Cratering by Explosives as an Earth Pressure Problem

Formation de cratères au moyen d'explosifs considérée comme un problème de poussée des terres

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SUMMARY

Factors governing the sizes and shapes of craters formed by conventional and nuclear explosions in earth media are analysed in this paper. Based on observations made during full-size and model tests, a new theory of cratering is proposed which enables a rational analysis to be made of craters that are of interest for engineering purposes. The most important simple solutions are outlined. Consequences of the proposed theory are investigated and found to be in general agreement with experience. A modified scaling law for crater dimensions, which takes into account the strength and deformation characteristics of cratered media, is presented.

SOMMAIRE

On présente une analyse des facteurs qui influent sur les dimensions et les formes des cratères produits par explosions conventionelles ou nucléaires dans le sol. Se basant sur les observations faites pendant des essais à l'échelle naturelle ainsi que sur les modèles, on propose une nouvelle théorie de formation des cratères pour excavations. Les plus importantes des solutions simples sont décrites. Les conséquences de la théorie proposée sont examinées et sont généralement trouvées en accord avec l'expérience. On arrive à une loi d'échelle modifiée, pour les dimensions de cratères, qui tient compte des caractéristiques de résistance et de déformation du sol en question.

WHEN AN EXPLOSIVE CHARGE is detonated in an earth medium, a sequence of phenomena can be observed which, at the end, results in a depression in the surface known as the *apparent crater* (Fig. 1). A close examination reveals that a part of the medium near the surface has been separated from the original mass and displaced to sometimes considerable distances. The lower limit of the remaining loose material defines a larger depression known as the *true crater* (Fig. 1). At the same time, other parts of the surrounding medium may have been fractured and plastically deformed with significant changes in structure.

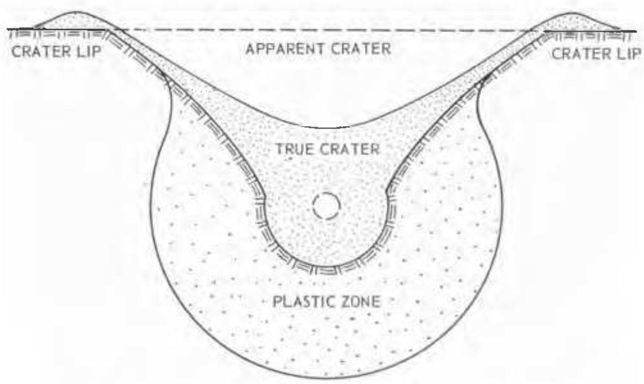

FIG. 1. Apparent and true crater formed by explosion.

Among the many questions that may be of interest in connection with the mentioned sequence, the following may be listed as most important when cratering is considered for engineering purposes.

1. What will be the size and shape of true and apparent craters formed by a given explosive charge placed at any depth of the earth medium in question?
2. What intensity of peak and residual stresses and displacements can be expected in the earth medium as well as in its pore fluid?
3. What will be the magnitude and extent of volume changes (compression, expansion) and structural changes (fracturing, remoulding) of the earth medium?

Although a few preliminary theories dealing with some aspects of this problem have been proposed (Pokrovskii and Fedorov, 1957; Maenchen and Nuckolls, 1961), the answers to these questions have been sought in the past primarily by establishing empirical relationships between charge size, depth of burst, and apparent crater dimensions for some cratered media (e.g., Waterways Experiment Station, 1961). However, the extrapolation of these empirical relationships to other media is full with uncertainty. It is known that crater sizes in media tested so far may vary for identical charge conditions by a factor of three or even more. Which physical characteristics of the cratered medium significantly influence crater sizes and shapes, or sizes of fracture zones and plastic zones, is largely unknown. Only a rational theory of cratering which recognizes that this is a very complex dynamic earth pressure problem may be able to fill this gap. Elements of such a new theory are exposed herein.

MECHANISMS OF CRATERING

Numerous observations and studies of cratering by both conventional and nuclear explosives (Johnson, *et al.*, 1959; Whitman, 1959; Nordyke, 1961; Violet, 1961; Townsend, *et al.*, 1961; Colorado School of Mines, 1961) have led to a fair understanding of the processes and phenomena involved. The following is a simplified interpretation of the basic mechanisms that lead to formation of craters by explosives (Vesić and Barksdale, 1963).

Consider the problem of an explosive charge deeply buried inside a cavity in a medium of known properties (Fig. 2a). If any such cavity were exposed to steadily increasing internal pressure p, there would be an ultimate cavity pressure, p_u, which would cause very large plastic deformation of the surrounding medium. Detonation pressures of all the explosives greatly exceed this ultimate pressure; this is why the cavity is expanded by an explosion. However, the expansion of the cavity by confined gases causes a drop of the internal pressure. It is reasonable to admit that expansion will take place as long as this internal pressure exceeds the ultimate cavity pressure p_u. When the cavity reaches its ultimate radius R_u, the internal pressure will be p_u and equilibrium will exist, at least for a while.

Further events depend greatly on the nature of the cratered medium, and differ in cohesionless materials as compared to cohesive materials and rock. In any case, after the gradual escape of gaseous by-products, the roof of the cavity may collapse, involving partial (Fig. 2c) or total subsidence (Fig. 2d) of the overlying material.

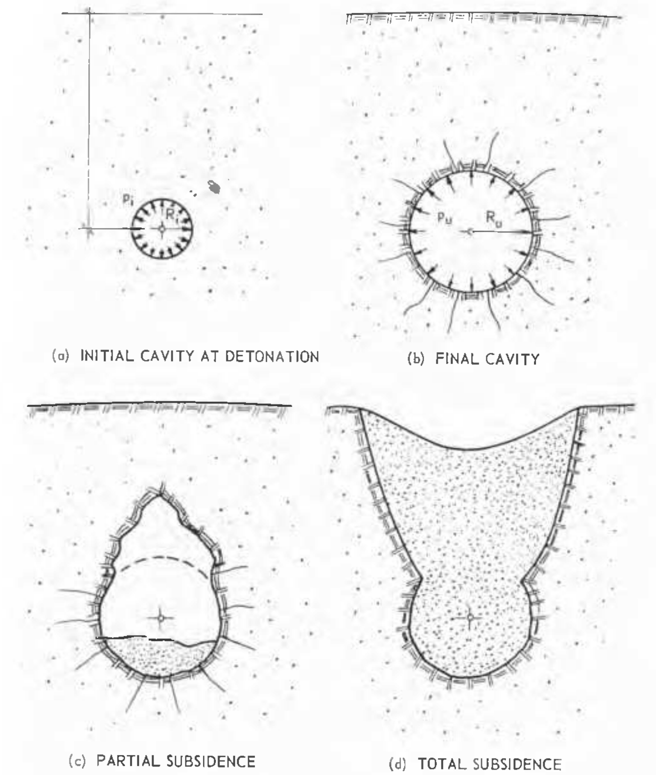

FIG. 2. Mechanisms of cratering at greater depth.

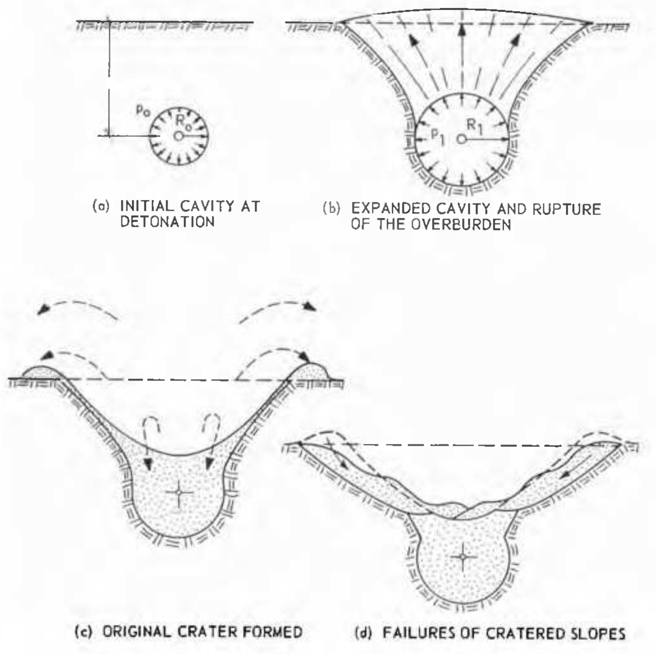

FIG. 3. Mechanisms of cratering at moderate depth.

A somewhat modified sequence of events takes place if the explosive charge is detonated at a moderate depth close enough to the free surface (Fig. 3a). Here the expansion of the cavity proceeds until the remaining overburden, in many cases already fractured by spalling, is sheared away (Fig. 3b). This happens at an internal pressure p_1 which is generally greater than or equal to p_u. Due to high kinetic energy and considerable velocity of escaping gases the sheared and fractured material is lifted from its original position and deposited either inside the crater or at the lips (Fig. 3c). Subsequent events, depending again on the nature of the cratered medium and groundwater conditions, consist of different kinds of slope failures (Fig. 3d).

Finally, a basically different sequence of events takes place if the explosive charge is set off at a very shallow depth or at the surface of the cratered medium. In such a case the gas sphere is barely formed or not formed at all. Depending on deformation and strength characteristics of the medium, plastic deformation or fracturing associated with the scouring action of gases becomes the predominant mechanism. Compression of the underlying soil is of only minor significance.

BASIC ASSUMPTIONS

Since satisfactory results have been obtained by essentially static approaches to so many dynamic problems in engineering, it is proposed to consider the problem of cratering as a problem of expansion, by a variable static pressure, of a cavity inside a solid.

Thus, it is assumed that, after an instantaneous detonation, a spherical cavity filled with gas at high pressure exists within the solid. In the case of conventional (chemical) explosives, this initial cavity will have the size of the explosive chamber and will be filled with gaseous combustion by-products. In the case of nuclear explosives, the initial cavity will have the size of the gas ball and will be filled largely by vapourized cratered medium. As the cavity expands, the inside pressure, assumed uniform, varies with cavity volume according to an equation of state for the gas in question. Such equations for the gaseous products of different chemical explosives or different earth materials are available (Jones and Miller, 1948; Armour Research Foundation, 1958).

With the cratered medium it is assumed for all basic computations that a part of it, adjacent to the cavity, behaves as a rigid-plastic solid, defined by a Mohr's envelope, or, more simply, by a shear-strength intercept (cohesion), c, and an angle of shearing resistance, ϕ. At a sufficient distance from the explosive charge the medium is assumed to behave as a linearly deformable, isotropic solid defined by a deformation modulus, E, and a Poisson's ratio, ν.

OUTLINE OF PROCEDURE

Based on preceding considerations the following procedure is proposed for crater analysis:

Step 1. Determine the ultimate cavity pressure, p_u, and the corresponding cavity radius, R_u, as well as radius of plastic zone, R_p. This is a known earth pressure problem, for which several solutions, dealing with special cases, have been known in the past (Bishop, *et al.*, 1945; Ménard, 1957; Kérisel, 1958; Ladanyi, 1961; Gibson and Anderson, 1961). A general solution of that problem for spherical and cylindrical cavities in a medium possessing both a cohesion, c, and an angle of internal friction, ϕ (Vesić, 1964) indicates that

$$p_u = cF_c + qF_q \qquad (1)$$

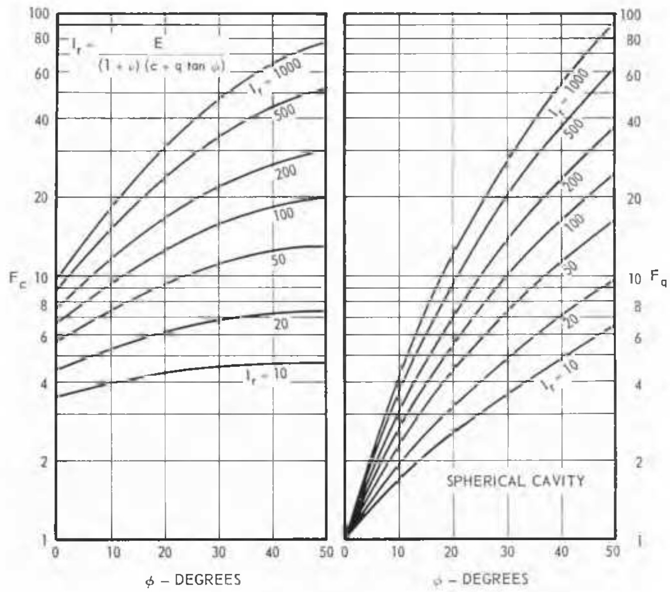

FIG. 4. Spherical cavity expansion factors.

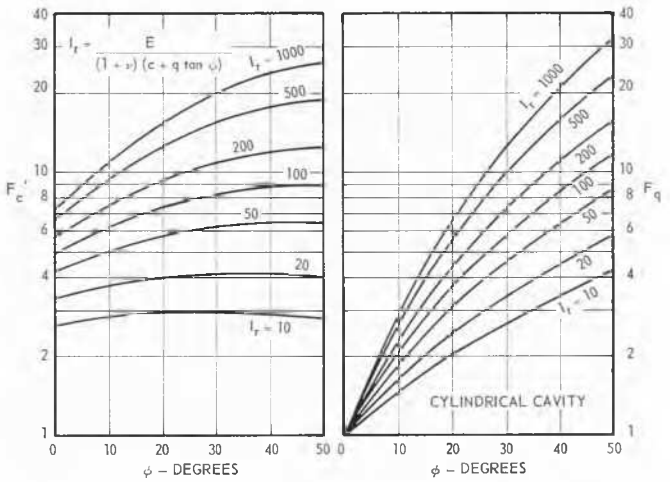

FIG. 5. Cylindrical cavity expansion factors.

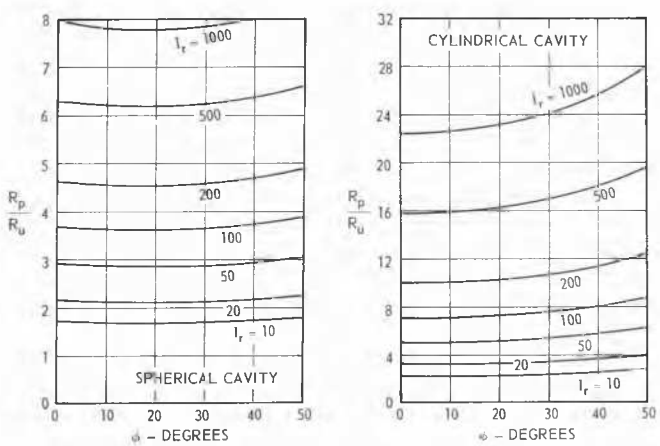

FIG. 6. Ratio R_p/R_u.

where q is the overburden pressure and F_c, F_q are dimensionless *cavity expansion factors.* These factors are shown in Figs. 4 and 5 as functions of the angle of internal friction and a quantity I_r, called *rigidity index*, and expressed by

$$I_r = E/(1 + \nu)(c + q \tan \phi). \tag{2}$$

The radius of the plastic zone R_p around the cavity is found to be governed by the rigidity index, as shown in Fig. 6. Typical values of rigidity indexes for different known media and static loading conditions are given in Table I, together with typical resulting values of the ratio of the radius of the plastic zone, R_p, to the radius of the spherical cavity, R_u. The agreement of these values with experimental observations is good and will be discussed later.

TABLE I. TYPICAL VALUES OF RIGIDITY INDEX, I_r

Medium	Rigidity index I_r (Eq 2)	R_p/R_u
Rock	250 to 700	5 to 7
Sand (loose to dense)	100 to 500	3.5 to 6.5
Saturated clay (soft to stiff)	20 to 600	2 to 7
Micaceous silt	20 to 50	2 to 3
Mild steel	525	6.4

Step 2. Check the stability of the mass overlying the cavity (Fig. 3b). For this, a solution of the problem of expansion of a cavity close to the surface of a semi-infinite solid needs to be developed. Such a solution has been proposed and programmed for an electronic computer (Vesić, Wilson, Clough, and Tai, 1964).

Step 3a. If the analysis outlined in Step 2 shows a stable cavity, investigate the short- or long-term stability of this cavity against subsidence, whether local or general (Fig. 2c, d). Such an investigation represents a familiar problem of soil and rock mechanics.

Step 3b. If, on the other hand, the analysis in Step 2 shows that the overlying ground will be sheared off, continue the analysis by estimating the dimensions of the apparent crater and checking the immediate and long-term stability of cratered slopes (Fig. 3d). The latter again represents a familiar problem of soil and rock mechanics. Peculiar aspects of the problem of cratered slopes have been discussed by Whitman (1959).

The theory exposed in the preceding paragraphs is based on a series of simplifying assumptions which, to a greater or lesser degree, deviate from reality. Thus, in each particular case there are limitations in application of the theory that should be taken into consideration. Some of the deviations, such as the effects of appreciable compression or expansion of the plastic zone, or the effect of non-isotropy of initial stress conditions, can be handled analytically. Many other effects, such as fracturing or cracking of some media, need to be evaluated on the basis of judgment and experience, as in any other soil engineering problem. Still, if the major premises of the theory are essentially correct, the major consequences should agree with reality, and the numerical answers should be reasonably close to observations. Some evidence to that effect is shown in the following text.

INFLUENCE OF THE PROPERTIES OF CRATERING MEDIUM ON CRATER DIMENSIONS

According to considerations presented earlier, the ultimate cavity radius, R_u, is related to the ultimate cavity pressure, p_u, through the equation of state of the gaseous products of the explosion. Such an equation can generally be presented in the form:

$$p_u(V_u/W)^n = C. \tag{3}$$

Here V_u is the ultimate cavity volume in cubic feet, W is the explosive weight or yield in kilotons, n is a dimensionless number analogous to the adiabatic exponent, and C is a constant for the gaseous product in question.

If we consider a point charge and a spherical cavity, Eq 3 can be transformed into:

$$R_u = C_1(W^{1/3}/p_u^{1/3n}), \tag{4}$$

where C_1 is another constant. Assuming that the true crater dimensions increase in direct proportion with the ultimate cavity radius, the following statement can be made: *True crater dimensions are proportional to the cube root of the energy yield of the explosive and inversely proportional to the 3nth root of the ultimate cavity pressure* p_u.

The exponent $3n$ generally varies with p_u, with the explosive, and, for nuclear explosives, with the cratered medium. For T.N.T. explosive and low ultimate pressure range ($p_u <$ 500 kg/sq.cm.), the exponent $3n$ is approximately equal to four, increasing to about eight in the 10,000 kg/sq.cm. pressure range. In the case of nuclear explosions, the exponent is again around four in the very low pressure range ($p_u <$ 10 kg/sq.cm.) increasing to about eight in the 10,000 kg/sq.cm. pressure range. It may be said that the exponent $3n$ is generally higher for nuclear than for chemical explosives.

CRATERING MEDIA AND SCALING LAWS

The ultimate cavity pressure, p_u, generally varies with the shear strength and rigidity index of the cratered medium. In some media, such as overconsolidated saturated clays and certain rocks, which have a practically constant shearing strength over a considerable depth, the pressure p_u will be independent of depth. According to Eq 4, the crater dimensions in such media will scale as $W^{1/3}$, as evident from numerous observations (Vaile, 1961; Vortman, 1961). A different situation occurs in conditions where the shearing strength increases with depth. This would be the case of deep deposits of normally consolidated, saturated clays, or of sand and gravel, as well as of rock such as sandstone or tuff. According to Eq 1 a significant increase of p_u with depth occurs in such media.

To examine the scaling laws in such cases more closely, consider what happens in a deep homogeneous medium in which the ultimate cavity pressure increases from a finite value, $\bar{p}_u$, at the surface as a linear function of charge depth z:

$$p_u = \bar{p}_u + \beta z. \tag{5}$$

For very small yields and therefore relatively shallow depths, βz will be small compared with $\bar{p}_u$, p_u will not vary significantly, and the crater dimensions may scale close to $W^{1/3}$. However, for very large yields, and corresponding large depths, $\bar{p}_u$ will be small compared with βz. Neglecting the former quantity and assuming that the charge depths z are taken proportional to $W^{1/m}$, where m is a scaling exponent, we find from Eq 4 that crater dimensions scale as:

$$1/m = n/(3n + 1). \tag{6}$$

Thus, if $3n = 4$, the scaling exponent m for very large yields would tend to 3.75; if $3n = 8$ it would tend to 3.37.

These findings are in general agreement with observations (cf. Nordyke, 1961). A similar trend of the scaling exponent in desert alluvium of Nevada has been detected and qualitatively explained by Chabai and Hankins (1960). However, it was suggested that at very high energy yield m would tend to four. Since at very large depths the ultimate cavity pressure, p_u, increases to the order of magnitude where $3n$ would be closer to eight, the possibility of m reaching the value of four, as suggested, is open to serious doubt. According to the proposed theory the ultimate value should remain around 3.4.

It is of interest to point out that, according to the proposed theory, the highest possible scaling exponent, in an extraordinarily soft deposit where shear strength would increase uniformly with depth, should be $m = 3.75$.

CRATER COMPARISONS

To check the value of the proposed theory it is of greatest interest to make comparisons of actual crater dimensions with those evaluated theoretically. Unfortunately, few cratering investigations have reported usable data on the mechanical properties of the cratered medium. To our knowledge, there are no data on the dynamic high-pressure triaxial strength of the media in question, which, in principle, is to be introduced for cratering investigations. Therefore, the comparisons presented herein are tentative in nature: they are made, whenever appropriate, on the assumption that the soil strength is equal to the static low-pressure strength.

One large-scale cratering investigation for which data on the static triaxial strength of the medium has been secured is the Rainier event. This was a 1.7-kiloton nuclear explosion, 895 ft deep, in a thick formation of bedded tuff (Johnson, *et al.*, 1959). Standard triaxial tests on 0.5-in. diameter samples of that rock material were made with confining pressures up to 1,500 tons/sq.ft., furnishing the following data: angle of shearing resistance, $\phi = 31°$; strength intercept (cohesion), $c = 84.2$ tons/sq.ft.; natural unit weight, $\gamma = 125$ lb/cu.ft.; Poisson's ratio, $\nu = 0.10$; Young's modulus, $E = 26{,}650$ tons/sq.ft.

The ultimate cavity pressure computed from Eq 1 with factors from Fig. 4 is $p_u = 2{,}777$ tons/sq.ft. The corresponding cavity radius is $R_u = 72$ ft and the radius of the plastic zone $R_p = 337$ ft. The actually observed radii are $R_u = 62$ ft, $R_p = 280$ ft. Both results are remarkably close to reality. This is particularly true if we consider that the ultimate cavity pressure is computed on the basis of static shear strength. By assuming a reasonably higher dynamic shear strength even better agreement would be found.

Another large-scale cratering test which could be investigated by using the same data is the Logan event (Bennett, *et al.*, 1960). This was a 5.0-kiloton nuclear explosion 932 ft deep in a tuff stratum similar to that in which the Rainier device was exploded. Assuming that all the material properties were the same, the computation gives: $p_u = 2{,}812$ tons/sq.ft., $R_u = 103$ ft. The actually observed average cavity radius was $R_u = 90$ ft, which is also remarkably close.

A few other comparisons also gave very encouraging results. For instance, the reported ratios of the plastic zone radius, R_p, to the cavity radius, R_u, vary from 2.5 to 6.7 (Pokrovskii and Fedorov, 1957; Townsend, *et al.*, 1961) a result which agrees very well with the theoretical prediction (Fig. 6).

In summary, comparisons that were possible on the basis of data available substantiate the proposed theory. Additional observations and a thorough analysis of past cratering experiments, using properly determined physical characteristics of the cratered media, are in progress.

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