The Resistance of Soils in Vibro-Sinking of Precast Reinforced Concrete Pipe Piles of Large Diameter

Résistance des sols dans le coulage par vibration de pieux-tuyaux de grand diamètre en béton précontraint

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SUMMARY

A mathematical model of resistance of soils during the vibro-sinking of large-diameter pipe piles based on data from measured curves of acceleration and stress in pile walls is proposed by the author. Approximate solutions for steady-state motion and stress are derived. It is shown how equations of steady motion and stress in pile walls can be used to calculate the soil constants characterizing the resistance of soils in vibro-sinking.

SOMMAIRE

L'auteur suggère un modèle mathématique de la résistance des sols dans le coulage par vibration de pieux-tuyaux de grands diamètres. Ce modèle est basé sur des faits significatifs tirés de courbes mesurées d'accélération et de contrainte dans un mur-écran. Des solutions approximatives pour un état permanent de mouvement et de contrainte sont déduites. On démontre dans cet article comment les équations de mouvement permanent et de contrainte dans un mur-écran peuvent être employées pour calculer les constantes caractéristiques de la résistance des sols dans le coulage par vibration.

PRECAST REINFORCED CONCRETE PIPE PILES of large diameters (i.e. thin-walled caissons) have been used extensively in the construction of the foundations of deep-water piers of many large bridges in China. These open-end pipe piles are made of a series of sections joined together by bolted steel flanges which are welded to the main longitudinal reinforcing bars. The lowest section is provided with a cutting shoe. Large-diameter pipe piles are sunk by powerful vibrators with eccentric weights rotating in opposite directions. Vibration with the help of water jetting and dredging by air ejectors inside has proven to be an efficient method of sinking the pipe piles.

The vibro-sinking of the large thin-walled pipe pile presents some problems to both foundation engineers and designers of the vibrators. No data are available for determining the optimum size of the pile that can be sunk with a particular type of vibrator to the desired depth in different soil strata. Methods for computing vibro-sinking stress in the wall of the pile in order to facilitate the structural design of the pile and control of vibration during sinking have yet to be devised. With the solution of some of these problems in view, Chinese engineers and research workers have set up schemes for experimental study of vibration modes and stress in the walls of piles during vibro-sinking.

During the years 1958–1960 measurement of acceleration, average sinking velocity, and stress in the reinforcing bars of the wall during vibro-sinking of several pipe piles has been carried out. Bonded wire strain gauges attached respectively to the reinforcing bars in pile walls and the cantilever beam of accelerometers were used to measure stress and acceleration. The calibrated stress and acceleration *versus* time curves from multichannel oscillographs were recorded. The average sinking velocity was computed from the penetration of the pile recorded every 15 seconds with a level.

The author considers that of the information which can be deduced from these data, the actual values of the resistance of soils during the vibro-sinking of pipe piles are the most valuable, since only after the resistance of soils is quantitatively known can the analysis of the problems mentioned above be made in terms sufficiently general to be of use to foundation engineers. First of all it is necessary to establish a mathematical model of resistance of soils on which a serious study of the mechanics of vibro-sinking can be based. The author will first suggest such a model based on the over-all nature of measured curves and then show how the constants characterizing the vibro-sinking resistance of soils can be calculated.

RESISTANCE OF SOILS IN STEADY MOTION OF VIBRO-SINKING OF PILES

Let G and M denote submerged weight and mass of the pile with attached vibrator and $P \sin \omega t$ the periodic pulsating force of the vibrator. The motion of vibro-sinking of the pile may be considered as forced vibration with one degree of freedom under the disturbing force of $G + P \sin \omega t$, the downward direction being taken as positive. Starting from rest, the process of vibro-sinking will undergo a short period of transient state, which indicates very quick sinking and small stress in the pile wall and is therefore unimportant. Only steady motion will henceforth be studied. The position of the end of the pile at any time t is specified by its downward vertical distance X at any instant after the motion becomes steady. The differential equation of steady motion of the pile can be written as

$$M(d^2X/dt^2) + F = G + P \sin \omega t, \qquad (1)$$

where F denotes the resistance of soil. In steady motion F also becomes periodic, i.e. $F(t) = F(t + T)$, where $T = 2\pi/\omega$ is the period of vibrating force. The resistance of the soil F may be reasonably resolved into the usual two parts—the damping on the outside surface of the pile F_q and the restoring force at the end of pile, or end resistance, F_r: $F = F_q + F_r$.

The author agrees with Barkan (1959) that damping in vibro-sinking can be considered viscous, i.e. F_q is proportional to velocity dX/dt. If the depth of sinking in one stage of steady motion is not great, the average length L_0 of the pile embedded in the soil strata may be taken as constant. If D denotes the outside diameter of the pile and q the damping factor per unit surface, we have

$$F_q = \pi D L_0 q (dX/dt). \tag{2}$$

The end resistance is a rather complicated thing, for which a simplifying mathematical representation is necessary. As the distinguishing feature of vibro-sinking is the combination of a linear motion with a periodic one, it is assumed here that only the latter determines the magnitude of F_r. Let X' denote the vertical downward displacement of the pile from the position of equilibrium with respect to the vibratory part of motion, $F_r = F_r(X')$. The simplest form of $F_r(X')$ that can be adapted to represent measured data of motion and stress in the pile wall is a piecewise linear function of X'. Let k be the end resistance factor per unit circumferential length of cutting shoe, the following representation of end resistance is suggested:

$$\left.\begin{aligned} F_r(X') &= \pi D k X' \text{ when } X' \geqslant 0 \\ &\qquad \text{or } -X'_0 \leqslant X' \leqslant 0,\ dX'/dt < 0; \\ F_r(X') &= 0 \text{ when } X' \leqslant -X'_0,\ dX'/dt < 0 \\ &\qquad \text{or } X' \leqslant 0,\ dX'/dt > 0. \end{aligned}\right\} \tag{3}$$

The form of end resistance (Eq 3) is established to account for a very significant fact about the measured stress of the pile wall. All field data indicate that the tensile stress in a section of pile wall a little above the shoe is considerable, which is possible only in the case of a fairly large negative restoring force, or pull, on the end of pile when it rises from the equilibrium position. This negative end resistance should be attributed to the highly concentrated damping force in the neighbourhood of the cutting shoe and the possible presence of an "earth plug" at the end of the pile. In fact, Eqs 2 and 3 should be considered as equivalent damping and equivalent end resistance only, as the former actually includes a part of the latter in linear motion while the latter includes a part of the former in the neighbourhood of the cutting shoe.

Substitute F_q and F_r from Eqs 2 and 3 into Eq 1 and introduce the following dimensionless quantities,

$$\tau = \omega t,\ x = \gamma(\omega^2/g)X,\ x' = \gamma(\omega^2/g)X',\ \gamma = G/P,$$
$$\mu = (\pi D L_0/M\omega)q,\ \lambda = (\pi D/M\omega^2)k,$$

we have the equation of motion:

$$\ddot{x} + \mu\dot{x} + f_r(x') = \gamma + \sin\tau, \tag{4}$$

where the dot denotes differentiation with respect to τ.

APPROXIMATE SOLUTION OF STEADY MOTION

Judging from the measured curves of acceleration, in the steady motion of vibro-sinking, besides the fundamental harmonic with the same frequency as the vibrator, harmonics of higher orders do occur, as the non-linear end resistance assumed above should predict. However, in most cases only harmonics of very high orders with small amplitudes appear. To seek an approximate steady-state solution, we shall consider only the fundamental harmonic. With the phase angle φ written in the disturbing force of the vibrator, the equation of motion (4) becomes

$$\ddot{x} + \mu\dot{x} + f_r(x') = \gamma + \sin(\tau + \varphi). \tag{5}$$

When the motion is purely vibratory, or average sinking velocity $v = 0$, the steady-state solution should be

$$x = w + a\sin\tau$$

where w is the average displacement which remains the same in every cycle of motion. In steady vibro-sinking with an average sinking velocity v, w will be the incremental average displacement for each cycle, or with

$$v = \frac{1}{2\pi}\int_0^{2\pi} x(\tau)d\tau = \frac{w}{2\pi},$$

the solution for $v \neq 0$ will be

$$x = v\tau + a\sin\tau. \tag{6}$$

Eq 6 is the solution that will be dealt with here. It may be mentioned in passing that the solution with an average acceleration b will be in the following form:

$$x = \tfrac{1}{2}b\tau^2 + a\sin\tau.$$

From Eq 2, the end resistance is determined by the periodic motion, which from Eq 6 is $x' = a\sin\tau$. Let $U(\tau - \tau_0)$ denote the step function, i.e. $U(\tau - \tau_0) = 0$ when $\tau < \tau_0$, $U(\tau - \tau_0) = 1$ when $\tau > \tau_0$, the end resistance can now be written as

$$f_r(x') = \lambda a \sin\tau[U(\tau) - U(\tau - \tau_0)].$$

The approximate solution of Eq 6 and end resistance are shown in Fig. 1.

Substituting $\ddot{x}$, $\dot{x}$ from Eq 6 and $f(x')$ into Eq 5, we have

$$[R] = \gamma - \mu v + (a + \cos\varphi)\sin\tau$$
$$- (\mu a - \sin\varphi)\cos\tau - f_r(x') = 0.$$

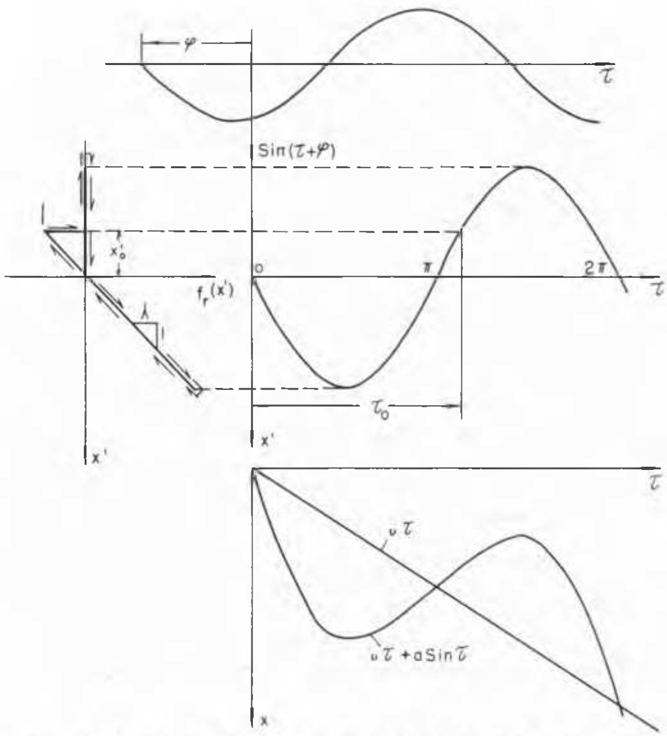

FIG. 1. Approximate steady-state solution and end resistance.

We shall use the Ritz-Galerkin method (Cunningham, 1958) to derive equations for solving the unknowns. Multiplying $[R]$ successively by three weighting functions 1, $\sin\tau$, and $\cos\tau$ and integrating for one complete cycle, we get three equations as follows:

$$\left.\begin{aligned} 2\pi(\gamma-\mu v)-\lambda a(1-\cos\tau_0)&=0,\\ 4\pi(a+\cos\varphi)-\lambda a(2\tau_0-\sin 2\tau_0)&=0,\\ 4\pi(\mu a-\sin\varphi)+\lambda a(1-\cos 2\tau_0)&=0. \end{aligned}\right\} \quad (7)$$

For our purpose Eq 7 can be considered as a system of three equations with three unknowns μ, λ, and τ_0. If there are measured values of v, a, and φ, Eq 7 can be solved to give three constants (dimensionless) characterizing the resistance of soils at that particular depth of penetration. In the measurements mentioned before, v and a (which can be computed from acceleration) were measured, but a reasonably accurate value of φ could not be assessed from the data at hand. As the stress in the reinforcing bars of the pile wall had been measured fairly completely, the additional equations required can be obtained as described below.

STRESS IN REINFORCING BARS OF PILE WALL

Fig. 2 shows the vibro-sinking pile and the forces acting on a section dZ in length. The distance of any cross-section of the pile from its end is designated by Z. The total displacement $\xi(Z,t)$, positive upward, is composed of two parts, i.e.,

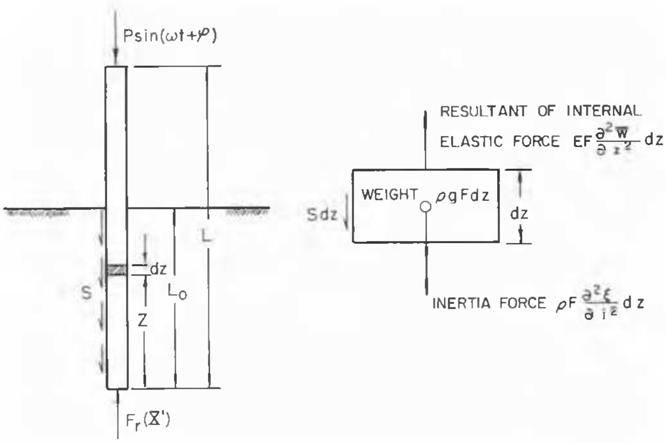

FIG. 2. Forces on pile during vibro-sinking.

that of rigid body motion $X(t)$ and that due to elastic deformation $\bar{W}(Z,t)$:

$$\xi(Z,t)=\bar{W}(Z,t)-X(t).$$

Let F = cross-sectional area of the pile wall, ρ = density of the reinforced concrete, E = modulus of elasticity of reinforced concrete and $c^2 = E/\rho$. Writing down the differential equation of motion of a section dZ high, we have:

$$c^2\frac{\partial^2\bar{W}}{\partial Z^2}-\frac{\partial^2\xi}{\partial t^2}-g+\frac{\pi Dq}{\rho F}\frac{dX}{dt}[U(Z)-U(Z-L_0)]=0.$$

To every term of the above equation carry out $\partial/\partial Z$, we get the differential equation for strain ϵ ($\epsilon=\partial\bar{W}/\partial Z$) as follows:

$$c^2\frac{\partial^2\epsilon}{\partial Z^2}-\frac{\partial^2\epsilon}{\partial t^2}+\frac{\pi Dq}{\rho F}\frac{dX}{dt}\frac{\partial}{\partial Z}[U(Z)-U(Z-L_0)]=0.$$

Introduce the following dimensionless quantities,

$$z=(\omega/c)Z,\ l=(\omega/c)L,\ l_0=(\omega/c)L_0,$$

the equation becomes

$$\epsilon''(z,\tau)-\ddot{\epsilon}(z,\tau)+\frac{\pi DqP}{\rho c\omega^2 FM}\dot{x}(\tau)\frac{\partial}{\partial z}[U(z)-U(z-l_0)]=0,$$

where the prime denotes partial differentiation with respect to z. Let $p=P/EF$, then $\pi DqP/\rho c\omega^2 FM=(\mu/l_0)p$. Also $(\partial/\partial z)U(z)=0$ and $(\partial/\partial z)U(z-l_0)=\delta(z-l_0)$, where δ is the impulse function, $\delta(z-l_0)=0$ when $z\neq l_0$,

$$\int_{-\infty}^{\infty}\delta(z-l_0)dz=1$$

when $z=l_0$ the equation of strain would be in simpler form. As initial conditions do not affect the steady-state strain very much, we assume $\epsilon(z,0)=0$, $\dot{\epsilon}(z,0)=0$. The boundary conditions can be written down immediately. Finally, we have the equation of strain of the pile wall:

$$\left.\begin{aligned} \epsilon''(z,\tau)-\ddot{\epsilon}(z,\tau)&=(\mu/l_0)p\dot{x}(\tau)\delta(z-l_0);\\ \epsilon(z,0)&=0,\ \dot{\epsilon}(z,0)=0;\\ \epsilon(l,\tau)&=p\sin(\tau+\varphi),\\ \epsilon(0,\tau)&=-p\lambda a\sin\tau[U(\tau)-U(\tau-\tau_0)]. \end{aligned}\right\} \quad (8)$$

Eq 8 can be expediently solved by the method of linear system analysis (Pfeiffer, 1961). For that purpose we shall first find the impulse response of ϵ by solving the following equation, i.e., with inputs within and on the boundaries in Eq 8 all replaced by impulse function $\delta(\tau)$,

$$\left.\begin{aligned} \epsilon''(z,\tau)-\ddot{\epsilon}(z,\tau)&=\delta(\tau)\delta(z-l_0);\\ \epsilon(z,0)&=v,\ \dot{\epsilon}(z,0)=0;\\ \epsilon(l,\tau)&=\delta(\tau),\ \epsilon(0,\tau)=\delta(\tau). \end{aligned}\right\} \quad (9)$$

Changing the variable τ in the above equation to s through Laplace transform, we get an ordinary differential equation for the transformed strain $\bar{\epsilon}$,

$$\left.\begin{aligned} \bar{\epsilon}''(z)-s^2\bar{\epsilon}(z)&=\delta(z-l_0);\\ \bar{\epsilon}(l)=1,\ \bar{\epsilon}(0)&=1. \end{aligned}\right\} \quad (10)$$

The solution of Eq 10 gives us the transfer or system function $H(s)$, which can be considered as the sum of three terms,

$$H(s)=H_p(s)+H_q(s)+H_r(s),$$

$$H_p(s)=\sinh sz/\sinh sl;$$

$$H_q(s)=\begin{cases}-\sinh sz\sinh s(l-l_0)/s\sinh sl, & z\leqslant l_0;\\ -\sinh s(l-z)\sinh sl_0/s\sinh sl, & z\geqslant l_0;\end{cases}$$

$$H_r(s)=\sinh s(l-z)/\sinh sl.$$

From the transfer function the steady-state response of strain of the pile wall to constant and sinusoidal excitations, i.e., strains caused by damping on the outside surface and disturbing forces of vibrator and end resistance on the upper and lower boundaries of the pile, can be easily evaluated from $H(O)$ and $H(i)$, $i^2 = -1$. Replace sine by its argument approximately, we have

$$H_p(i) = Z/L;$$

$$H_q(o) = H_q(i) = \begin{cases} -z(1 - l_0/l), & z \leqslant l_0; \\ -l_0(1 - z/l), & z \geqslant l_0; \end{cases}$$

$$H_r(o) = H_r(i) = 1 - Z/L.$$

Let $f_r(T)$ denote the end resistance expanded into a Fourier series of one term, we have finally the steady-state strain

$$\left.\begin{aligned} \epsilon &= p[Z/L \sin \tau - \eta\mu\dot{x}(1 - Z^2/L)f_r(T)], \\ \eta &= \begin{cases} (1/L_0 - 1/L)Z\cdot, & Z \leqslant L_0 \\ 1 - Z/L, & Z \geqslant L_0. \end{cases} \end{aligned}\right\} \quad (11)$$

Let σ_s = steady-state stress of reinforcing bars of the pile wall and C = transformed area of the pile wall, as $\epsilon/p = \epsilon EF/P = \sigma_s C/P = \sigma_s/p_s$, where $p_s = P/C$, Eq 11 can be changed into that for σ_s,

$$\sigma_s = p_s[(Z/L)\sin\tau - \eta\mu\dot{x} - (1 - Z/L)f_r(T)], \quad (12)$$

with η as in Eq 11. Substitute in $\dot{x}$ and $f_r(T)$,

$$\begin{aligned} \sigma_s &= p_s(-A_k + A_s \sin\tau + A_c \cos\tau) \\ &= p_s[-A_k + \sqrt{(A_s^2 + A_c^2)}\sin(\tau + \psi)], \end{aligned}$$

where

$$\left.\begin{aligned} A_k &= \eta\mu v + (\lambda a/2\pi)(1 - \cos\tau_0)(1 - Z/L), \\ A_s &= (Z/L)\cos\varphi - (\lambda a/4\pi)(2\tau_0 - \sin 2\tau_0) \\ &\qquad \times (1 - Z/L), \\ A_c &= (Z/L)\sin\varphi - \eta\cdot\mu\cdot a - (\lambda a/4\pi) \\ &\qquad \times (1 - \cos 2\tau_0)(1 - Z/L). \end{aligned}\right\} \quad (13)$$

Let σ_- and σ_+ denote the magnitudes of maximum compressive and tensile stresses respectively, the fundamental harmonic of stress wave in the reinforcing bars can be obtained completely from the following equation:

$$\left.\begin{aligned} &\left.\begin{matrix}\sigma_- \\ \sigma_+\end{matrix}\right\} = p_s(\sqrt{(A_s^2 + A_c^2)} \pm A_k), \\ &\tan\psi = A_c/A_s. \end{aligned}\right\} \quad (14)$$

CALCULATION OF CONSTANTS CHARACTERIZING RESISTANCE OF SOILS IN VIBRO-SINKING OF PILE

If stresses in the reinforcing bars at any cross-section of the pile are measured, Eq 14, in addition to Eq 7, can be used in calculation of the dimensionless constants μ, λ, and τ_0. For that purpose it is better to use the difference and sum of σ_- and σ_+. Let $\alpha = (\sigma_- - \sigma_+)/2p_s$, $\beta = (\sigma_- + \sigma_+)/2p_s$, we have,

$$\alpha = A_k \text{ and } \beta^2 = A_s^2 + A_c^2. \quad (15)$$

Substituting

$$(\lambda a/2\pi)(1 - \cos\tau_0) = \gamma - \mu v,$$

$$(\lambda a/4\pi)(2\tau_0 - \sin 2\tau_0) = a + \cos\varphi,$$

$$(\lambda a/4\pi)(1 - \cos 2\tau_0) = \sin\varphi - \mu a$$

from Eq 7 into Eqs for A_k, A_s, and A_c we obtain:

$$\left.\begin{aligned} A_k &= \eta\mu v + (\gamma - \mu v)\cdot(1 - Z/L), \\ A_s &= \cos\varphi[2(Z/L) - 1] - a(1 - Z/L), \\ A_c &= \sin\varphi[2(Z/L) - 1] + \mu a[1 - (Z/L) - \eta]. \end{aligned}\right\} \quad (16)$$

Eq 15 together with Eq 16 furnishes two additional equations for the calculation of soil constants.

From the dimensionless constants μ and λ the damping and end resistance factors can be easily calculated. It is appropriate to use

$$\nu = \sin\tau_0 = x'_0/a = X'_0/A$$

as the third constant characterizing the resistance of soils in the vibro-sinking of piles. ν can be defined as the percentage of rise with tensile restoring force acting. A typical set of constants for deep vibro-sinking in sand strata is

$$q = 3 \text{ ton-sec/cu.m.}, \quad k = 1 \text{ ton/sq.cm.}, \quad \nu = 17 \text{ per cent.}$$

CONCLUSION

A reasonable mathematical model of the resistance of soils in vibro-sinking of large-diameter pipe piles can be set up by assuming viscous damping and piecewise linear end resistance with hysteresis loop. The presence of superharmonics of higher orders and fairly large tensile stresses in reinforcing bars of pile walls at cross-sections a little above the cutting shoe of the pile in measured acceleration and stress curves testify to the feasibility of such a representation of the resistance of soils.

The approximate solution for steady-state motion of vibro-sinking gives a system of three equations involving three vibration parameters and three soil constants. If the average sinking velocity V, amplitude of vibratory displacement A or acceleration B, and phase angle φ are measured in a stage of steady vibro-sinking, the system of equations can be solved to give three soil constants. If one of the parameters is not measured or is not available for calculation, additional equations of stresses in reinforcing bars at a cross-section of the pile wall must be used.

With resistance of soil as the disturbing force on the pile, the steady-state solution for stress waves in reinforcing bars can be found without much difficulty.

Using observed field data, numerical values of constants characterizing the resistance of soils can be readily computed. When soil constants for different soil strata are available, it is anticipated that an analytical and more reasonable approach to the design of foundations on large-diameter pipe piles and the vibrators will then be possible.

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