Method of Determining the Lateral Bearing Capacity of Single Piles Based on Test Loading

Méthode de mesure de la capacité portante latérale d'un pieu basée sur des essais de chargement

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SUMMARY

This paper deals with the method of determining the lateral bearing capacity of single piles. The method is based on test loadings during which measurements are made of the horizontal load P, the pile displacement y_p at the level of the applied load, and the tangent of the angle of rotation tan φ_p of the upper part of the pile.

SOMMAIRE

Cet article présente la méthode pour déterminer la capacité portante latérale d'un pieu de fondation. Cette méthode est basée sur des essais de chargement du pieu, durant lesquels on mesure la charge latérale P, le déplacement du pieu y_p au niveau de la charge, et la tangente de l'angle de rotation tan φ_p du bout supérieur du pieu.

HORIZONTAL TEST LOADINGS of piles with measurements of the horizontal force P, the displacement y_p of the pile at the level of the application of the force P, and of the tangent of the angle of rotation, tan φ_p, of the pile head (Fig. 1) enable the lateral bearing capacity of single piles to be found without finding the lamination or enquiring into the character of the physico-mechanical qualities of the soil into which the piles are driven.

The method presented here is based on two alternative general assumptions: one regards the pile as an elastic beam on an elastic foundation, and the other, as a cantilever beam fixed in the ground at a certain penetration depth. For both these assumptions the author proves that, irrespective of the value of force P, the ratio of the displacement to the tangent of the angle of rotation, $y_p/\tan\varphi_p$, is a constant value, determining the pile behaviour in the soil.

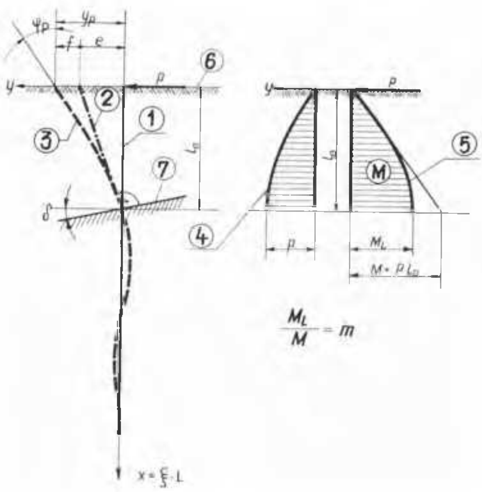

FIG. 1. Lateral loading of test pile: 1, pile axis before loading; 2, pile axis in the first stage of loading; 3, final position of the loaded pile axis; 4, passive earth pressure diagram; 5, bending moment diagram of the pile; 7, conventional level of fixing.

ASSUMPTION: PILE EQUIVALENT TO AN ELASTIC BEAM ON AN ELASTIC FOUNDATION

With the above conditions assumed (Fig. 1), the deflections of the pile and the pressures on the lateral surface of the pile are bound by the differential equation of the deflection curve, that is,

$$\frac{EI}{b}\frac{d^4y}{dx^4} = -p,$$

where EI = the stiffness of the pile, b = the width of the pile at right angles to the direction of the acting force, y = the deflection of the pile, x = the distance measured from the ground level, p = unit pressure of the lateral surface of the pile against the surrounding soil. We now assume in accordance with the Winkler hypothesis, that $p = c \,.\, y$, where c = the value of the lateral soil reaction.

With this relationship, three assumptions can be considered with respect to the modulus in question, that is: c is a constant, c varies linearly with increasing depth of penetration, c varies parabolically. For each of these assumptions, the ratio $y_p/\tan\varphi_p$ was found, the minimum depth of pile penetration in the soil (Titze, 1943) being as shown in Table I. From the values of the $y_p/\tan\varphi_p$ ratio, found by

TABLE I. VALUES OF $y_p/\tan\varphi_p$ FOR VARYING VALUES OF c AND MINIMUM DEPTH OF PILE PENETRATION, t

Values under analysis	Lateral soil modulus c		
	c = constant	$c_x = c_t x/t$	$c_x = c_t(x/t)^{1/2}$
$y_p/\tan\varphi_p$	L	$1.50\,L$	$2.75\,L$
for $\lambda = t/L$	$\geqslant 2.5$	$\geqslant 4$	$\geqslant 3$
where L	$\sqrt[4]{(4EI/cb)}$	$\sqrt[5]{(EIt/c_t b)}$	$\sqrt[4.5]{(EIt^{1/2}/16c_t b)}$

means of test loading, the values of L, and hence the unknown values of the lateral soil modulus c corresponding to the actual pile and soil conditions, can now be deduced for any of the assumed variations of c. In this way we can dispense with the rather haphazard values of the modulus c, the finding of which usually presents considerable difficulties, as for example, in laminated soils.

ASSUMPTION: PILE EQUIVALENT TO A CANTILEVER BEAM FIXED IN THE GROUND

Conclusions drawn from numerous tests favour this assumption, as the maximum bending moments in the piles appear at a nearly constant depth of penetration, independent of the load value, the numerical values of the moments being in direct proportion to the respective horizontal forces. On the other hand, it was found that the displacements of the free end of the pile to a considerable degree exceeded the deflections calculated for cantilever beams, equal in length to the distance between the point of application of the force and the point of maximum bending moment. This discrepancy may appear contrary to the assumed static pattern, but can be explained if we assume that the conditions of fixing develop gradually as a result of the increasing force (Fig. 1).

The deflection curve of a pile is wave-shaped, with its amplitude tending to decrease. For this reason, when discussing the working conditions of the upper part of the pile, we can assume that in the initial stage of loading, it rotates slightly in the ground and a passive earth pressure forms on the lateral surface of the pile so that fixing conditions develop gradually. The upper part of the pile starts to act as a cantilever beam, elastically fixed at a certain, nearly constant depth. However, the conventional level of fixing will undergo rotation through an angle δ when fixing conditions develop, and for this reason the displacement of the pile, y_p, as measured at the level of the ground surface, is due partly to the effect of the pile rotation, e, and partly to its deflection, f (Fig. 1): $y_p = e + f$.

It has been assumed (Gołubkow, 1950) that the depth at which the maximum bending moment in the pile occurs coincides with the depth at which the upper part of the pile with its l_0 length practically does not deflect. The distribution of passive earth pressure is assumed to follow the pattern indicated in Fig. 1, and we find the value of $y_p/\tan \varphi_p$ for these conditions; then, using the geometrical relations in Fig. 2, we determine the position of the conventional level of fixing, l_0, from the formula

$$l_0 = 1.15\, y_p/\tan \varphi_p.$$

At the depth thus found, the maximum bending moment will be given by the formula $M_l = mPl_0$, where the coefficient m is decreasing due to the passive earth pressure in the upper part of the pile (regarded as a cantilever).

For a parabolic distribution of passive earth pressure to a depth of l_0, the coefficient $m = 5/8$, for a triangular distribution $m = 2/3$, and for a rectangular pattern $m = 1/2$.

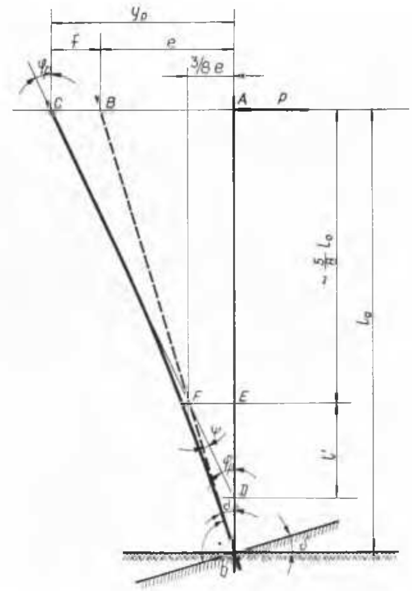

FIG. 2. Key sketch of symbols used.

SURVEY AND ANALYSIS OF TECHNICAL LITERATURE RELATING TO TEST LOADINGS OF PILES

In order to examine the conclusions made above, surveys were done of all available publications describing horizontal test loadings of piles during which measurements had been made of the horizontal force, the displacement and the rotations of the pile head, and the location and value of the maximum bending moment. The actual depth of the maximum bending moment, l_{rz}, was compared with its calculated depth, l_0, as follows: $n = l_{rz}/l_0$. Similarly, comparison was made between the actual bending moment, M_{rz}, and its calculated counterpart, $M_0 = Pl_0$, that is, $m = M_{rz}/Pl_0$. The results of this analysis are given in Table II.

TESTS CARRIED OUT BY THE AUTHOR

Tests were carried out using 48 model piles, comprising 24 wood and 24 steel model piles, in a total of nine test groups. Each test pile was 70 cm long.

The cross-sectional dimensions of the pine model piles were 15 mm in the direction perpendicular to the applied force and from 2.0 to 12.0 mm in the direction parallel to this force. The corresponding dimensions of the steel model piles were 15 mm and from 1.55 to 4.00 mm, respectively.

TABLE II. ANALYSES OF PUBLISHED DATA ON HORIZONTAL TEST LOADING OF MODEL AND NATURAL SIZE PILES

Serial no.	Test by	Piles	Soil	n	m
1	Loos and Breth (1949)	Steel model piles diam. 62–60 mm l = 200 cm	Sand of medium granulation	1.0	0.485
2	Słomianko and Jednorał (1955)	Wooden piles 27 × 27 cm l = 700 cm	Fine, laminated sand	0.83–1.05	0.47–0.67
3	Hückel (1963)	Wooden model piles 1 × 2–4 × 2 cm l = 80 cm	Beach sand from Sopot	1.03–1.07	0.69
4	Sieriebro (1960)	Prestressed concrete diam. 120–100 cm l = 1300 cm	Laminated soil	tan φ_p not measured	0.43–0.65
5	McCammon and Ascherman (1954)	Prestressed concrete diam. 137–112 cm l = 5200 cm	Soft clays	1.01	M_{rz} not measured

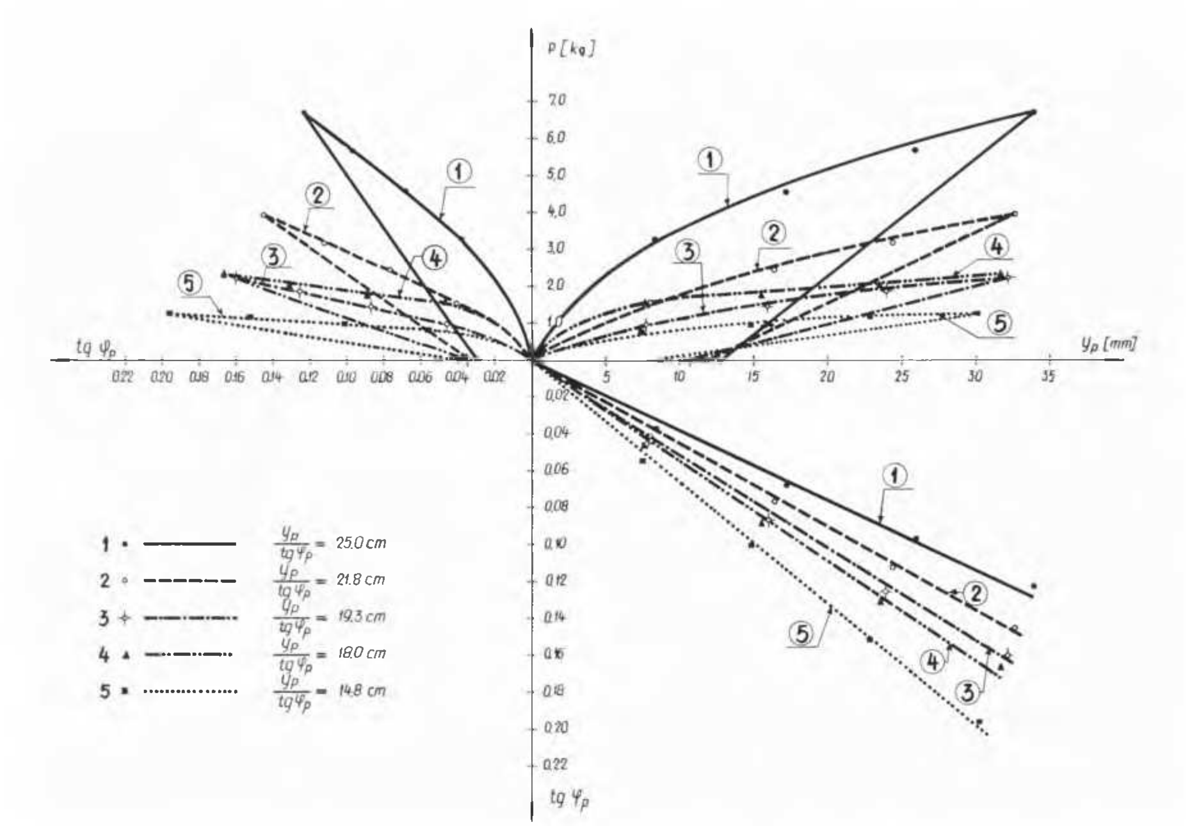

FIG. 3. Test results in group V. Pile dimensions: 1, 15.0 × 12.0 mm; 2, 14.4 × 9.8 mm; 3, 13.6 × 7.8 mm; 4, 13.9 × 6.2 mm; 5, 14.7 × 3.8 mm.

Brass model piles, circular and tubular in cross-section, with diameters of from 4 mm to 10 mm, were also tested. In addition, two prestressed concrete piles of natural size were test loaded. The problem of the scale of models used in laboratory research has been disregarded here, as the tests were carried out with a view to checking the actual relationships existing between the measured coefficients, and not to extrapolating the values obtained from model tests to natural scale.

The model piles were tested in a rectangular wooden container 86 cm high and 21 by 70 cm in area. All the tests were carried out using dry sand from the beach of Sopot (bulk density, $\gamma = 1.66$ tons/cu.m.; angle of internal friction $\varphi' = 35°$). The sand was compacted in layers 4 cm thick by means of an electrical surface vibrator, operating for 4 min.

The model tests also included an attempt to measure the displacement of the pile foundations in the soil by X-ray pictures. X-ray pictures were examined by negatroscopy, their scale being taken into consideration (Węgrzyn, 1961). During the loading of the model piles, measurements were taken of the horizontal force, P, and of the displacement, y_p, at the point of application of the force P. The value of $\tan \varphi_p$ was also determined.

Tests in groups IV, VIII, and IX provided additional measurements of the magnitude and distribution of the bending moments. These readings were accomplished by means of strain gauges. Some examples of these tests will be found in Fig. 3, in which a constant value $y_p/\tan \varphi_p$, determined for five piles of various dimensions of test group V, can be seen. Strain gauge readings for model pile 3, test group VIII are presented in Fig. 4 illustrating the constant character of the depth at which the maximum bending moments in piles occur when loaded with the increasing lateral force P. Fig. 4 also presents an example of the method of establishing the values of the coefficient m.

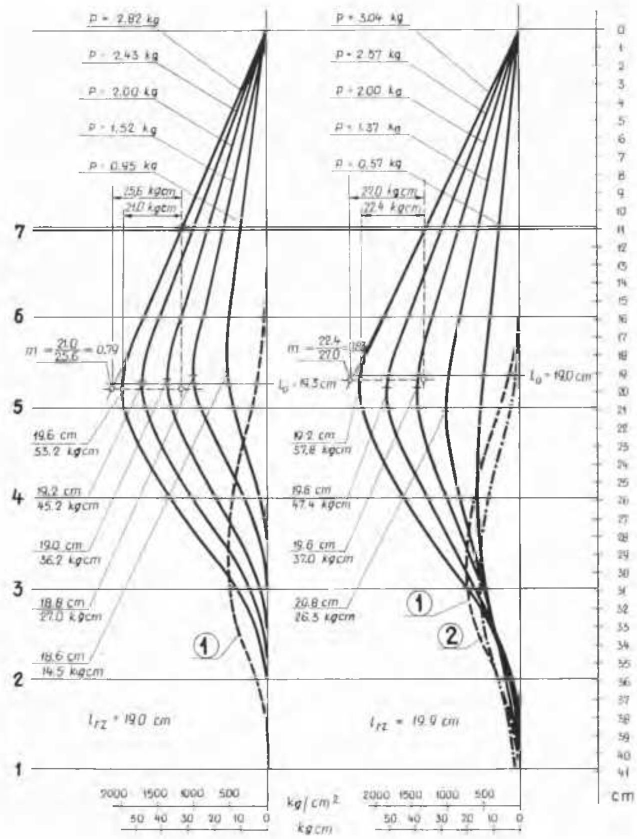

FIG. 4. Bending moment diagram for pile No. 3 test group VIII. Values in numerators represent the conventional level of fixing, in denominator, the bending moment values. l_{rz} = mean value of the conventional depth of fixing.

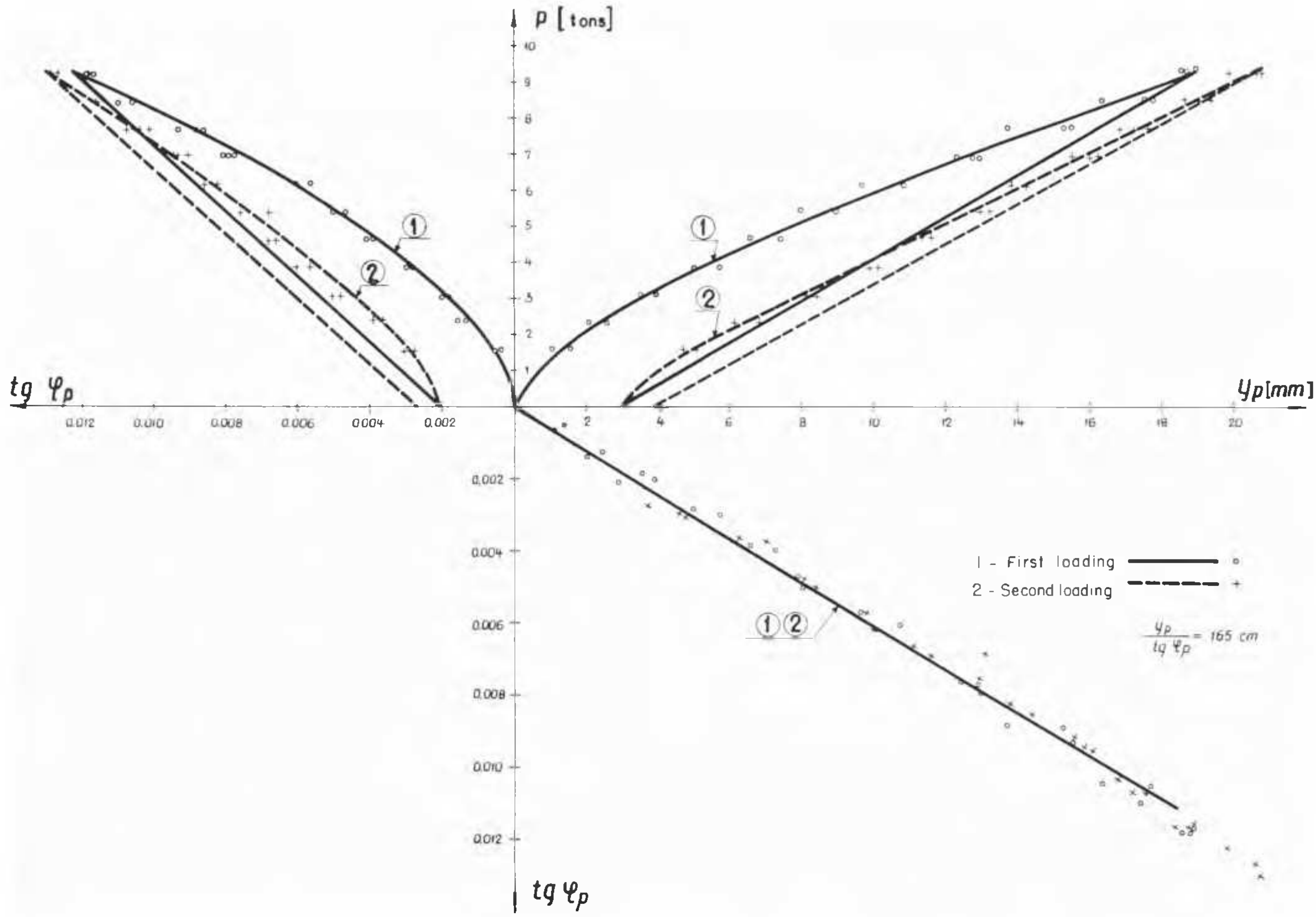

FIG. 5. Test results from loading of the prestressed concrete pile of natural size.

In addition to the model tests described above, horizontal test loadings were also carried out on two prestressed concrete piles of natural size. Fig. 5 shows a set of measurements relating to one of them.

CONCLUSIONS

1. An analysis of pile tests published by other authors as well as tests carried out by the writer on model piles and on piles of natural size, confirm the conclusion of the author that the value of $y_p/\tan \varphi_p$ is a constant.

2. The constant character of the depth at which the maximum bending moment in piles will occur was confirmed in the tests made by the author. The tests of the two final groups all show that, in unloaded piles, the stresses existing in the material do not dissipate completely, but that 30 to 40 per cent of the maximum stresses remain after unloading. This fact favours the author's theory that the conditions of fixing develop gradually as the loading force increases.

3. The location of the conventional level of fixing, calculated from $l_0 = 1.15\ y_p/\tan \varphi_p$, in most cases coincides with the depth at which the maximum bending moments occur in the pile. The differences in the cases examined do not exceed 5 per cent for the most part.

4. The value of the coefficient m lies between 0.5 and 0.7 for piles of natural size, and between 0.7 and 0.8 for model piles.

5. As we may infer from the X-ray examinations performed at a depth l_0, calculated from the formula $l_0 = 1.15\ y_p/\tan \varphi_p$, the pile displacements are either equal to zero, or are so small that they cannot be noticed at all.

6. The method of test loading described here, along with the method of determining the constant value $y_p/\tan \varphi_p$, can also be applied to piles regarded as elastic beams resting on an elastic foundation, as described above.

REFERENCES

GOŁUBKOW, W. N. (1950). *Bearing capacity of pile foundations.* Moscow, Maszstrojizdat.

HÜCKEL (1963). Resistance des pieux à la force horizontale. *Recherches dans le domaine de la mecanique des sols sur modèles réduits*, chap. III. Warsaw, PWN.

LOOS, W., and H. BRETH (1949). Model tests on laterally loaded piles and sheet piles. *Der Bauingenieur*, No. 9, pp. 264–75.

MCCAMMON, G. A., and J. C. ASCHERMAN (1954). Resistance of long hollow piles to applied lateral loads. American Society for Testing Materials, Preprint.

SIERIEBRO, A. JA. (1960). Experiments on the bearing capacity of untapered piles in the harbour of Kalinigrad. *Transportnoje stroitielstwo*, No. 9, pp. 43–5.

SŁOMIANKO, P., and T. JEDNORAŁ (1955). *Bearing capacity of laminated wooden piles.* Publication of the Maritime Institute in Gdansk.

TITZE, E. (1943). *The passive earth pressure by lateral loading of pile foundations.* Berlin, W. Ernst u. Sohn.

WĘGRZYN, M. (1961). Sur l'application de la radiographie au cours des essais effectués sur les modèles de fondations. *Archiwum Hydrotechniki*, No. 3, p. 455.