirals with poles in points a and a′.

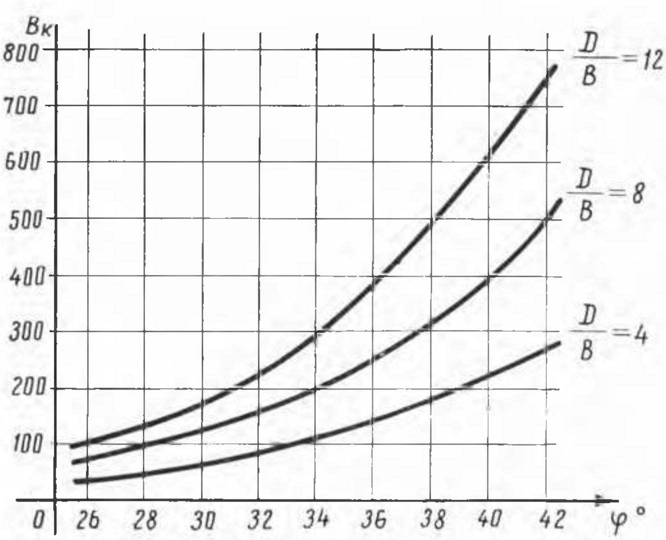

FIG. 6. Curves for determining coefficient B_k in Eq 5.

With these geometrical dimensions of the sliding zones the normal component of surcharge intensity in points d and d′ is expressed by a formula:

$$q_d = \gamma B\left[\frac{D}{B} + \frac{\cos\phi}{\sqrt{2}}\, e^{(\pi/4+\varphi)\tan\varphi}\right]$$

and in points a and a′: $q_a = \gamma B(D/B)$. The solution of the limiting stress state equation and conditions of equilibrium in the triangular compacted core lead to the following value of the mean intensity of the "critical" load under the foundations:

$$\sigma_{kp} = \gamma B\left[\frac{D}{B} + \frac{\cos\varphi}{2\sqrt{2}}\, e^{(\pi/4+\varphi)\tan\varphi}\right] \times \frac{\cos\varphi(\sin\varphi + \cos\varphi)}{1 - \sin\varphi\cdot\cos\varphi}\, e^{(\pi/2+2\varphi)\tan\varphi} \tag{4}$$

In the case of an axial-symmetrical problem (circular foundations), the solution of one of the equations of the limiting stress state (Berezantzev, 1952) gives, if approximate shapes of sliding lines are introduced (Berezantzev, *et al.*, 1963), for the same "critical" load the following formula:

$$\sigma_{k.k} = B_k \gamma B \tag{5}$$

where B_k is the coefficient obtained as a result of numerical integration (Berezantzev, *et al.*, 1963) and is a function of φ and D/B. B_k can be determined from the curves in Fig. 6.

Repeated comparison of experimental data with the results obtained from formulae 4 and 5 showed that theoretical values $\sigma_k(\sigma_{k.k})$ are some 35 to 40 per cent lower than the experimental ones. This becomes evident if columns 8 and 10 of Table II are compared. Hence, the loads determined with formulae 4 and 5 correspond to somewhat smaller values of relative settlement, approximately 0.15 instead of 0.20. Relative settlement of 0.06 to 0.1 is permissible for the majority of structures. Thus, for an approximate determination of an allowable load with formulae 4 and 5, a coefficient of 0.6 to 0.7 should be introduced. For designs requiring more accuracy in the estimation of expected settlement, it is necessary to develop a load-settlement curve taking into consideration the influence of the sliding zones (Eq 2).

The progress of design methods depends on the development of an accurate solution of the combined problem of the theory of elasticity and the limiting stress state theory. Also, careful experimental study of the deformation characteristics in relation to the load value and the anisotropy of soils should be undertaken.

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