6/26

An Approximate Solution for Drainage Rates under Drawdown Conditions in Sand

Solution approximative pour la vitesse de drainage dans le sable sous conditions d'affaissement

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SUMMARY

The grain size characteristics of the upstream sand filter zone of the Sesquilé Dam raised the question of whether its drainage facilities were adequate under drawdown conditions to prevent any instability in this zone. This paper presents the laboratory testing programme, which consisted of both instantaneous drawdown and constant rate of drawdown tests, and the analysis developed thereof. Simple and approximate equations were derived for the level of 100 per cent saturation, which fit the experimental curves some time after the start of each test, and their applications to stability calculations are briefly outlined.

SOMMAIRE

Les caractéristiques de la granulométrie du sable des filtres amont du barrage de Sesquilé ont posé la question de savoir si les caractéristiques de drainage étaient adéquates dans la condition de vidange rapide pour éviter les problèmes de stabilité dans cette zone. Cet article présente le programme d'essais en laboratoire qui ont consisté en des essais de vidange instantanée ou à vitesse constante, et l'analyse développée de ces essais. Des équations simples et approximatives ont été dérivées pour le niveau de 100 pour cent de saturation, lesquelles s'accordent avec les résultats expérimentaux quelque temps après le début de chaque essai. On indique brièvement les applications de ces équations aux calculs de stabilité.

THE RESULTS PRESENTED IN THIS PAPER were obtained during a laboratory investigation which was performed in order to determine the water lag in the material used for the upstream fine sand filter zone of Sesquilé Dam. This earth dam, of 52 m maximum height, is of the inclined impervious core type. The results and analysis reported herein are valid only for vertical one-dimensional gravity drainage. Due to space limitations only the essential information is given.

NOMENCLATURE

a, b, c, f, m = arbitrary constants
A = area of steel test cylinder = 330 sq.cm.
G_a = air-water space ratio (Terzaghi, 1943)
H = elevation of top of water level in water tank above datum plane
ΔH = total drawdown produced by water tank
$dH/dt = V$ = drawdown rate applied by water tank
H_c = capillary head above atmospheric pressure line
H_o = elevation of atmospheric pressure line above datum plane
H_{oo} = initial elevation of atmospheric pressure line, at start of test
H_p = pressure head on bottom of steel test cylinder, referred to the same datum plane. Values obtained from piezometer data.
ΔH_p = total effective drawdown indicated by piezometers
$dH_p/dt = V_p$ = effective constant drawdown rate
H_s = elevation of 100 per cent saturation line above datum plane
H_{so} = initial elevation of atmospheric pressure line, at start of test
$dH_s/dt = V_s$ = rate of change of elevation of 100 per cent saturation line
i = hydraulic gradient
K = coefficient of permeability
Q = total discharge
q = rate of discharge = dQ/dt
t = time
W_d = specific drainage based on change in elevation of 100 per cent saturation line (H_s)
W_{dc} = specific drainage for constant drawdown tests, based on change in elevation of 100 per cent saturation line (H_s)
W_{do} = specific drainage based on change in elevation of atmospheric pressure line (H_o).

Although the same nomenclature is used for specific drainage as that proposed by Kellogg (1948), the expression used in this paper is different from Kellogg's and the two should not be confused. In this paper specific drainage is defined as the volume of drainable water per unit volume of soil; Kellogg defined it as drainable volume of water per unit volume of soil, per unit change in head. Kellogg's specific drainage has units; the one used in this paper is dimensionless.

TEST APPARATUS

Figs. 1 and 2 show the general and some detailed aspects of the test apparatus used. Basically, the drawdowns were produced by lowering the water tank manually. The water tank raising or lowering system consists of a pulley system connected by rope to a drum which is welded to the lower front of the steel cylinder. This drum has a steel handle, and a hole and pin system to secure the drum at any position.

TESTING PROGRAMME

Sand (85 per cent passing the No. 40 U. S. standard sieve, and 15 per cent passing the No. 200 sieve), obtained from a test pit in the fine sand filter zone, was placed and compacted to the top of the steel cylinder. The dry density and water

FIG. 1. Photograph of test apparatus.

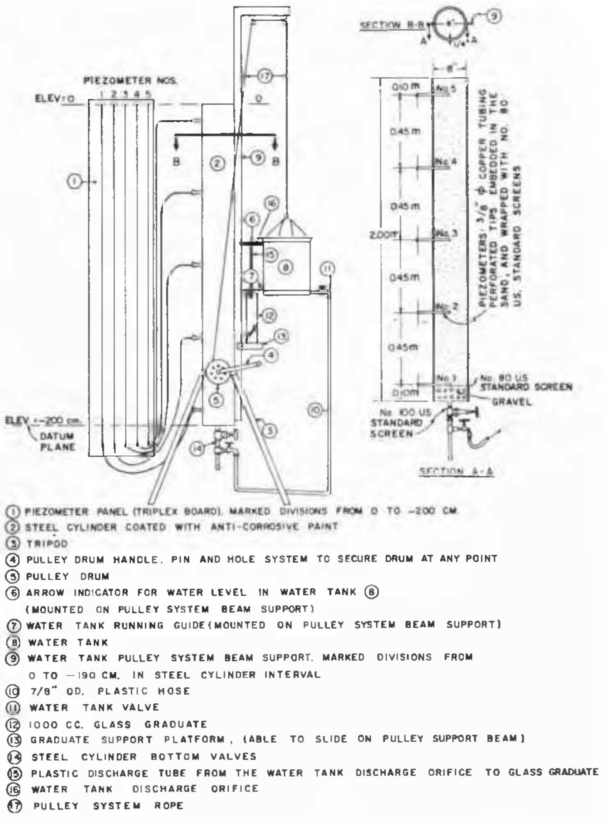

FIG. 2. Sketch of test apparatus.

content obtained for the sample, are representative of 43.7 per cent of the construction control test results and fall in the optimum −3 per cent water content range. The same sample was used for all the tests.

Saturation was accomplished by raising the water tank to an elevation somewhat higher than the predetermined initial water level for each test. Usually, good saturation was obtained in a period of one to two days. Saturation was verified by the absence of air bubbles in the piezometer connecting systems, by the rapid reflection of any movement of the water tank on all the active piezometers, and by the lack of variation in piezometric levels when the bottom valves of the cylinder were closed. Washing out of fines through the connecting systems was prevented by placing No. 80 and No. 100 U. S. standard screens in critical places, as shown on Fig. 2.

The testing programme consisted of two groups of tests: (1) instantaneous drawdown tests, where the total head difference was applied instantaneously; (2) constant rate of drawdown tests, where the total head difference was produced by a constant rate of manually lowering the water tank.

The initial conditions, and the characteristics of each test, are indicated in Table I. Before the start of each test full saturation of the sand to the predetermined initial water level was verified. In all the tests total discharge and piezometer readings at small time intervals (usually of 1 or 2 min.) were obtained. Temperature readings in the water tank were made at frequent intervals. In addition, the time lag of the piezometers was determined. This lag was negligible and similar for the four piezometers tested, so this factor was neglected in the computations of the main test results.

TABLE I. TEST RESULTS

Test	ΔH (cm)	V (cm/min)	H_{00} (cm)	ΔH_p (cm)	V_p (cm/min)	K (cm/min)	
						(1)†	(2)‡
5	50	Instant.	0	49	Instant.	0.405	—
6	50	Instant.	−32.5	48	Instant.	0.328	—
1	100	Instant.	0	—	Gradual*	—	—
2	100	Instant.	−64.5	75	Instant.	0.344	0.268
3	150	Instant.	0	—	Gradual*	—	—
9	150	Instant.	0	140	Instant.	0.319	—
4	150	Instant.	−82	—	Gradual*	—	—
8	200	Instant.	0	150	Instant.	0.309	0.267
7	200	Instant.	−8	—	Gradual*	—	—
4A	100	0.5	−20	91	0.455	0.343	0.295
2A	100	1.0	−20	92	0.92	0.354	0.262
3A	100	5.0	−20	88	4.4	0.358	0.340
1A	100	10.0	−20	88	8.8	0.363	0.323

*Test results discarded, see text.

†(1) Calculated from each reading of each test, using

$$K = \frac{Q(n+1) - Q(n)}{[t(n+1) - t(n)]\dfrac{[i(n+1) - i(n)]\,A}{2}}$$

‡(2) Calculated on basis of $K = -dQ/dt \div i.A$, using Q and i, *versus* t curves.

All the tests were performed in the laboratory of the Colombian Ministry of Public Works in Bogotá. A minimum of two people operated the apparatus in all the tests.

TEST RESULTS

Due to the small temperature variations (between 63 F and 57 F), no temperature corrections were applied to the test results.

Although the data obtained from the piezometer lag tests did not justify application of any correction to the test results, there was a definite lag with respect to the applied drawdown in the whole system, and in all the tests. This lag is given by the difference between the applied drawdown, and applied drawdown rate, *versus* the effective drawdown, and effective drawdown rate as shown on Table I. The

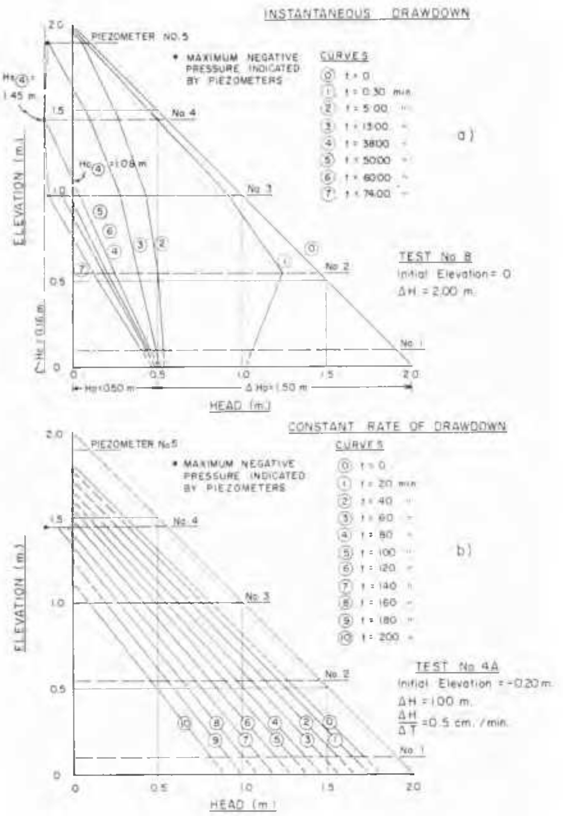

FIG. 3. Head *versus* elevation curves.

effective drawdown and effective drawdown rates for all the tests were obtained on the basis of extrapolation to the zero elevation line on the head *versus* elevation curves (Fig. 3).

Those results of the instantaneous drawdown tests, where the instantaneous drawdown was not reflected in the piezometers after the application of the drawdown, were discarded because they were not instantaneous in the true sense of the word. All the results of the constant drawdown tests were interpretable. Methods of obtaining the values of i, H_c, H_s, and H_p were identical for all the tests and are described below.

The hydraulic gradient i shown is an average value for all the piezometers and is based on a straight line gradient between each and every piezometer. Its value was computed on the basis of the following equation:

$$i = 1 - (H_p + H_c/H_s) \qquad (1)$$

which is basically equal to Terzaghi's equation (1943) for the hydraulic gradient during the drainage of a stratum of ideal sand through its base.

The value of H_c (height of capillary rise) varied between 14 and 20 cm, 16 cm being representative of 78 per cent of the test results, so this value of $H_c = 16$ cm was used for every test. H_s, H_p, and H_0 values were determined extrapolating the straight line gradient to the 16 cm head line, to the zero elevation line, and to the zero head line, respectively, using for each test the head *versus* elevation curves such as those shown on Fig. 3*A*.

Values of permeability K were computed as indicated in Table I (notes 1 and 2), and represent the average of all the computed points for each test. Apart from Table I, results are also shown on Figs. 3, 4, 5, and 6.

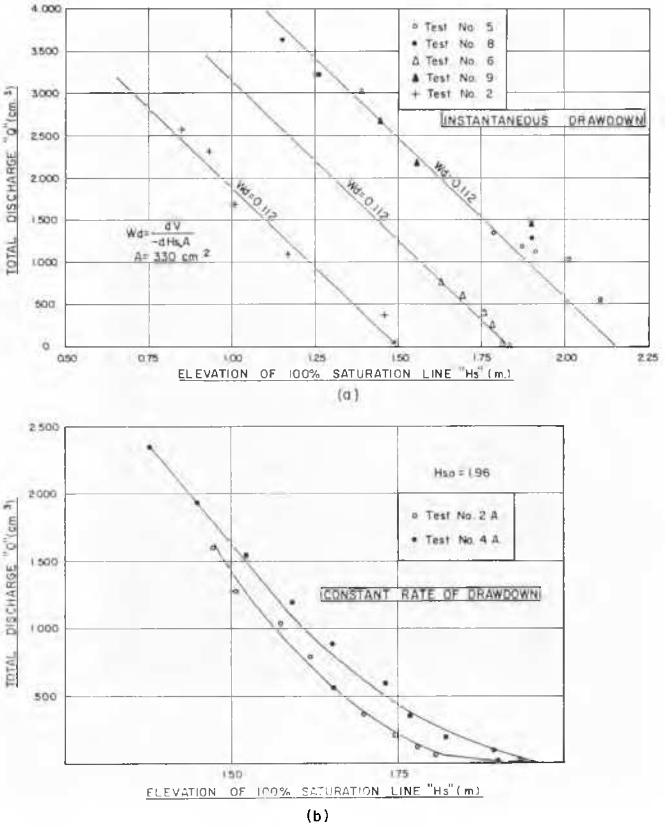

FIG. 4. Elevation of 100 per cent saturation line *versus* total discharge.

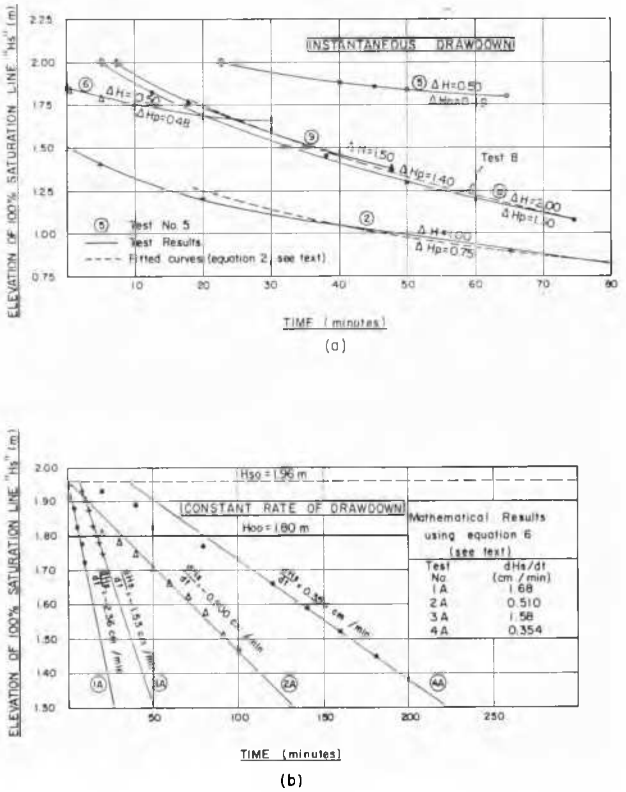

FIG. 5. Elevation of 100 per cent saturation line *versus* time.

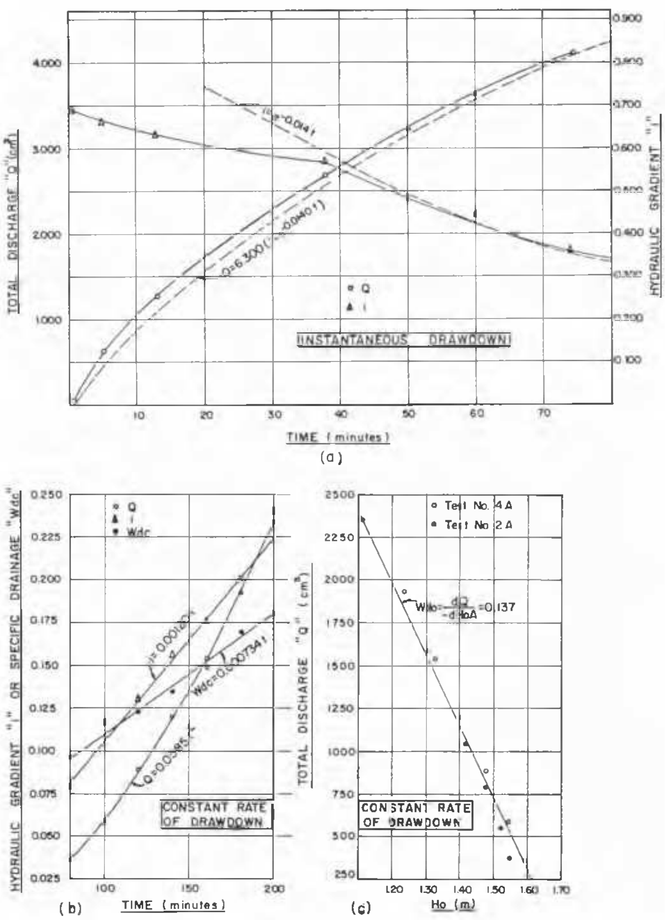

FIG. 6. (a) Hydraulic gradient and total discharge *versus* time for instantaneous drawdown Test 8. (b) Hydraulic gradient, total discharge, and specific drainage (W_{dc}) *versus* time for constant rate of drawdown Test 4A. (c) Elevation of atmospheric pressure line (H_0) *versus* total discharge for constant rate of drawdown Test 4A.

ANALYSIS OF INSTANTANEOUS DRAWDOWN TEST RESULTS

The specific drainage W_d has a constant value during all the tests and can be considered equal to 0.112 for all the tests (Fig. 4a). Although not shown, the slope of H_0 *versus* Q curves is exactly equal to the H_s *versus* Q curves, so that $W_{do} = W_d$. The constancy of W_d implies that Terzaghi's G_a (the ratio of air to water space in the unsaturated zone) is a constant in this type of test.

After the line of 100 per cent saturation (H_s) has passed the atmospheric pressure line's initial position (H_{oo}), H_s *versus t* can be fitted by a curve giving the following relationship:

$$H_s = H_{so} - \Delta H_p(1.00 - e^{-at}), \qquad (2)$$

where $a = K/W_d \cdot \Delta H_p$. Tests 2 and 8 (Fig. 5a) are shown fitted with curves of this type.

Starting from the fundamental equation (Terzaghi, 1943)

$$-dH_s/dt = K/W_d i \qquad (3)$$

where $W_d = nG_a$, it can be shown that:

$$i = e^{-at} \qquad (4)$$

$$Q = W_d \cdot \Delta H_p \cdot A(1.00 - e^{-at}). \qquad (5)$$

Curves of i and Q *versus* time for Test 8 (Fig. 6a) are shown fitted with curves whose values are closely approximated by Eqs 4 and 5. The discrepancy between the gradient obtained from test results and Eq 4 is especially indicative of the inadequacy of the fitted curve during the time interval when H_s has not passed H_{oo}.

ANALYSIS OF CONSTANT RATE OF DRAWDOWN TESTS

Specific drainage W_d is not a constant for the actual test period interval (Fig. 4b), nor is it equal for all the tests. It has a linear relationship with time as shown on Fig. 6b. Due to this variation with time, and to distinguish this specific drainage from the constant specific drainage found in the instantaneous drawdown tests, its nomenclature was changed to W_{dc}, so that:

$$W_d = \text{constant}$$

$$W_{dc} = b \cdot t \text{ and } dW_{dc}/dt = b.$$

Although not shown graphically the coefficient b varies linearly with the effective drawdown rate.

The specific drainage W_{do} based on H_o is a constant equal to 0.137 as shown on Fig. 6c (H_0 *vs.* Q). Since H_c is constant, the difference between W_{dc} and W_{do} implies that G_a is not a constant in the unsaturated zone and that the amount of water from the unsaturated zone which contributes to the total discharge increases linearly with time.

After the 100 per cent saturation level H_s has passed the initial atmospheric pressure level H_{oo}, H_s *versus t* is given by a linear relationship such that:

$$dH_s/dt = -V_p(1.00 - e^{-c}) = \text{constant (Fig. 5b)} \qquad (6)$$

where $c = K/(\pi \cdot W_{do} \cdot V_p)$, and by integration one obtains:

$$H_s = H_{so} - V_p(1.00 - e^{-c})t \qquad (7)$$

and using the fundamental relationship (Eq 3) it can be shown that:

$$i = (W_{dc}/K)V_p(1.00 - e^{-c}) = f \cdot t \qquad (8)$$

where $W_{dc} = b \,.\, t$, and

$$Q = A \cdot V_p(1.00 - e^{-c}) \int_0^t W_{dc} \cdot dt = m \cdot t^2. \qquad (9)$$

Mathematical values of the rate of change of the 100 per cent saturation line obtained using Eq 6 are shown on Fig. 5b. Except for test 1A, the mathematical values compare favourably with the experimental results, once H_s has passed H_{oo}.

COMPARATIVE ANALYSIS WITH PREVIOUS INVESTIGATOR'S RESULTS

No theoretical solutions were attempted in this investigation. As mentioned by Lambe (1948), none of the theoretical solutions applied to instantaneous drawdown tests of the type performed for this investigation fit the experimental curves for their entire length: some diverge at the initial parts of the curve, and some diverge at the end. The equations deduced experimentally in this paper fit into the first category, until the elevation of the 100 per cent saturation line passes the initial position of the atmospheric pressure line (H_{oo}).

The equations derived by Kellogg (1948) in his pioneering work, were tried on the results of the instantaneous drawdown tests, with negative results: his curve diverges for both the initial and end portions of the curves. Kellogg's solution is rather complicated, and the fact that his definition of W_d is different from that used in this paper makes it much harder to use in practical work.

No results for tests similar to the constant rate of drawdown tests performed for this investigation were found in the literature available to the authors.

APPLICATION TO THE FIELD PROBLEM

Based on a wedge type of stability analysis suggested by Mr. P. T. Bennett, which uses the hypothesis of a linear saturation line during drawdown similar to that proposed by Cedergren (1948), stability analyses were set up using Eqs 2 and 6 to compute the water lag by assumptions of daily instantaneous drawdown increments and of a constant rate of drawdown respectively, both of them given by the operational characteristics of the dam. The water lag obtained was completely negligible, due to the very small drawdown rates of Sesquilé Dam, and did not affect the static factor of safety originally computed for the upstream sand filter zone.

Piezometer installations in the field were not made, because the experimental results could not be extended to be included in a flow net theory.

CONCLUSIONS

It is believed that the equations presented in this paper are a good first approximation at the present stage of knowledge for use in the stability analysis of upstream sand filter zones, subjected to drawdown. Further research, especially on constant rate of drawdown tests, should clear up the unknowns.

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