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Soil Properties – Shear Strength and Consolidation

Propriétés des sols—Résistance au cisaillement et consolidation

Chairman/Président: L. F. Cooling (Great Britain)
Deputy Chairman/Président adjoint: C. B. Crawford (Canada)
General Reporter/Rapporteur général: O. Moretto (Argentina)

Members of the Panel/Membres du Groupe de discussion
G. D. Aitchison (Australia)
A. W. Bishop (Great Britain)
H. Leussink (Germany)
R. J. Marsal (Mexico)

Chairman: L. F. COOLING (Great Britain)

WE ARE VERY PRIVILEGED TODAY to hear a special lecture on the geology of Canada, and it is my pleasant duty to introduce the speaker, Dr. James M. Harrison, who is Assistant Deputy Minister, Research, of the Department of Mines and Technical Surveys of Canada. Dr. Harrison is a former Director of the Geological Survey of Canada and a scientist of international repute. As a scholar, lecturer, and author, Dr. Harrison has contributed substantially to the growth of geological science in Canada, and to a better understanding of Canadian geology. He is an authority on the geology of the Precambrian, having spent several years studying the Canadian Shield and in particular the Quebec-Labrador iron ore region. He is a Fellow of the Royal Society of Canada, and a Past President of the Geological Association of Canada. This spring he was elected a foreign associate of the National Academy of Sciences of the United States of America. We are looking forward to hearing from one who is a special authority on this subject. Dr. Harrison.

(Dr. Harrison's lecture appears on pp. 97–103.)

CHAIRMAN COOLING

We are very much indebted to Dr. Harrison for his fascinating talk. To me it is indicative of his mastery of the subject that he can take such a broad topic as the geology of Canada and make such an interesting and informative talk from it. We are very grateful to Dr. Harrison, and although you have already shown your appreciation, I would ask you to reinforce it in a formal way. [Applause]

Now may I introduce the members of the panel. They are: Professor H. Leussink of Germany, Dr. A. W. Bishop of the United Kingdom, Mr. R. J. Marsal of Mexico, and Dr. G. D. Aitchison of Australia.

At this time I would like to invite the General Reporter, Dr. O. Moretto, Professor of Soil Mechanics and Foundation Engineering of the University of La Plata, and Consulting Engineer of Buenos Aires, Argentina, to present a summary of his general report. Dr. Moretto.

General Reporter: O. MORETTO (Argentina)

Design for Stability in Civil Engineering, when reduced to its simplest form, can be visualized as a comparison between stressing and the ability of materials to withstand stress. The ability of a material to withstand stress is measured mainly by its mechanical properties, determined under stress conditions similar to those prevailing in the structure. It is only natural then that one of the first tasks of any branch of civil engineering dealing with a given material should be the thorough study of its mechanical properties.

For this reason, the sections of soil mechanics conferences, meetings, and technical publications referring to mechanical properties always have the most papers. The variety and complexity of the mechanical properties of soils make these studies imperative at a time when knowledge of them is still limited. The emphasis on this facet of soil mechanics was maintained in the interim between the Fifth and the Sixth International Conferences. And the Sixth International Conference is no exception to this trend; sixty-one papers have been assigned to Division 2, even though it has been limited to the shear strength and deformation of soils.

When a design for stability is confined to problems of strength and deformation in engineering practice, the contrast between stressing and material properties can frequently be separated into two independent or nearly independent operations. First, the stressing load is compared with the failure strength of the material to ensure that a reasonable margin or factor of safety exists between them; secondly, deformations are computed and checked against some previously established allowable values, usually empirical in nature. This simple scheme may often be complicated by the interdependence between failure strength and deformation which sometimes occurs, for example, in statically indeterminate problems. Yet, in soil mechanics, the distinction is frequently clear and solutions are said to be controlled either by failure or by settlement. Based on this assumption, the papers assigned to Division 2 have been divided into those concerned mainly with the shear strength of soils under static loading and those dealing mainly with soil deformation under static loading (including soil behaviour in general under dynamic loading). This session refers to shear strength of soils under static loading.

As in other branches of civil engineering dealing with non-elastic, creeping materials, strength problems are much easier to solve than deformation problems. By the same token, the determination of the strength parameters also proves easier.

Soil mechanics has done much in devising methods to determine these strength parameters in the field and, through adequate samples, in the laboratory. Study of the various typical soil materials and correlation of the results to form rational laws which permit extrapolation and understanding of the behaviour of a material are essential, if reliable solutions to practical problems are to be found. Limitations of time and expense make for solutions based on a few tests, comprising only simple stress-strain conditions. For them to provide efficient answers, however, the engineer must have a wide general knowledge of soil behaviour, with sufficient insight to extrapolate beyond the bounds of limited testing.

Much has been done to forward this aim, but the variety of soil types and conditions and the diversity of states of stress make the task ahead still formidable. Twenty-nine of the papers presented in Division 2 deal with some aspect of the shear-strength behaviour of soils. They are, of course, only a small fraction of all the papers on the subject published since the last international conference and cover the following subjects: laboratory testing devices, torsional field shear tests, pore pressure build-up in laboratory tests, shear-strength components, analysis of the standard triaxial test, anisotropic states of stress, time effects, shear strength of sand, high confining pressures, typical soils, stabilized soils, and angle of internal friction *versus* void ratio.

Earlier I said that soil mechanics has done much in devising methods to determine the shear-strength parameters in the field and, through adequate samples, in the laboratory. As a branch of civil engineering, it has matured sufficiently to have passed through the early stage of spectacular advancement. Most of the fundamental knowledge of its framework is well developed; progress is now measured in the gradual transformation of theories and of the furthering of the experimental determination of material properties, with the aim of a better interpretation of the variety of stressing conditions that are produced in nature and of the manner in which they are withstood by soils. All this work has as its main purpose the closing of the enormous gap still existing between theory and practice.

The rational framework built up by the science of soil mechanics indicates solutions for problems controlled by failure under static loading, which constitute by far the great bulk of practical problems.

These solutions require the determination of the shear strength parameters of the soils involved. Yet, the use made of progress in sampling and testing is rather limited and a really alarming amount of practical work is performed ignoring it entirely. Non-scientific solutions, such as many of the widely popular penetration tests, take their place.

This situation arises partly from sampling difficulties caused by the nature of soil and the impossibility of making rational field tests, partly from a need for extended studies of the behaviour of typical soils in nature and of the representativeness of both field and laboratory strength tests. A great number of the laboratory studies on the shear strength of soils ignore the possible relation of their results to field behaviour. It is hoped that those contributing discussions to this session will keep in mind these shortcomings. Soil mechanics has now become an integral part of everyday civil engineering. The profession expects it to provide efficient solutions of their problems, whether simple or complicated, routine or spectacular, dull or dramatic, with scientific effectiveness, so as to provide the cheapest safe solution which would make good use of existing natural possibilities.

The following subjects are proposed for panel discussion.

1. Shear-strength behaviour of soils under anisotropic states of stress, with special reference to plane strain, making comparisons with the behaviour of the same soils when subjected to standard triaxial tests.
2. Shear-strength behaviour of soils under high confining pressures, differentiating cases where particle degradation may be a factor (sand and gravel) from those in which negligible or no degradation takes place (silts and clays).
3. Effect of very large sample size on the shear strength of fine soils, coarse soils, and rockfill materials, with reference to the effect that large particle-size degradation may have on that strength when tests are run under sufficiently high confining pressures.
4. Shear-strength behaviour of non-saturated soils with special reference to methods of testing to obtain the strength parameters to be used in foundation design, taking into account possible change in strength due to consolidation, expansion, and/or wetting during or after construction, but excluding consideration of problems related to small buildings on expansive clays.

These subjects constitute some of the many topics where information appears to be in one of the three following states: (*a*) sufficient knowledge exists to allow a collection of information which would prepare the way for further advances; (*b*) a certain amount of work has been done on the subject and a résumé of conclusions is necessary to orient whatever new investigations are convenient; (*c*) so little is known about the problem that a start should be made to induce research to cover the gap. This last statement applies especially to the fourth subject suggested for discussion. A large number of the problems faced in soil mechanics practice refer to the design of simple foundations placed on unsaturated soils. However, not much is known of the shear-strength behaviour of such soils, a problem that constitutes one of the fields of practical research that needs more attention.

(*Dr. Moretto's General Report appears on pp. 220–41.*)

CHAIRMAN COOLING

Thank you very much Dr. Moretto. Dr. Moretto has already itemized the subjects proposed for panel discussion and without more ado I will invite the discussions from the panel.

Panelist: H. LEUSSINK (Germany)

ANISOTROPIC STATE OF STRESS

First of all, having been asked by the General Reporter to do so, I would like to make a few remarks on cohesionless soils with respect to models of such soils. I do so because, in the field of shearing behaviour, we in Karlsruhe have worked, and presently are working, mainly with these types of soils. We chose to do so because we agree with Dr. Moretto's statement that specific knowledge of the various factors influencing the shear strength of sand "should prove very useful in clarifying the behaviour of such materials."

Laboratory Testing Devices

At the first conference in 1936 in Cambridge, Massachusetts, Kjellman (1936), described a shear apparatus in which samples of cubic shape were investigated. The three principal normal stresses acting perpendicular to the surfaces of the cube could be varied independently. It seems, that *Lorenz, et al. (2/34)* have constructed a new—and very probably improved—version of this type of apparatus. Dr. Scott, of the California Institute of Technology and his co-workers have been working for several years with similar equipment. Dr. Scott gave some indications of this work at the Ottawa Conference in 1963. Other research workers try to avoid the great difficulties resulting from the complexity of these types of apparatus by making one side of a prismatic specimen considerably longer than the other (1 : 5 and 1 : 15 respectively). This holds true for the investigations of Bjerrum and Kummeneje (1961) and

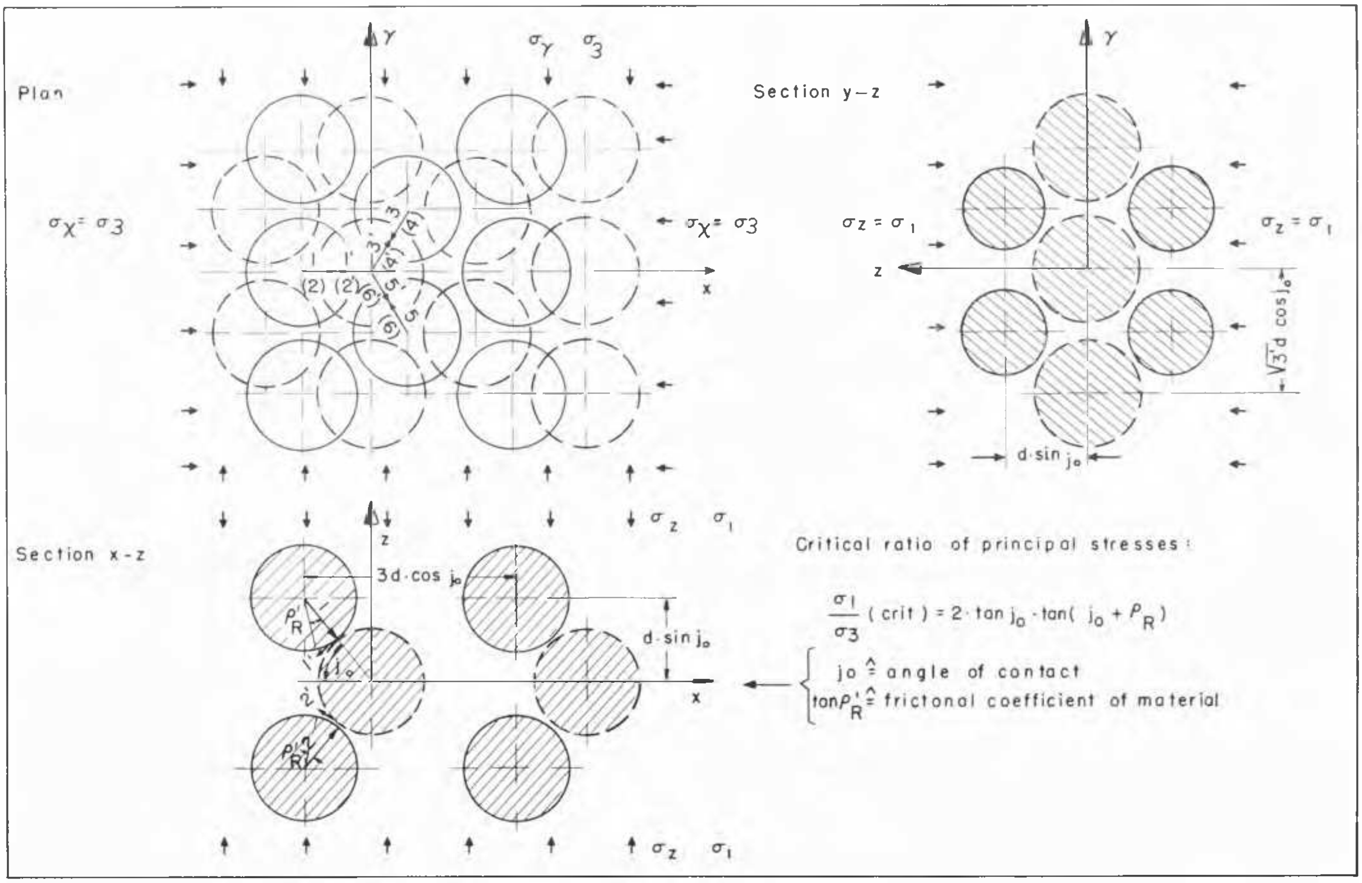

FIG. 1. Hexagonal packing in triaxial test (Wittke, 1962).

Cornforth (1964), and for our own apparatus (Leussink, 1965).

Theory

Theories describing the shearing behaviour and the stress-strain relationships for granular masses as a function of the soil parameters have of necessity rigourously simplified the problem. These theories generally assume an assembly of balls of equal diameter, arranged regularly in the form of the so-called face-centred cubic quadratic or hexagonal array. With these assumptions, Thurston and Dersiewicz (1959), Idel (1960), Dantu (1961), Wittke (1962), and Rowe (1962) derived equations for the critical ratio of principal stresses in the three-dimensional case. Most of the other derived relations seem to be contained in the equations of Wittke (1962), who considers porosity a further variable.

Fig. 1 gives an example of the arrangement of the spheres for hexagonal packing as well as the equation for the critical ratio of principal stresses in the state of failure. Mathematical relations for the law of deformation for ball packings during and after failure have been given, to my knowledge, only by Rowe (1962) and Leussink and Wittke (1963).

The only theoretical relations for the critical ratio of the principal stresses as well as for the deformations, which apply to the plane strain state are to my knowledge, given by Wittke. The outcome is that in certain cases the critical ratio of principal stresses σ_1/σ_3 is considerably higher in the plane strain case, whereas the deformations in the direction of the major principal stress are smaller than in the case of a triaxial compression test.

Experiments

The tests with natural sand given by Bjerrum and Kummeneje (1961) and Cornforth (1964) reveal that the triaxial state differs from the plane strain state by a difference in the angle of internal friction in the order of 0.5 to 4°. Fig. 2 shows test results from our laboratory for a well-rounded, uniform quartz sand (grain sizes between 0.1 and 2.0 mm; coefficient of uniformity $U \approx 2.2$). The difference is 2.5 to 3° in the angle of internal friction, ϕ respectively for dense and loose sand; or about 10 per cent difference in the tan ϕ value.

Fig. 3 shows test results with glass balls as a model for a cohesionless soil in the densest hexagonal and quadratic packing. A considerable difference between the triaxial and plane strain states could be observed for the hexagonal packing. Fig. 4 shows the results plotted for steel balls. At the same time the theoretical relations according to Leussink (1965) and Leussink and Wittke (1963) are given. The agreement seems to be fairly good, though the experiments show generally lower values than the theory.

Conclusions

In natural sands there are, differing from place to place, more hexagonal and more quadratic or other situations. Moreover, natural sands have different dimensions and irregular shapes of particles. Other properties (a total of 14) which may have an influence have been described by Wittke (1962). Nevertheless, the existing and the current theoretical and experimental investigations for cohesionless soils already give some information about the difference between the

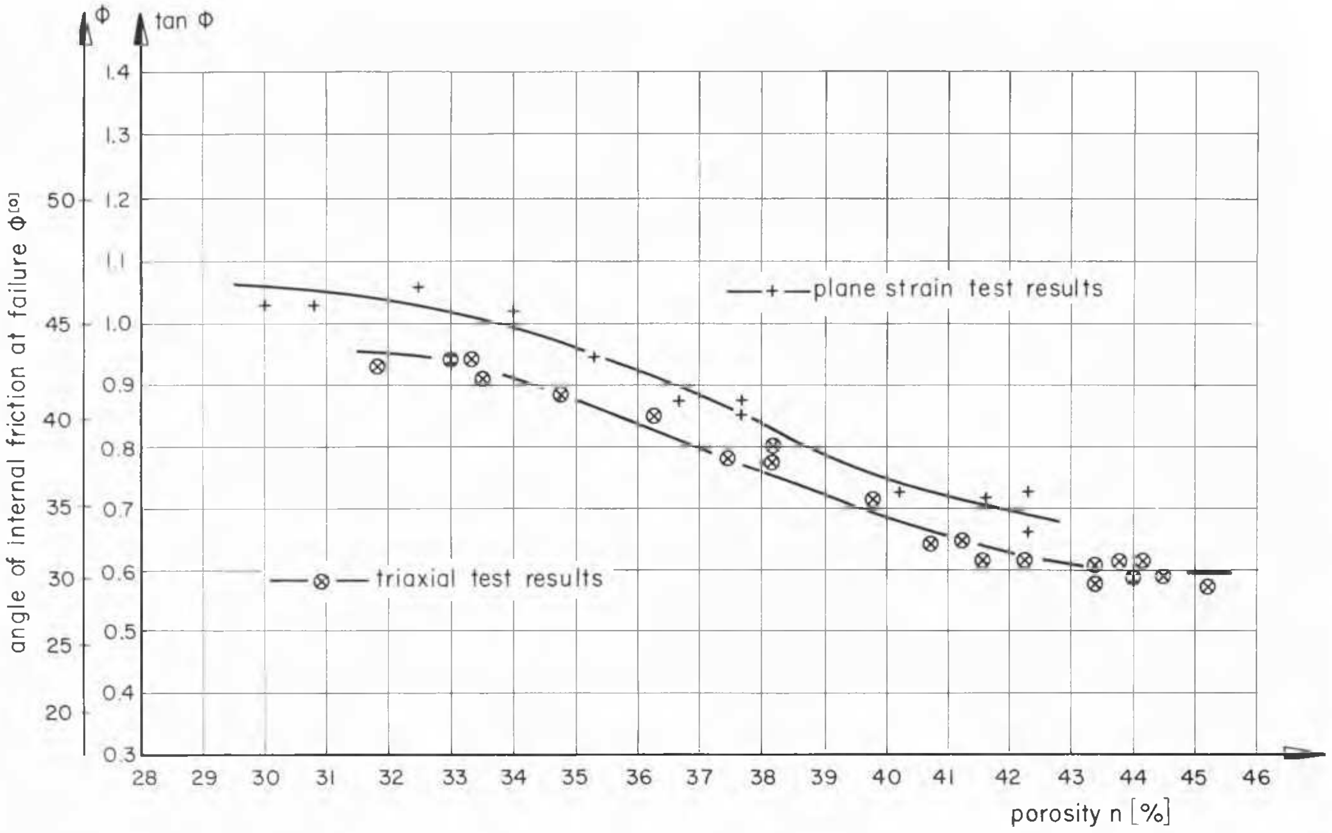

FIG. 2. Results of plain strain and triaxial tests on sand samples (Leussink, 1965).

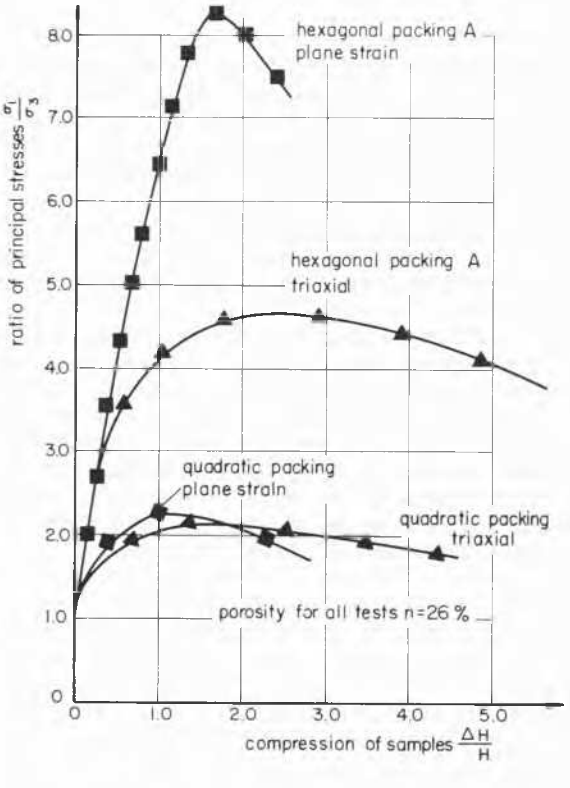

FIG. 3. Triaxial and plane strain tests on glass ball packings (Leussink, 1965; Leussink and Wittke, 1963).

triaxial and plane strain states. For very simple regular arrangements of particles the difference can even, as we saw, be postulated theoretically. Corresponding tests prove the reliability of the theory.

It seems that the tan ϕ value for sand is about 10 per cent higher in the plane strain state than in the triaxial case, though this may be not quite in agreement with the findings of *Lorenz, et al.* (*2/34*). With respect to sand alone I would like to ask if enough tests have already been made, so that this could be taken as a first proposition in the sense the General Reporter is thinking about when he asks for "establishing corrections to be applied to the results of triaxial tests?" This proposition does not consider the differences in deformation behaviour between samples in plane strain and triaxial tests. But, even if the members of this meeting should tend to a positive answer to my question, it is beyond any doubt that the statement of The General Reporter fully holds true, namely that "investigations on the effect of anisotropic consolidation and non-cylindrical states of stress on stress-strain relation and shear strength of soils constitutes one of the most promising and most needed topics of immediate research."

FRICTION OF THE LOADING-PLATES

The second topic I would like to mention is friction in contact surfaces. The paper of *Broms and Jamal (2/10)* deals with the influence of non-uniform stress distribution within the standard triaxial test. Without intending to discuss the findings of this interesting investigation, I only want to draw attention to a fact which also may have an influence

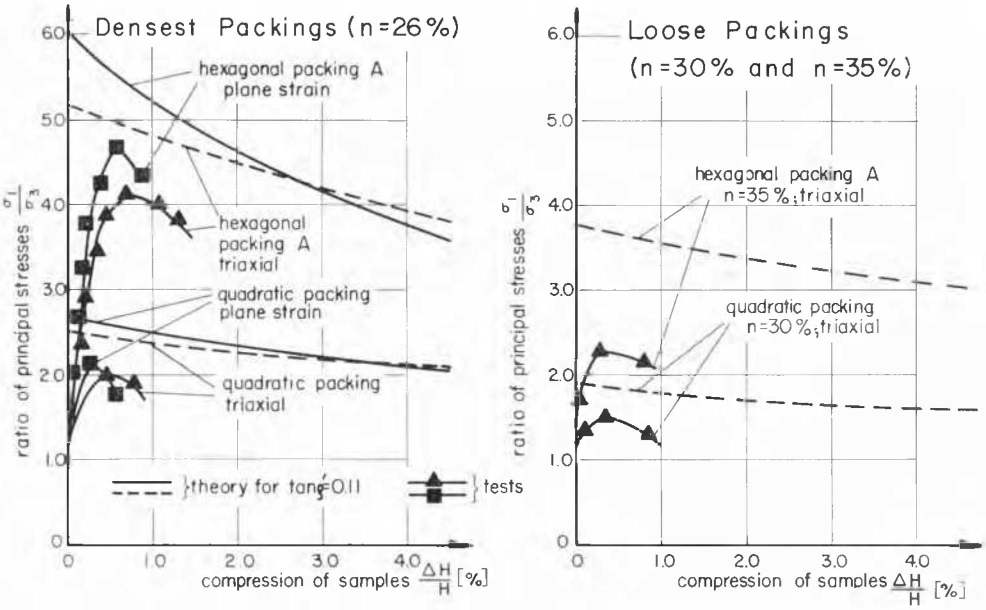

FIG. 4. Triaxial and plane strain tests on steel ball packings (Leussink, 1965; Leussink and Wittke, 1963).

on the phenomenon described here. This is the influence of the radical frictional forces between the end plates of the triaxial testing cell and the sample, caused by lateral deformation of the test specimen. This deformation occurs in most one-dimensional compression tests for soil and other material. Bishop and Green (1963), as well as Rowe and Barden (1964), have brought some light into the conditions prevailing in the contact surfaces by lubricating the end faces in order to prevent friction as far as possible.

I only want to give a hint of theoretical and experimental investigations just finished by Egger (1965), who deals with the magnitude and the distribution of the radial stresses for different combinations of deformability of the tested material and the end plates, and for different friction properties in the contact surfaces. Egger has also measured the radial stresses in the contact surfaces directly. Fig. 5 gives the yield stress according to the Hencky-von Mises criterion for rigid end plates with full frictional contact. The figure shows the lines of equal relation between the real yield stress τ_0, as influenced by the friction in the contact surface, and the stress $\tau_{0,\mathrm{id}}$ in the same specimen with no friction in the contact surface.

The results, as indicated in Fig. 5, may at the same time give new elements of understanding as to why we observe failure planes inclined at less than 45° more often in tests with non-lubricated loading plates. Furthermore, these investigations may lead to the conclusion that perhaps the occurrence of inclined failure planes is no absolute proof for the

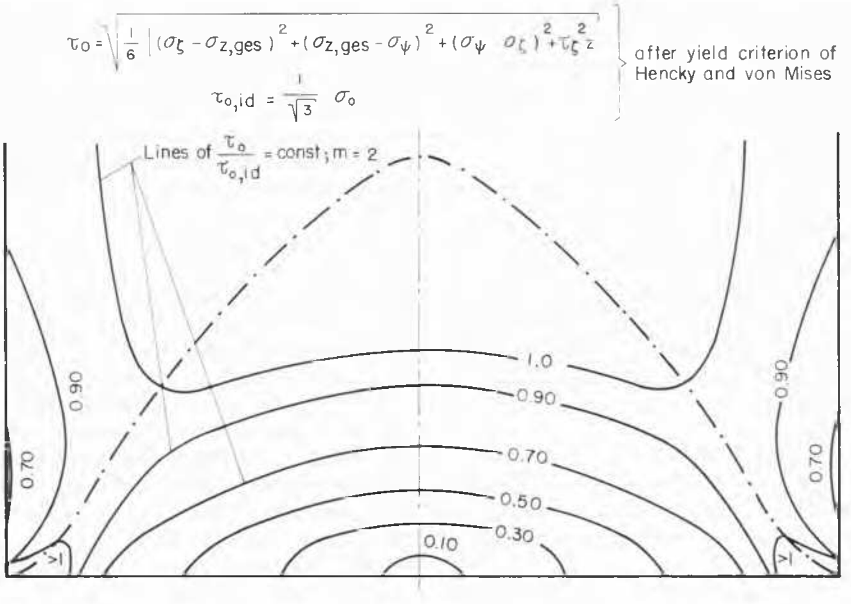
$\tau_o = \sqrt{\frac{1}{6}\left[(\sigma_\xi - \sigma_{z,ges})^2 + (\sigma_{z,ges} - \sigma_\psi)^2 + (\sigma_\psi - \sigma_\xi)^2\right] + \tau_{\xi z}^2}$
$\tau_{o,id} = \frac{1}{\sqrt{3}}\sigma_o$
after yield criterion of Hencky and von Mises
Lines of $\frac{\tau_o}{\tau_{o,id}}$ = const; m = 2
1.0
0.90
0.70
0.50
0.30
0.10
0.90
0.70
>1
0.90
0.70
>1

FIG. 5. Lines of equal reaction between yield stress τ_0 in a sample with full contact friction and stress $\tau_{0,\mathrm{id}}$ in a sample without any end plate friction (Egger, 1965).

validity of the conception of the failure behaviour of materials, which forms the basis of Mohr's failure criterion.

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Panelist: A. W. BISHOP (United Kingdom)

I have been asked by the General Reporter to introduce for discussion the subject of the shear-strength behaviour of soils under high confining pressures, with special reference to the influence of particle degradation.

The expression "high confining pressures" is a purely relative one, but within the context of this Conference it can be taken to refer to series of tests in which the cell pressures lie mainly in the range 100 lb/sq.in. (7 kg/sq.cm.) to 1000 lb/sq.in. (70 kg/sq.cm.). The upper limit of this range represents a little over twice the maximum lateral stress likely to be encountered in the highest earth- or rockfill dam at present under construction (*c.* 1000 ft). Tests in this range reported to the present Conference by *Insley and Hillis (2/23)*, and to the Ottawa Conference on Laboratory Shear Testing of Soils by Hall and Gordon (1963) and Hirschfeld and Poulos (1963) were in fact carried out specifically for the design of high earth dams. Mr. Marsal will discuss tests in this range on rockfill for similar purposes.

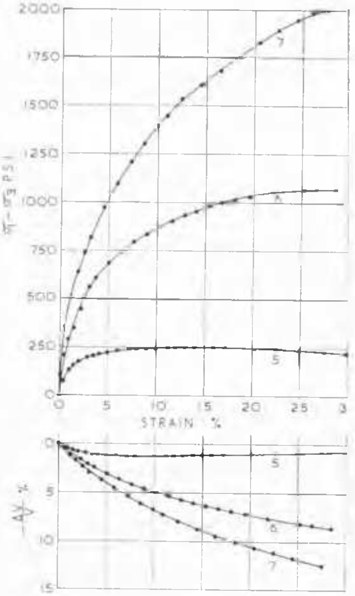


FIG. 6. Results of drained tests on saturated Ham River sand.

Tests carried out by Vesić and Barksdale (1963) in relation to the bearing capacity of deep foundations in sand and to cratering by deeply buried nuclear devices have been extended to cell pressures of 10,000 lb/sq.in. (700 kg/sq. cm.). Tests on St. Peter sand by Borg, Friedman, Handin, and Higgs (1960) extend to 30,000 lb/sq.in. (200 kg/sq. cm.). Also relevant to this discussion are tests on sand and undisturbed clay in the range 100–1000 lb/sq.in. reported to this Conference by *Bishop, Webb, and Skinner (2/7)*, tests on compacted and stabilized soils reported by *Wissa, Ladd, and Lambe (2/60)*, and tests on undisturbed and remoulded clay published in *Géotechnique* by Bishop, Webb, and Lewin (1965).

One problem which arises immediately in drained tests at high pressures is the definition of strength. This is illustrated by data obtained by Bishop, Webb, and Skinner (2/7) (Fig. 6) which show that failure in a loose, narrowly graded sand is defined by a slight peak at "low" pressures (test 5, at $\sigma'_3 = 100$ lb/sq.in.), is probably defined by the flattening in test 6 (at 500 lb/sq.in.) though the rate of volume change is not zero, but is not defined even at 27 per cent axial strain in test 7 (at $\sigma'_3 = 990$ lb/sq.in.) where the volume change is very large and is still continuing at the maximum travel of the apparatus. Similar stress-strain curves are reported by Borg, *et al.* (1960) (Fig. 7), and have presented a problem in rock mechanics since the work of von Karman in 1911 (for example, Schwartz, 1964).

It is of interest to note that Hirschfeld and Poulos (1963), testing a dense sand of wider grading at a maximum $\sigma'_3 =$

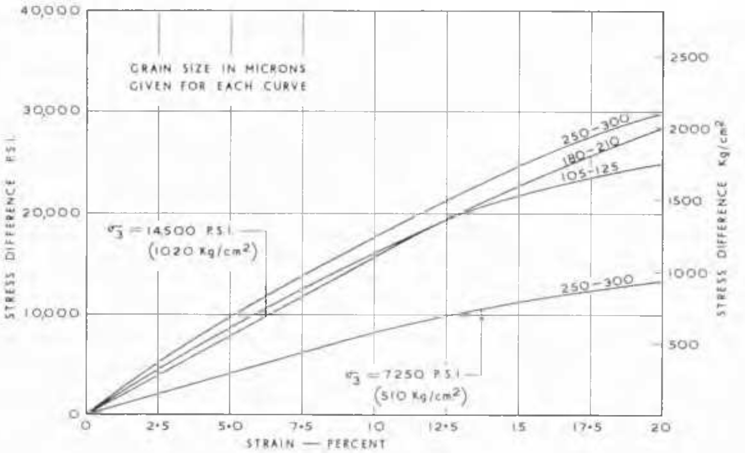
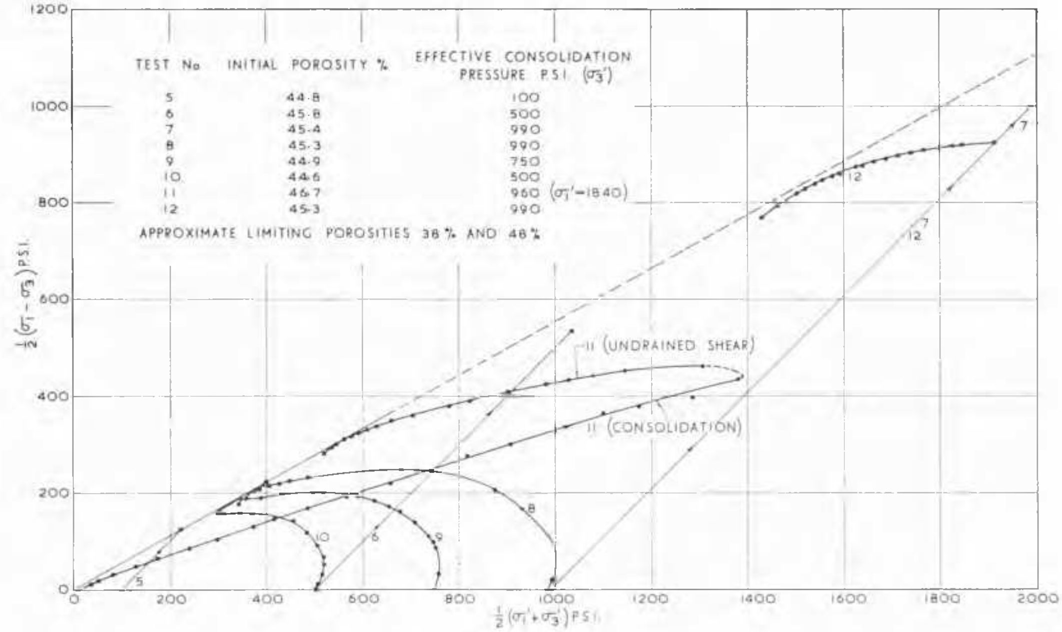


FIG. 7. Stress-strain curves for sand at very high confining pressure (after Borg, *et al.* (1960).


TEST No	INITIAL POROSITY %	EFFECTIVE CONSOLIDATION PRESSURE P.S.I. (σ_3')
5	44·8	100
6	45·8	500
7	45·4	990
8	45·3	990
9	44·9	750
10	44·6	500
11	46·7	960 (σ_1'–1840)
12	45·3	990

APPROXIMATE LIMITING POROSITIES 38 % AND 48 %

FIG. 8. Stress paths for tests on Ham River sand.

40 kg/sq.cm. (570 lb/sq.in.), just defined a peak at 17 per cent axial strain. Vesic and Barksdale (1963) report failure strains from about 13 per cent at σ'_3 = 18 kg/sq.cm. (250 lb/sq.in.) to about 25 per cent at 72 kg/sq.cm. (1000 lb/sq. in.) and beyond, also for a dense, widely graded sand. The presentation of the corresponding stress-strain and volume change curves would be of considerable interest. From the values of stress ratio given by Vesić and Barksdale (their Fig. 2) the value of ϕ' in the high pressure range (defined by the tangent to the origin) is almost constant at 31° for values of σ'_3 from 1000 lb/sq.in. to 10,000 lb/sq.in.

Borg, *et al.* (1960) report ultimate strength values in the range 1000–2000 bars (14,500–29,000 lb/sq.in.) confining pressure for tests on quartz sand of 250–300 micron grain size, which, although not defining peaks (Fig. 2) except in one case, give an average ϕ' of 32° at axial strains of 19–28 per cent. The material had in the process of shear altered from a single-size sand into a well-graded material about 50 per cent in the silt range or finer. It is therefore of interest that in the tests on glacial till containing initially 42 per cent silt and clay size particles as a result of alteration under natural conditions of shear, Insley and Hills (2/23) found no difficulty in defining the peak values and obtained a remarkably straight envelope, dropping only from $\phi' = 35\frac{1}{2}°$ to ϕ' = 34° over a range of σ'_3 = 100 to 450 lb/sq.in. in drained tests.

The difficulty of defining a peak stress is thus clearly associated in cohesionless soils with the rate at which particle breakdown is continuing at large shear strains. This has a parallel in drained tests on sensitive clays (e.g. Crawford, 1960), though the nature of the structural breakdown is probably different. The drained compression test with increasing axial stress is perhaps the most unfavourable test from this point of view, but even a test run under decreasing average stress (Fig. 8, test 12; data from Bishop, Webb, and Skinner) showed a small rate of volume decrease at the maximum strain applied, though ϕ' rose from 30.2° (end of test 7) to 33.0° for this test. Vesić and Barksdale (1963)

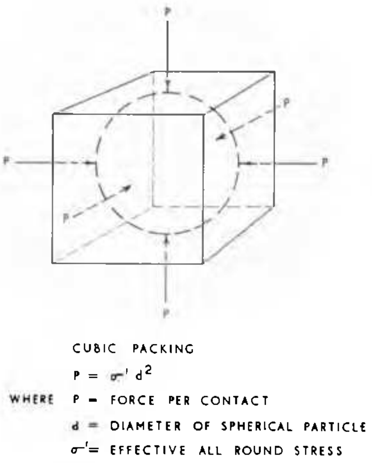

P
P
P
P
P
P
CUBIC PACKING
P = σ' d²
WHERE P - FORCE PER CONTACT
d = DIAMETER OF SPHERICAL PARTICLE
σ' = EFFECTIVE ALL ROUND STRESS

FIG. 9. Relation between effective stress and contact force for idealized packing.

reported that above a cell pressure of 750 lb/sq.in. a volume decrease occurred during shear at constant mean stress for their dense sand.

In a dry or partly saturated material this volume decrease is not in itself undesirable if the resulting strength is adequate. In a saturated granular material it may be highly dangerous as the resulting undrained stress path becomes very similar to that of a quick clay. Two examples are given by Bishop, Webb, and Skinner (2/7) of isotropically and anisotropically consolidated sand which failed with mobilized values of $\phi' = 21°$ due to the high rate of increase of pore water pressure (the value of $A_f = 1.13$ in the first test), the full value of ϕ' only being mobilized in the ultimate state (Fig. 8, tests 8 and 11). Pervious material under high stresses on the upstream side of very high dams may for this reason be dangerous under shock loading, rockfill, owing to its tendency to particle breakdown, not excluded.

It is of direct interest, therefore, to engineers to ask how particle breakdown is related to (*a*) particle size, (*b*) particle strength, (*c*) particle shape, (*d*) shape of the grading curve, (*e*) the magnitude of the applied stress, (*f*) the relative values of the mean and deviatoric components of stress, and (*g*) the magnitude of the consequent shear strain. Clearly these question cannot be answered in a brief discussion, but I can draw attention to some of the factors at work.

If we assume for simplicity a regular cubical pack of equal spherical particles subjected to an equal all round stress

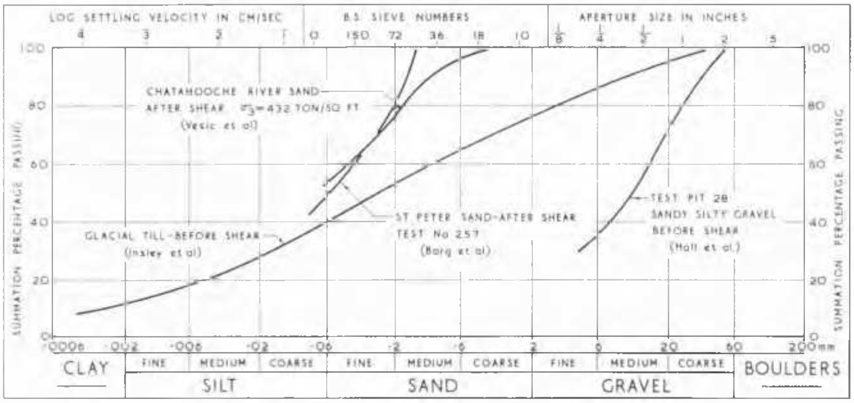


FIG. 10. Particle size distribution for various soils.

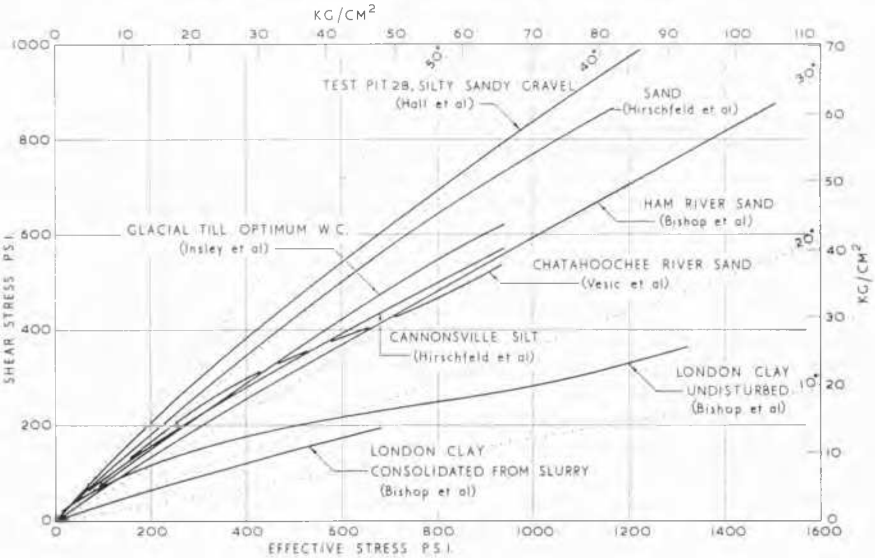


FIG. 11. Mohr envelopes for various soils under high confining pressures. (All tests drained except London clay consolidated from slurry.)

(Fig. 9) it is apparent that the force P per contact is given by the expression:

$$P = \sigma d^2 \tag{1}$$

where d is the diameter of the particle. If the particle is of quartz it is likely to remain elastic up to the point of failure except possibly within a few local asperities (this aspect is discussed by Bromwell, 1965). Hence, from the solution for two equal spheres in contact (Timoshenko and Goodier, 1951), the maximum contact pressure is σ_m where:

$$\sigma_m = 0.6\ [4PE^2/d^2]^{\frac{1}{3}}, \tag{2}$$

where E is Young's modulus and a Poisson's ratio of 0.3 is assumed. From Eq (1) we therefore obtain

$$\sigma_m = 0.6\ (4E^2\sigma)^{\frac{1}{3}}. \tag{3}$$

The maximum shear stress in the particle is 0.3 σ_m, and the maximum tensile stress is 0.13 σ_m, and these stresses are thus independent of particle size and depend only on ambient stress. Taking $E = 1 \times 10^6$ kg/sq.cm. for quartz, a tensile strength of 830 kg/sq.cm., and a shear strength of 11,000 kg/sq.cm., we find that local tensile failure around a contact would occur when $\sigma = 0.3$ kg/sq.cm. (4 lb/sq.in.)* and shear failure beneath it when $\sigma = 43$ kg/sq.cm. (600 lb/sq.in.).

While this analysis is very over-simplified, and ignores the confining effect of other particles on the stress within the particles and on their strength and also the variations of shape and strength with particle size, it is of interest to note that some particle damage has been reported in narrowly graded sand sheared at confining pressures as low as 40 lb/sq.in. (*c.* 3 kg/sq.cm.) (Bishop and Green, 1965) and that considerable fracturing has been measured at 500 lb/sq.in. (35 kg/sq.cm.) (Bishop, Webb, and Skinner, 2/7).

This analysis does at least suggest that it is the strength and deformation characteristics of the particles, rather than their size, which matter in relation to fracturing of particles, and that limited damage occurs even at relatively low confining pressures.

It is also apparent from Table I that soils with a relatively narrow initial grading, and hence a high porosity on placement (A and B), undergo a much larger volume change during shear than very widely graded soils (C and D) which can be placed at a very low initial porosity. This difference in volume change is probably directly related to particle fracture. It is perhaps significant that the *initial* porosities of the compacted dredger tailings TP.28 reported by Hall and Gordon (1963) and the till reported by Insley and Hillis (2/23) (compacted at optimum water content) are about 16 per cent and 22 per cent respectively. The *final* porosity of Borg, *et al.*'s (1960) 250–300 micron sand was in the same range (19 per cent) after being sheared at a cell pressure of 1020 kg/sq.cm. (14,500 lb/sq.in.). The grading curves of this sand after shear and of soils C and D are shown in Fig. 10 together with Vesić and Barksdale's sand after shear. They may suggest an ideal grading for high dam fills and raise the question of the best grading of rockfill for this purpose. There may be an optimum proportion of fines to reduce particle fracture without too great a reduction in the angle of friction.

TABLE I. DECREASE IN VOLUME DURING DRAINED TRIAXIAL COMPRESSION TESTS (σ_1^1 INCREASING)

Soil	Initial porosity (%)	Confining pressure, σ_3' (p.s.i.)	Decrease in volume during shear (%)
A—Loose sand (Bishop, Webb, & Skinner, 2/7)	46	500	8.5
B—Dense sand (Hirschfeld & Poulos, 1963)	34	570	6
C—Compacted dredger tailings (Hall & Gordon, 1963)	*c.* 16	650	2
D—Compacted till (Insley & Hillis, 2/23)	22	450	2

The only real clay for which we have detailed data is the London Clay illustrated in the paper by Bishop, Webb, and Skinner (Fig. 4 of 2/7). The slope of the effective stress envelope for the undisturbed soil changed dramatically from 30° near the origin to 10° at very high stresses.* The slope of the tangent from the origin to the furthest stress circle represents a ϕ' of 15°. This is close to the residual value given by Skempton (1964) and suggests that the curvature is due to progressive breakdown of particle aggregates and orientation of the platey particles. Evidence of actual particle damage is not available. For the same clay consolidated from a slurry the drop in ϕ' from undrained tests is only from 21° to about 16° (tangent from the origin). Undrained tests on compacted Vicksburg Buckshot Clay referred to by Wissa, *et al.* (2/60) and described in more detail by Wissa (1965) show a corresponding drop from approximately 23° to 18°.

In conclusion, therefore, it appears that for sands, gravels, and rockfill, the failure envelope will be moderately curved if initially dense, and only slightly curved if loose† (Fig. 11), the origin ϕ' not falling below about 30° for sand even if taken to extremely high stresses‡ provided a technique adequate to define failure is used. However, particle damage appears to start at quite low stress and major fracturing of the particles occurs in the high stress range, which can lead to dangerous undrained shear characteristics.

For a till having a percentage of fines similar to sand sheared at high pressure little curvature of the envelope is observed. However, for an undisturbed clay with a clay fraction of 57 per cent the slope of the effective stress envelope shows a very marked curvature which suggests that at high pressures particle orientation is well developed by the time the peak stress difference is reached. For a clay consolidated from a slurry and tested under undrained conditions the curvature of the effective stress envelope is less marked, but is nevertheless significant if wide stress ranges are conidered (Fig. 11).

The General Reporter has raised the question of the effect of shearing under conditions of plane strain at high pressures. It is apparent that at high pressure the behaviour of granular material whether placed initially densely or loosely approximates to that normally associated with a loose sand.

Tests carried out at the Imperial College by Cornforth (1961, 1964) showed that for loose sand there was little difference either in strength or volume change in the plane strain apparatus, though the failure strain tended to be smaller. The same tendency may be expected in general in granular materials at high confining pressures.

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*For cast steel shot this limit would rise to approximately 130 kg/sq.cm. or 1,850 lb/sq.in.

*Fuller details are given in the paper by Bishop, Webb, and Lewin (1965).

†Curves plotted by *de Beer* (2/6) show marked curvature in the low stress range, but the curvature may be exaggerated since the stress circles compared appear to correspond to equal porosities after consolidation, not to equal porosity as placed.

‡Whether the effect of ram friction under non-uniform loads when the large strains are reached has led to an overestimate of ϕ' at very high pressures does, of course, need to be examined.

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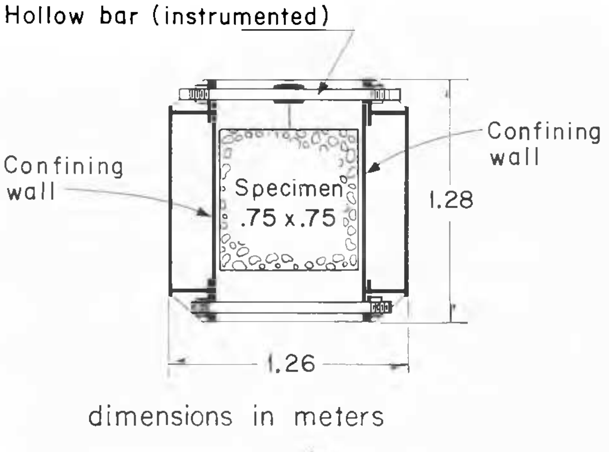
Panelist: R. J. Marsal (Mexico)

The General Reporter has asked me to discuss plane strain triaxial tests of rockfill samples. At the Comisión Federal de Electricidad (C.F.E.) Laboratory, we have just started to work on plane strain triaxial tests with compacted rockfill, the maximum particle size being 20 cm. The specimen has a cross-section of 75 × 75 cm and a height of 180 cm. To test these samples we built the frame shown in Fig. 12A,



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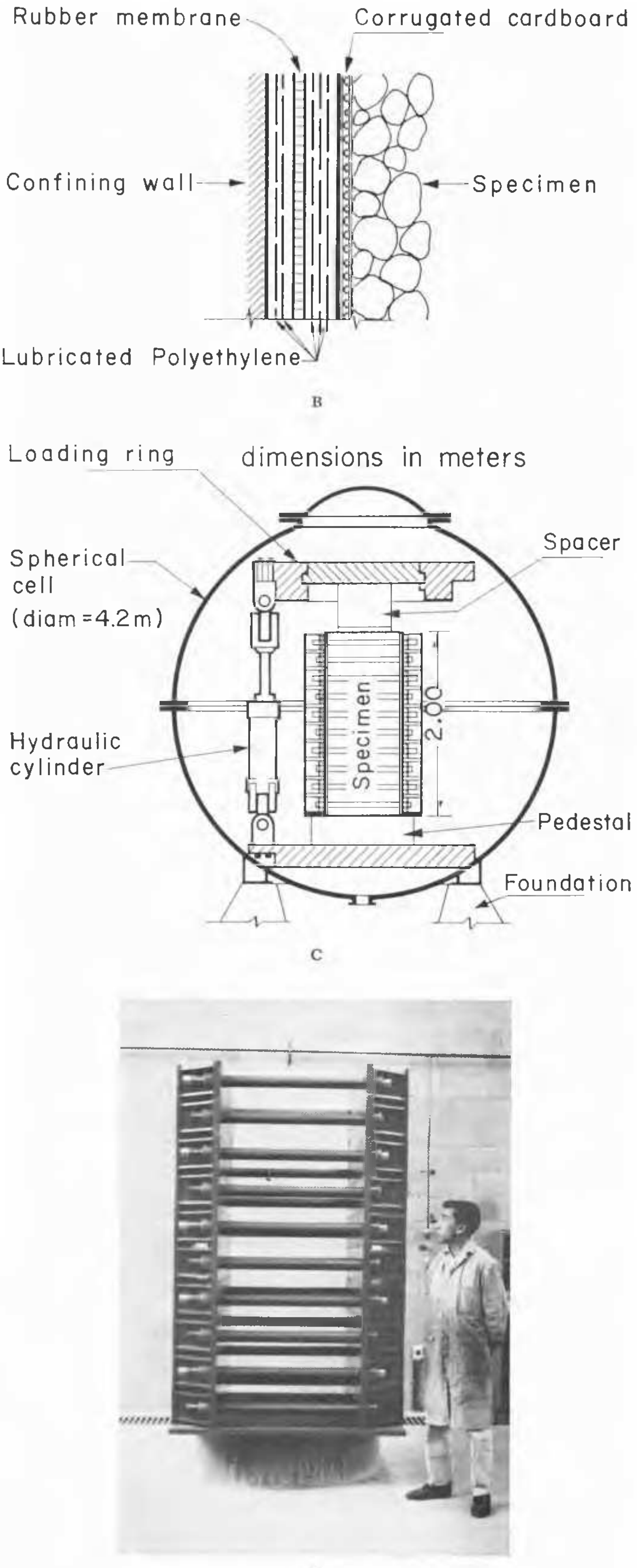


FIG. 12. Device for testing of rockfill under plane strain. A, horizontal section; B, anti-friction cover; C, assembly inside triaxial cell; D, photograph of device.

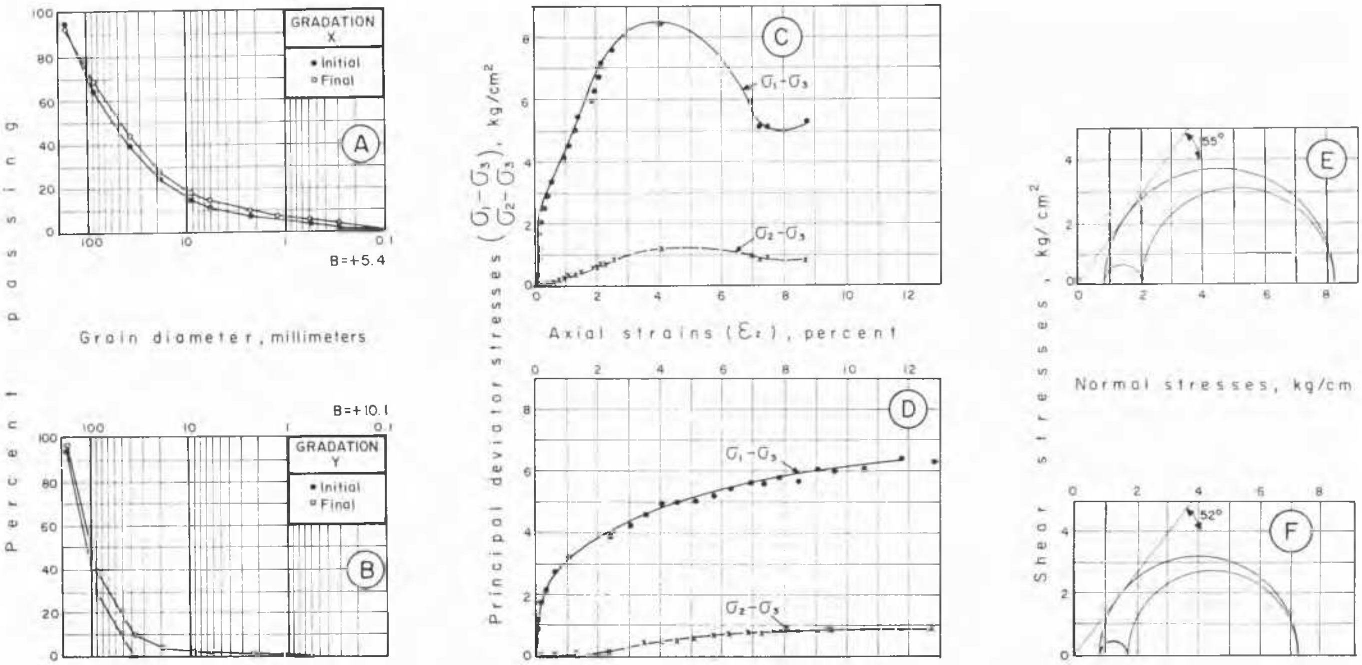


FIG. 13. Plane strain tests with two gradations of rockfill ($\sigma_3 = 0.9$ kg/sq.cm.).

composed of a steel base and two parallel walls connected by means of 20 hollow bars. Each wall is made of ten box girders. Ten of the bars are instrumented with linear variable differential transformers, so that reactions developed by the specimen against the girders can be measured with an accuracy of one per cent.

To minimize friction on the walls, the specimen cover is built with a rubber membrane and several layers of polyethylene scales which are lubricated with grease; a sketch of this cover is shown in Fig. 12B. Direct shear tests on this type of cover, performed between a steel plate and a layer of crushed gravel, revealed that the coefficient of friction varies between 0.03 and 0.05, for normal stresses in the interval of 1–30 kg/sq.cm. and loading times ranging from 15 minutes to 6 hours.

The tests I will describe were run in a loading device with a capacity of 150 tons, the confining pressure being developed by means of vacuum inside the specimen. We are now installing the plane strain frame inside the rockfill triaxial apparatus (Fig. 12C) in order to perform tests at confining pressures of 2 to 25 kg/sq.cm.

Let us examine some results obtained with two rockfill samples tested in the plane strain device under a confining pressure of 0.9 kg/sq.cm. Both specimens, X and Y, are quarry-blasted and of the same origin. The essential difference between them lies in their gradations, which are presented in Figs. 13A and 13B. Both were compacted to 90 per cent relative density, void ratios being 0.32 and 0.62 for the X and Y samples, respectively. In Figs. 13C and 13D deviator stresses ($\sigma_1 - \sigma_3$) and ($\sigma_2 - \sigma_3$) are plotted against axial strains. The main features of this information follow. (1) In both examples strains at the beginning of the test were very small. (2) Specimen X exhibited a peak value in both deviator stresses for an axial strain of 4 per cent. (3) Deviator stresses in specimen Y kept increasing and rupture was not attained at a strain of 12 per cent. From

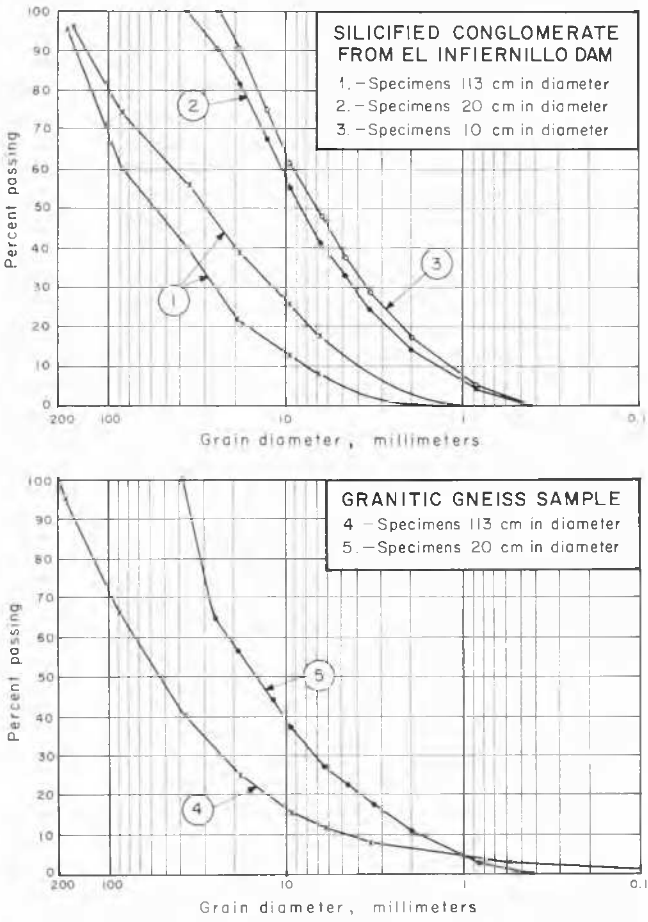


FIG. 14. Grain size curves.

the initial and final gradation curves it was found that breakage of grains was 5.4 and 10.1 per cent for specimens X and Y, respectively. Mohr circles at failure are plotted in Figs. 13E and 13F, the tangents through the origin having angles of 55° and 52° with the horizontal axis. If one compares these values with those found in triaxial cylindrical tests, the differences for both examples are 13°; they are greater than the discrepancies reported by Dr. Leussink. A better comparison of both types of triaxial tests may be obtained by computing a principal stress ratio that takes into account the three principal stresses at failure, namely: $\sigma_1/\sqrt{2}(\sigma_3)$ for the cylindrical tests and $\sigma_1/\sqrt{(\sigma_2^2 + \sigma_3^2)}$ for the plane strain test. Calculations applied to the tests described before show that $\sigma_1/\sqrt{(\sigma_2^2 + \sigma_3^2)}$ is 15 to 17 per cent greater than $\sigma_1/\sqrt{2}(\sigma_3)$.

We must run plane strain tests under high confining pressures to gain a better insight into problems connected to shear strength and stress-strain relationships for anisotropic stress conditions.

Dr. Moretto has asked about *the effect of specimen size and breakage of grains on the shear strength of granular materials* in the hope that I can produce conclusive information on this matter. But I am afraid this is impossible; all data herein presented must be considered preliminary. However, the results may be of great interest to engineers in charge of the design of rockfill dams. This information was obtained at the Rockfill Laboratory of the C.F.E. and at the Institute of Engineering, U.N.A.M.

In order to investigate the influence of specimen size on shear strength, a comparison of results obtained from testing two materials is shown in Tables II and III. The first material is a silicified conglomerate used at the rockfill shoulders of El Infiernillo Dam, and the second, a granitic gneiss rockfill sample. With the El Infiernillo material, tests were run in three different triaxial cells, specimens being 113, 20, and 10 cm in diameter with slenderness ratio of 2.5. A series of tests with granitic gneiss were made with specimens 113 and 20 cm in diameter. In all cases, the ratio between specimen diameter and maximum particle size was about five. The gradations of the materials were adjusted so as to have approximately the same coefficient of uniformity; granulometric distributions are shown in Fig. 14. Note that two curves are plotted for El Infiernillo rockfill sample (series 1), which enclose the gradations of specimens. Control of these tests was rather poor; however, the rest of the specimens (series 2 to 5) were prepared weighing all the component fractions for each layer. Another item of importance is the type of cover used for transmitting cell pressures to the specimens. A detailed description of the covers used is given elsewhere (Marsal, *et al.*, 1965). Deviator stresses shown in Tables II and II are the result of dividing the applied axial load by the total cross-section of the specimen. From a careful study of Tables II and III and Fig. 15 the following main conclusions are reached.

1. In both the silicified conglomerate and the granitic gneiss samples, the principal stress ratio σ_1/σ_3 decreases when the confining pressure is increased from 2 to 25 kg/sq.cm. Variations of σ_1/σ_3 are gradual, thus indicating that the failure envelopes are curved. (See Fig. 15).

2. Comparison of σ_1/σ_3 values in Table I shows that the principal stress ratio is systematically higher for smaller specimens. An investigation on the influence of the cover has disclosed that results of tests run with two rubber membranes and a layer of sand, as in tests 1 to 5, are conservative (error smaller than 5 per cent in σ_1), while those built with

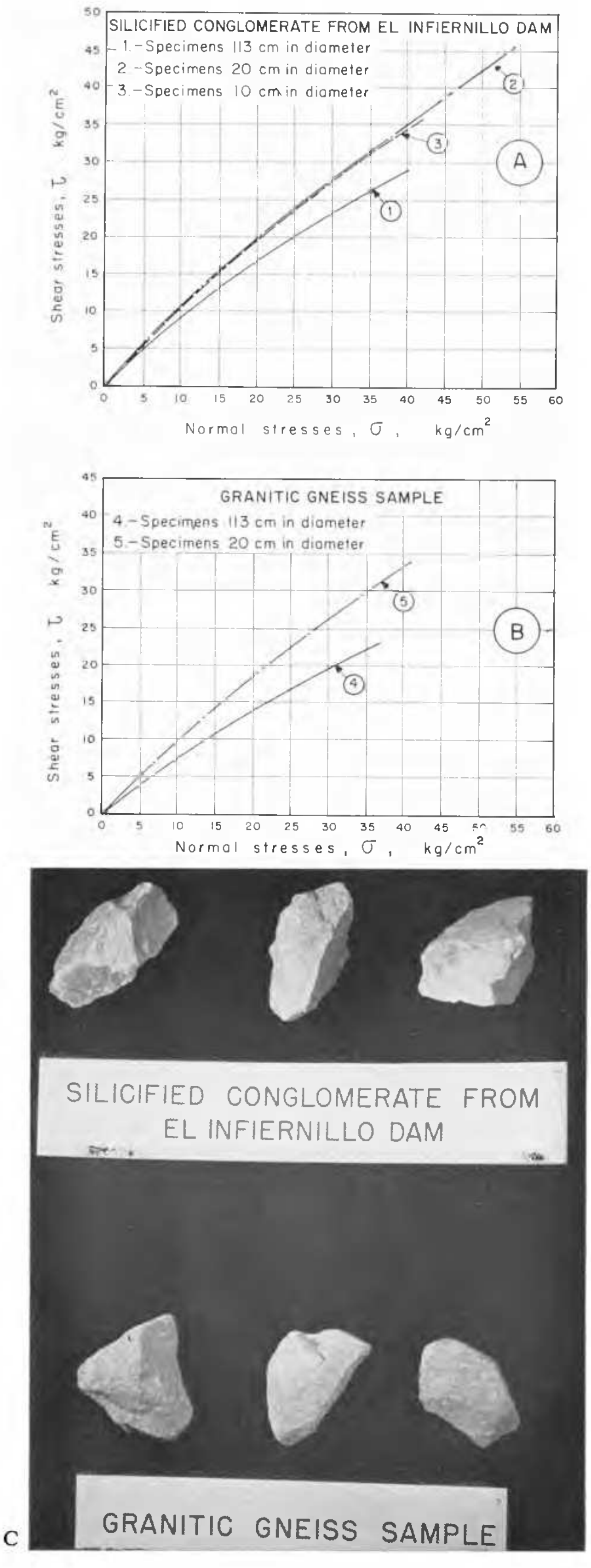


FIG. 15. Mohr envelopes: A, silicified conglomerate from El Infiernillo Dam; B, granitic gneiss; C, typical shapes of particles contained in samples.

TABLE II. TRIAXIAL COMPRESSION TESTS, COMPARISON OF RESULTS OBTAINED WITH SPECIMENS 10, 20, AND 113 CM IN DIAMETER, SILICIFIED CONGLOMERATE, EL INFIERNILLO DAM

	Specimen diameter	113 cm (44.5 in.)						20 cm (8 in.)				10 cm (4 in.)			
Line	Test No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
a	σ_3 (kg/sq.cm.)	1.88	4.88	9.98	16.88	24.89	5	10	17	25	50	2	5	10	17	25
b	γ_m (tons/cu.m.)	1.77	1.76	1.87	1.86	1.84	1.92	1.89	1.87	1.88	1.83	1.76	1.83	1.82	1.81	1.80
c	e_i	0.51	0.50	0.40	0.40	0.46	0.41	0.43	0.45	0.43	0.47	0.54	0.47	0.48	0.49	0.50
d	σ_1 (kg/sq.cm.)	11.32	24.92	44.50	70.10	97.25	32.9	52.4	91.2	119.5	217.7	21.4	32.1	59.8	90.9	117.6
e	σ_1/σ_3 at failure	6.01	5.10	4.46	4.15	3.91	6.58	5.24	5.36	4.77	4.35	10.7	6.42	5.98	5.35	4.70
f	φ to the origin	45.6°	42.2°	39.5°	37.6°	36.3°	47.3°	42.8°	43.3°	40.8°	38.8°	56.1°	47.0°	45.6°	43.2°	40.4°
g	Axial strain at failure, ϵ_{af} (%)	12.8	16.6	14.7	14.7	13.8	7.9	10.9	11.5	14.5	15.0	8.8	8.0	11.5	15.8	13.8
h	Volumetric strain at failure, ϵ_{vf} (%)	+0.1	−4.3	−3.1	−4.6	−5.4	−1.4	−3.9	−3.9	−5.6	−8.6	−1.8	−1.3	−2.9	−5.4	−6.1
i	Breakage of grains B (%)	—	6.3	15.3	14.3	14.1	4.3	6.6	7.9	10.9	10.9	2.7	4.2	5.3	7.6	9.0

TABLE III. TRIAXIAL COMPRESSION TESTS, COMPARISON OF RESULTS OBTAINED WITH SPECIMENS 20 AND 113 CM IN DIAMETER, GRANITIC GNEISS SAMPLE

	Specimen diameter	113 cm (44.5 in.)				20 cm (8 in.)			
Line	Test No.	16	17	18	19	20	21	22	23
a	σ_3 (kg/sq.cm.)	4.54	9.94	16.66	24.18	5	10	17	25
b	γ (tons/cu.m.)	1.94	1.90	1.90	1.90	1.89	1.95	1.90	1.86
c	e_i	0.29	0.32	0.36	0.33	0.38	0.35	0.38	0.40
d	σ_1 (kg/sq.cm.)	18.36	34.04	56.33	78.51	28.5	51.2	83.0	111.5
e	σ_1/σ_3 at failure	4.04	3.65	3.38	3.25	5.70	5.12	4.88	4.46
f	φ to the origin	37.1°	34.7°	32.8°	31.8°	44.5°	42.2°	41.3°	39.3°
g	Axial strain at failure ϵ_{af}, (%)	11.0	13.5	17.7	15.6	7.8	10.2	12.1	14.5
h	Volumetric strain at failure, ϵ_{vf}, (%)	−1.9	−4.1	−5.2	−5.4	−3.6	−4.4	−4.3	−5.3
i	Breakage of grains B (%)	10.5	16.8	19.6	23.9	13.6	16.4	17.7	17.1

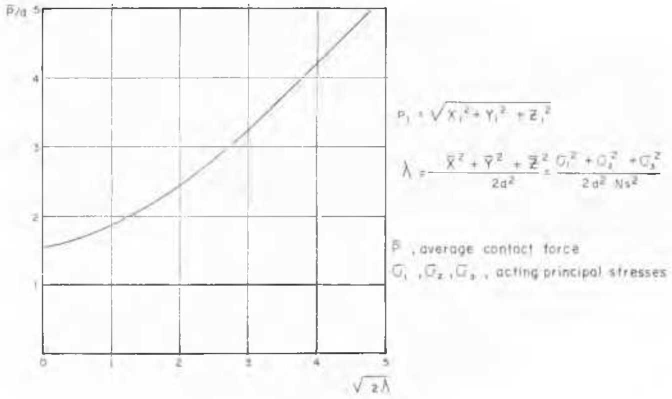


FIG. 16. Statistical analysis of contact forces.

a rubber jacket and polyethylene bands are on the unsafe side, the error being about 8 per cent in the major principal stress. Differences of σ_1/σ_3 in the granitic gneiss sample are much greater (Table III). The discrepancy found between curve 1 and curves 2 and 3 (see Fig. 16) was 7 per cent for a normal stress of 30 kg/sq.cm. The difference amounts to 20 per cent in the case of envelopes 4 and 5.

3. Axial strains at failure for the silicified conglomerate range from 8 to about 15 per cent when cell pressures vary between 5 and 25 kg/sq.cm. Except for test 1, volume changes undergone by the specimens during application of deviator stresses correspond to compression. Note that volumetric strains in tests 1 to 5 were measured with circumferential extensometers, whereas those of tests 6 to 15 were made by means of a burette; therefore, the two sets of measurements cannot be compared. The same type of result for the granitic gneiss shows that both axial and volumetric strains at failure increase with the confining pressure, although values recorded are not very consistent.

The analysis of particle breakage as well as its influence on the shear strength of a granular material requires a review of concepts already known. First of all we will work with contact forces, that is, with actions or reactions developing among grains (Fig. 16). Let us call X_i, Y_i, Z_i, the components of contact forces P_i, according to a Cartesian frame of reference. Average intergranular pressures σ_z, τ_{zx}, τ_{zy}, will be defined as follows:

$$\sigma_z = \sum_{i=1}^{N_s} Z_i, \quad \tau_{zx} = \sum_{i=1}^{N_s} X_i, \quad \tau_{zy} = \sum_{i=1}^{N_s} Y_i, \tag{1}$$

N_s being the average number of contacts per unit area (Marsal, 1963). Values of components X_i, Y_i, and Z_i vary from point to point and presumably are distributed at random. Assume for them statistically normal functions having a common standard deviation d. Then, the frequency function for the variable, $P_i = \sqrt{(X_i^2 + Y_i^2 + Z_i^2)}$ is:

$$\varphi(P/d) = \exp(-\lambda) \sum_{m=0}^{\infty} \frac{\lambda^m}{m!} \times \frac{(P/d)^{2(m+1)} \exp[-(P/\sqrt{2}d)^2]}{2^{m+1}\Gamma(m + 3/2)}, \tag{2}$$

and its average value is

$$\bar{P}/d = \sqrt{2} \exp(-\lambda) \sum_{m=0}^{\infty} \frac{\lambda^m (m+1)}{\Gamma(m + 3/2)}, \tag{3}$$

where

$$\lambda = \frac{\bar{X}^2 + \bar{Y}^2 + \bar{Z}^2}{2d^2} = \frac{\bar{\sigma}_z^2 + \bar{\tau}_{zx}^2 + \bar{\tau}_{zy}^2}{2N_s^2/d^2} \qquad (4)$$

in both expressions, and X, $\bar{Y}$, and $\bar{Z}$ are the mean values of X_i, Y_i, and Z_i.

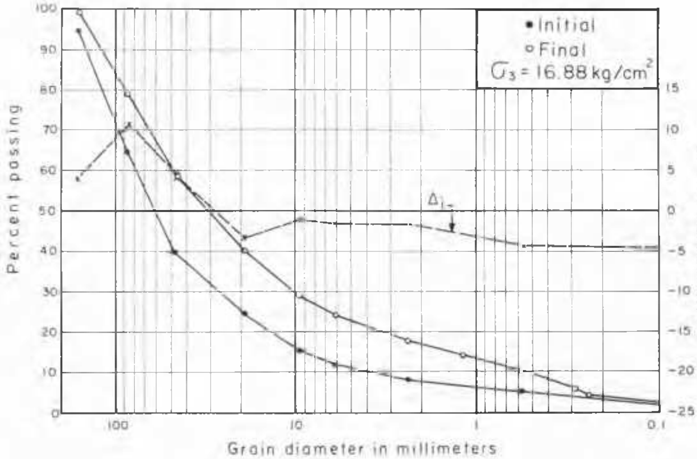
Eq (3) in terms of $\sqrt{2\lambda}$ is shown in Fig. 16. Then, the statistical average of $\bar{P}/d$ does not bear a linear relationship with the resultant stresses divided by the standard deviation and the number N_s.

Since particles break down under a very complicated state of stresses produced by contact forces, a test was developed to determine the crushing strength P_a. This test is run by placing three fragments of the rock, of approximately the same size, between two rigid plates. Attached to them, three dials permit measurement of the average deformation undergone by the particles. Loads are applied by means of a hydraulic jack and recorded with a proving ring. Results obtained with this device are presented in Table IV. In this table information is also shown on a quarry-blasted diorite from El Infiernillo and a natural gravel from Pinzandarán. Besides the crushing strength, data from unconfined compression tests are tabulated. Standard deviations of both P_a and q_u indicate a high degree of dispersion in these determinations. The average number of contacts recorded in the crushing tests are included in Table IV. Comparison of q_u and P_a values seems to indicate that there is no clear-cut relationship between both mechanical properties.

To investigate particle breakage, grain size determinations were made before and after testing specimens in triaxial compression; Fig. 17 shows results of one of these tests. Since some of the grains contained in the sample undergo fragmentation, the final curve moves toward the right of the plot. If differences, Δ_i, in per cent finer are computed, a graph like that drawn in Fig. 17 is obtained. Particle breakage, B, was defined as the sum of Δ_i having the same sign. In this way we have a parameter that takes care of the process as a whole. Values of B in terms of the resultant acting stresses (σ_m) for several materials are presented in Fig. 17. The stress σ_m is the arithmetic average given by the following expression:

$$\sigma_m = \tfrac{1}{2}\,[\sqrt{3}\sigma_3 + \sqrt{(\sigma_1^2 + 2\sigma_3^2)}], \qquad (5)$$

in which σ_3 is the confining pressure and σ_1 the major principal stress at failure. Scattering of the data is important and is probably caused by small variations in the gradation and in the crushing strength of rock particles (Table IV). All B *versus* σ_m graphs (Fig. 17) show a rather high breakage value close to the origin of stresses, a fact that would be hard to account for, unless we bear in mind relationships between contact forces and resultant stresses like those disclosed by the statistical analysis described before (Fig. 16). Based on the above experimental evidence and on results of



Sieve	Initial	Final	Differences Δi
	Retained in percent	Retained in percent	
6x8"	5.0	0.9	+4.1
3x4"	30.0	19.5	+10.5
1.5"	25.0	20.6	+4 4
3/4"	15.0	18.7	−3.7
3/8"	10.0	11.3	−1.3
1/4"	3.0	5.0	−2.0
4	—	—	—
8	4.0	6.0	2.0
14	—	4.2	—
30	3 0	3.6	−4.8
50	—	4 2	—
100	—	3.6	—
200	5.0	1.8	−4.6
minus 200	0.0	0.6	−0.6

B=+19.0

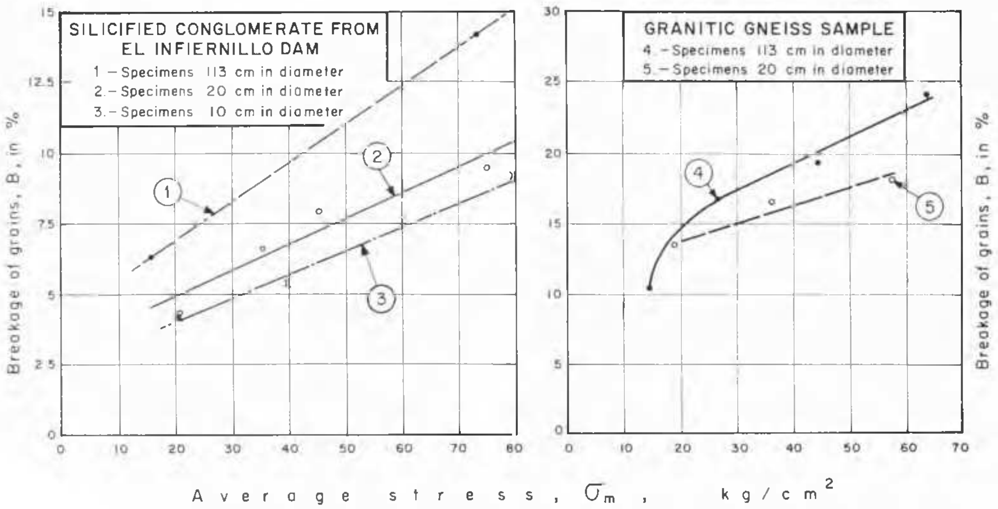
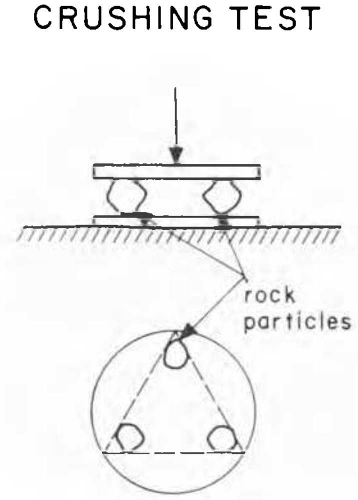


FIG. 17. Breakage of grains.

TABLE IV. CRUSHING AND COMPRESSION TESTS ON ROCKFILL SAMPLES

	Crushing tests						Unconfined compression tests			
Material	d_m (cm)	N_c	P_a (Kg)	S.D. (Kg)	δ (mm)	No.	diam (cm)	q_u (kg/sq. cm.)	S.D. (kg/sq. cm.)	No.
Silicified conglomerate, El Infiernillo	3.61	4.1	286	156	0.28	10	4.6	1751	537	13
Granitic gneiss	3.90	3.9	169	107	0.36	10	4.6	743	—	2
Diorite, El Infiernillo	4.13	3.9	243	194	0.33	10	4.6	1052	354	15
Pinzandarán, natural gravel	4.19	3.5	895	507	0.24	10	4.6	1192	502	8

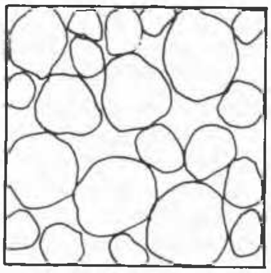
d_m, average diameter of particles; N_c, average number of contacts; P_a, force that produced failure in one of the particles; S.D., standard deviation; δ, deformation at failure; No., number of tests; q_u, un-confined compressive strength.



one-dimensional compression tests as well as on theoretical speculations of statistical nature, the following working hypothesis for particle breakage is proposed:

$$B = \alpha(\bar{P}/P_a), \qquad (6)$$

in which $\bar{P}$ and P_a have the same meaning as before and α is a constant. Taking into consideration variables involved in the computation of the average contact forces $\bar{P}$, one may conclude that B depends on the resultant acting stresses, the number of contacts per unit area N_s, the standard deviation for the assumed statistical distribution, and the average force P_a that produces fracture of the particles by crushing. On the other hand, N_s is a function of the mean diameter of grains and their shape, as well as the void ratio and number of contacts per particle (Marsal, 1963).



$$e = \frac{V_t - V_s}{V_s}$$

$$e' = e$$

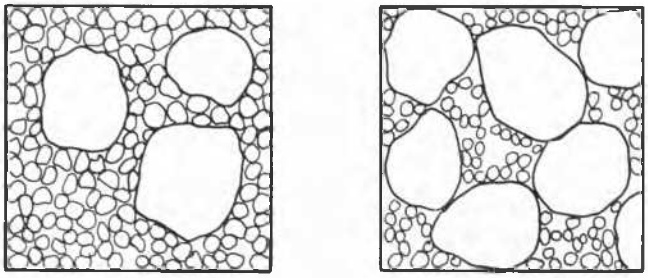
FIG. 18. Soil matrix for $d_{\min}/d_{\max} \sim 1/2$.
e = nominal void ratio;
e' = structural void ratio;
V_1 = volume of larger particles;
V_2 = volume of smaller particles;
$V_s = V_1 + V_2$ = volume of solids;
V_t = total volume.

The average void ratio of the specimen is not a representative value to be correlated with the shear strength, particularly in materials that undergo substantial particle breakage. Migration of broken grains was invariably noticed in triaxial tests and one is tempted to think that many of these particles remain idle (i.e., independent from the resisting matrix), thus not contributing to the work done by the skeleton while attaining failure. Therefore, the solid structure actually working is not properly represented by the gradation found after testing the specimen. Assume that βB is the number of broken particles which do not contribute to the solid skeleton of the specimen. Then, the corrected void ratio, or structural void ratio, e' will be,

$$e' = (e + \beta B)/(1 - \beta B), \qquad (7)$$

where e is the void ratio computed from the total volume of the specimen and the corresponding volume of solids. As an example, if $e = 0.60$, $B = 0.20$, and $\beta = 0.5$, then, $e' = 0.78$. This result discloses the influence that the particle breakage has on the average value of the parameter e, which is considered as the important one in correlations of shear strength of granular soils. Note that e' does not take into account conditions at the failure surface of the sample; these may be quite different from the average structural arrangement.

I agree with Dr. Bishop on his developments about the maximum contact pressure σ_m, provided they are applied



$$e = \frac{V_t - (V_1 + V_2)}{V_1 + V_2} \qquad e = \frac{V_t - (V_1 + V_2)}{V_1 + V_2}$$

$$e' = \frac{(V_t - V_1) - V_2}{V_2} \qquad e' = \frac{V_t - V_1}{V_1}$$

A B

FIG. 19. Soil matrix for $d_{min}/d_{max} \sim 1/14$. e = nominal void ratio; e' = structural void ratio; V_1 = volume of larger particles; V_2 = volume of smaller particles; $V_s = V_1 + V_2$ = volume of solids; V_t = total volume.

only to "a regular cubical pack of equal particles." However, the conclusion that σ_m is independent of the particle size is not valid for soils even of uniform gradation. In this particular case, the contact forces acting between particles can be written:

$$\bar{P} = [\pi r_v (1 + e)/3^{1/4} n_c]^{2/3} d^2 \sigma, \qquad (8)$$

in which r_v is the shape factor, e the void ratio, n_c the average number of contacts per particle, d the grain diameter, and σ the confining pressure (Marsal, 1963).

On the other hand, the maximum contact pressure according to the Hertz formula is:

$$\sigma_m = 0.6\ (\bar{P}E^2/r^2)^{1/3}. \qquad (9)$$

For the case of a uniformly graded soil, r is the radius of curvature of the surfaces in contact and not half the value d. Therefore, σ_m is not independent of the grain size, but a function of the ratio $(d/r)^2$. In actual soil particles, the radius of curvature of contact points may be considerably smaller than the diameter of the particle itself, the ratio d/r varying within wide limits, even in a single grain.

I believe that the strength and deformation characteristics of the particles as well as the particle size play an important role in the breakage process.

My contribution to this Session *(2/36)* which related to the effect of the specimen size on the shear strength of granular soils was based on results obtained from testing samples produced by quarry blasting of the same rock. However, corresponding gradations were different in order to have equal ratios between specimen diameter and maximum grain size. It was also decided that the specimens to be compared should have common coefficients of uniformity and void ratios, the latter parameter assumed to be the important one. Under these testing conditions a significant difference was found between the shear strengths of specimens 113 cm in diameter and those having diameters of 20 and 10 cm. Differences were higher in the material that showed greater particle breakage.

On the other hand, Dr. Leussink presents results of tests performed with materials of uniform gradation and the same origin, grain size being approximately 1/10 of the specimen diameter. Values of particle breakage and variations in σ_1/σ_3 for each series of tests are not reported. Were these tests run at a low confining pressure? From the information gathered at the Karlsruhe Institute, Professor Leussink contends that the sample size does not influence the shear strength.

Based on tests made with a quartz sand, specimens being prepared with mixtures of two different grain diameters, d_{min} and d_{max}, Professor Leussink concludes that "the flatter the gradation curves, the smaller the angle of internal friction, if we test soils with grains of similar shape and the same material friction at equal porosity." The results supporting this statement follow: for $d_{min}/d_{max} \sim \frac{1}{2}$, tan ϕ is equal to 0.9, whereas tan $\phi = 0.65$ for $d_{min}/d_{max} \sim 1/14$, the porosity being 35 per cent for both cases. Again, no mention is made of confining pressures and corresponding particle breakage nor of proportions of grain sizes in the mixtures. Therefore, it is not easy to evaluate the data presented. Let us assume that mixtures for d_{min}/d_{max} equal to 1/2 and 1/14 have the same proportions by weight and the same porosity. Then one can anticipate that (1) where $d_{min}/d_{max} \sim 1/2$ almost all grains are connected in a single solid matrix, and (2) for $d_{min}/d_{max} \sim 1/14$, either the small grains constitute the working solid skeleton (Fig. 19A) or this is built up by the larger particles leaving the smaller ones idle within the voids (Fig. 19B). In the first case, the porosity is a meaningful parameter for shear-strength correlations, whereas in the second case the measured n value may be quite different from the one representing the actual soil skeleton that works during shear (see definition of the structural void ratio in my previous remarks). These possibilities may explain the results presented by Professor Leussink.

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PANELIST LEUSSINK

EFFECT OF SPECIMEN SIZE ON THE SHEAR STRENGTH OF GRANULAR MATERIALS

Mr. Marsal postulates a diminishing of the shear strength with increasing size of specimen in triaxial tests. I find it difficult to agree with him, if we are speaking of absolutely the same material; possibly a great deal of the difference is due to a difference in definition.

Mr. Marsal's organization owns one of the two large triaxial cells (diameter $\geq$ 1.0 m) in the world (Marsal, *et al.*, 1965); the other one was constructed between 1957 and 1959 in Karlsruhe (Leussink, 1960). We tested a variety of granular soils in triaxial tests with diameters between 3.6 and 100 cm. The results of one of these series are shown in Fig. 20 where the tan ϕ values of a uniform moraine with grain sizes of d = 0.4 to 0.5 cm, 5 cm, and 10 cm from tests with sample diameters of D = 3.6, 5.0, 50, and 100 cm are plotted as a function of the porosity, n. The grains of the three soils consisted of the same material, had similar shapes, and the same surface roughness. All test results fit into a straight-line relationship between tan ϕ and n, which was found by Idel (1960). I could not find from these results that the sample diameter influences the shear strength.

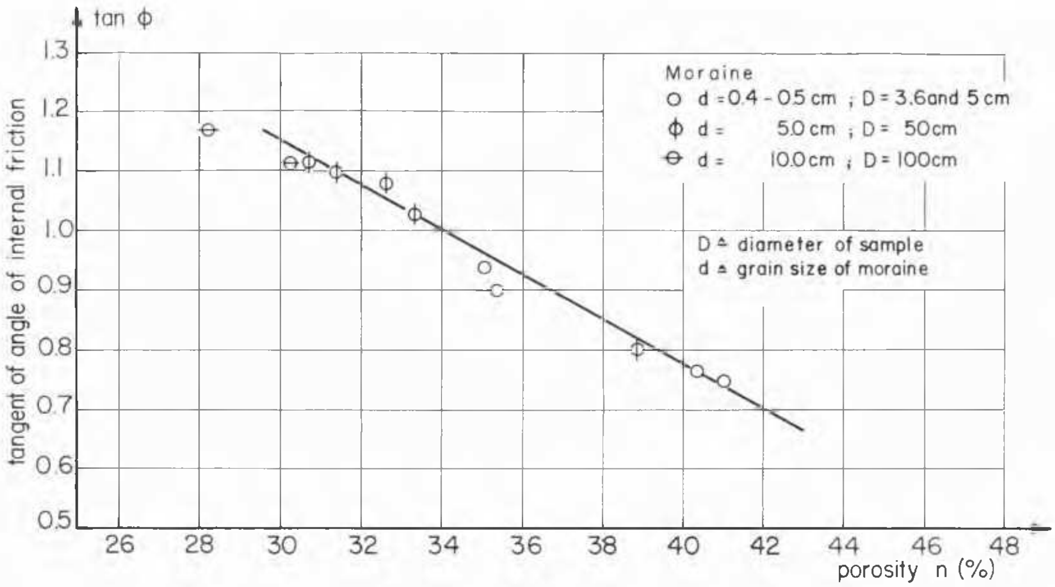

FIG. 20. Triaxial tests with different diameters of samples on uniform moraine (Idel, 1960).

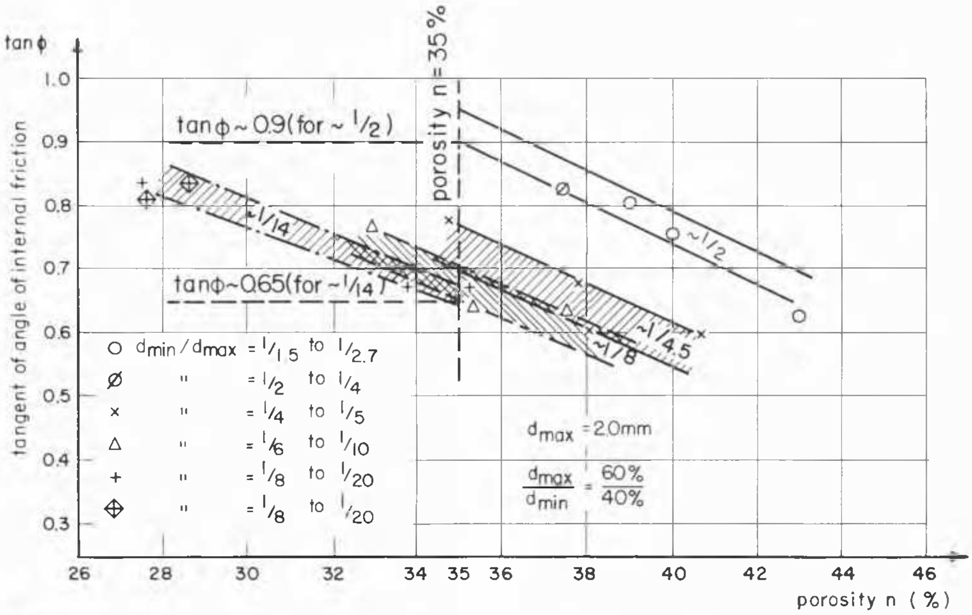

FIG. 21. Triaxial shear tests on quartz-sand mixtures of two different grain sizes ($d_{\min}$ and $d_{\max}$) (Idel, 1960).

Would it not be possible that the differences observed by Mr. Marsal are at least partially due to the fact that actually soils with different gradation curves have been investigated in the various cells? We found that the flatter the gradation curves the smaller the angle of internal friction, if we tested soils with grains of similar shape and the same material friction at equal porosity. Mr. Marsal, however, tested soils with flatter curves in the larger cells.

Fig. 21 demonstrates this finding for quartz sand mixtures of grains with two different diameters ($d_{\min}$ and $d_{\max}$). We realize that for a quotient of $d_{\min}/d_{\max} \sim \frac{1}{2}$ we get a tan ϕ value of 0.9 and for ~1/14 a tan ϕ about 0.65 for the same porosity in both cases of $n = 35$ per cent. The large specimen of Mr. Marsal contains about 50 per cent coarse grains. I believe that the reason for the observed behaviour is as follows. The structure of the coarse material, as far as transmission of stresses is involved, has a different function in the coarse and the fine specimens, even if the gradation of all specimens reveals the same uniformity coefficient. In the large specimen the load is mainly transmitted by the large grains, with the smaller grains more or less only filling the voids, without much transmitting of stresses. Therefore the large grains which have relatively sharp edges get higher concentrated stresses and consequently a higher percentage of grain breakage; the test results show that the breakage is 46 to 130 per cent higher in the large specimen.

It is well known, as Professor Bishop underlined a few minutes ago and as Fig. 22 (Leussink and Blinde, 1964) shows, that, with coarse gravels or rockfill the critical ratio of stresses diminishes when the confining pressure increases,

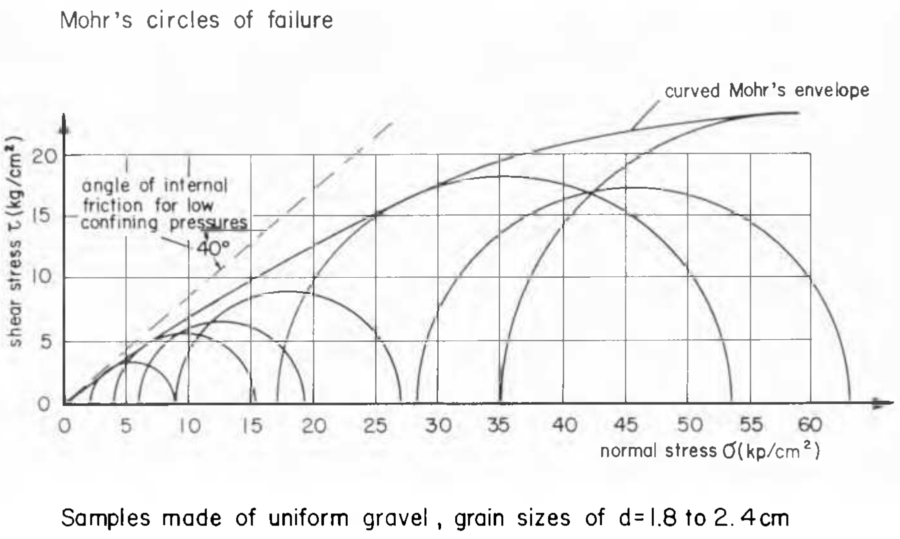

FIG. 22. Decrease of the angle of internal friction with increasing confining pressures as measured in triaxial tests (Leussink and Blinde, 1964).

and simultaneously the crushing of the particles increases. The considerable concentration of stress transmission in the largest grains can easily be explained by their very much higher modulus of deformability, compared with that of the total assembly of grains. It very often can be observed, when the specimen is examined after completion of the test that most of the largest grains show well developed shear planes through the middle of their bodies.

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Panelist: G. D. AITCHISON (Australia)

The General Reporter has drawn attention to the surprising fact that there is a dearth of papers on the subject of the shear strength of unsaturated soils. He has pertinently remarked that the few papers that have appeared in the literature of the past few years discussed aspects of the effective stress law or principle as applied to unsaturated soils. Comparatively little progress has apparently been made towards a clarified statement of the principles and technology of the definition of the strength parameters of unsaturated soils. The incompleteness of this knowledge does not reflect any lack of interest in the subject among soil mechanics workers. On the contrary, the topic is under consideration in a great number of research laboratories around the world. Many ingenious hypotheses are being pursued, each aimed at the solution of one or more facets of the total problem. The stage is therefore being set for the clarification of total knowledge on the soil mechanics of unsaturated soils—even if we are still far from this point of clarity.

Why is this so? Why are we still somewhat in the dark concerning unsaturated soils whereas we believe that we have a reasonably adequate knowledge of the behaviour of saturated soils? This is not an idle question, for to answer it honestly we must query the validity of a substantial part of our "established knowledge" of the mechanics of soils.

The first and major point to realize is that we must deal with an effective stress *principle* and not always with a simple effective stress *law*. It has been customary to consider Terzaghi's statement of effective stress as a law governing soil mechanics behaviour. This "law" was examined by Skempton during the Conference on Pore Pressure and Suction in Soils in London in 1960, and the conclusion was reached that it was a satisfactory approximation for many engineering studies of saturated soils. At the same Conference, however, several authors (Skempton, Bishop, Jennings, Coleman, Aitchison) pointed out the inadequacies of such a law in relation to unsaturated soils and an alternate expression,

$$\sigma' = \sigma + \chi\,(u_a - u_w) - u_a, \qquad (1)$$

was accepted (Aitchison and Bishop, 1960) for unsaturated soils.

Under certain idealized circumstances it was considered possible to substitute S_r for χ in Eq (1) giving

$$\sigma' = \sigma + S_r\,(u_a - u_w) - u_a. \qquad (2)$$

This substitution, although only approximate, serves to highlight the inherent errors associated either with the use of an incorrect effective stress law or with incorrect measurements of pore fluid pressure during a triaxial shear test. Donald (1963) has demonstrated the point that some types of test apparatus may measure only pore air pressure and not the true pore water pressure; whereas in other tests a measurement may be made of pore water pressure and not of pore air pressure. If either of these pressures is considered to be the pore fluid pressure in a simple effective stress law some serious errors in the inferred c' and ϕ' parameters will follow.

Returning to the accepted form of effective stress statement for unsaturated soils (Eq (1)), at first sight this statement appeared to differ from the traditional effective stress law

$$\sigma' = \sigma - u$$

only in the introduction of the suction term $(u_a - u_w)$ and the χ term. However, these two apparently innocent changes were in fact full of significance.

Many workers (e.g. Jennings and Burland, 1962; Aitchison, 1960, 1960a) immediately recognized that the χ term (which could normally be expected to have values between 0 and 1) would be strongly dependent upon a multiplicity of factors, most of which are interdependent. In a specific soil of specific composition and specific particle arrangement the χ term is primarily dependent upon soil suction and the related moisture expressions of water content and degree of pore space saturation. However, in any soil which may be subjected to changes in composition (through a change in the electrolyte) or to changes in particle arrangement (as a consequence of strains) the χ term may vary greatly even at a constant water content in the sample. Strain history is therefore suggested as a significant determinant of χ.

If a list is made of all of the factors which appear to influence the χ term, it is readily seen that this is essentially the same list as that of the factors known or believed to contribute, in a physico-chemical sense, to soil strength. Thus we have χ = function of: suction, water content, degree of pore space saturation, particle composition, surface charge on particles, electrolyte composition, particle arrangement, stress history, strain history, etc. The statement that χ is a function of these listed variables does not necessarily imply that the function is a continuous one. In fact it is abundantly clear that there are several circumstances in which χ must be considered as a discontinuous function, particularly the case of collapsing soils in which a sharp discontinuity exists in the functional relationships between χ and both soil suction and strain.

It should be noted that the χ term is not necessarily the same in statements of effective stress for volume change or for strength. However, this point is not under discussion here.

Referring back to the list of controls of χ, it follows that the use of the χ term in statements of effective stresses and hence of soil strength must be made with great care if one is to avoid confused or circular arguments. In the face of this complex situation, cognizance may be taken of the χ term in strength studies by following one of two courses of action. Either all of the factors contributing to the χ term must be evaluated quantitatively or χ must appear in any measuring process in precisely the form which is relevant to the condition being studied. For the latter alternative the value of χ must reflect the field conditions by virtue of the fact that every control of χ in the laboratory test must be identical with every control of χ in the field process supposedly represented by the laboratory test.

It is quite obvious that the acceptance of the dependence of the χ term on a multiplicity of controls leads to the acceptance of a multiplicity of χ values and hence of effective stresses (at specified values of applied stress and pore water pressure). This complexity highlights the mental obstacle which must be overcome in coping with unsaturated soils—or with other "unusual" soil conditions. We must accept the fact that no single simple law will suffice and we must therefore be prepared to be critical in our selection of the relevant factors to insert into our effective stress statements or we must look for a radically different form of expression of strength behaviour of such soils.

Turning now to the quantitative evaluation of χ we face a real problem. Of course in a simple idealized soil, involving groups of regularly-packed spheres, it is possible to compute the value of χ relevant to any known stress history and to any desired physical property, that is a χ term for the frictional component of strength at a particular stage of a moisture cycle may be computed and this may be shown to differ from the χ term for volume change in the same soil. An example of such an elementary computation in this form was presented at the London Pore Pressure Conference (Aitchison, 1960), but this has been recognized (Bishop and Blight, 1963) as inadequate for general purposes. A more sophisticated model was adopted (Aitchison, 1960a) for the suggested form of computation of effective stresses in multi-phased materials and it would seem that this approach of considering quantitatively the structural arrangement of soil particles may hold promise for the future. Indeed many research workers are now involved in attempts to quantify descriptions of soil fabric (e.g. Lafeber, 1964), and progress suggests that this gap in knowledge may be closed in the near future. In the meantime the computation of the χ term from presumed arrangements of soil particles cannot give quantitative answers for real soils.

A different approach to the quantification of the χ term lies in sets of parallel tests, involving on the one hand a measurement of a stress-dependent property of a soil in the unsaturated state and on the other hand the measurement of the same property in the saturated, or dry, state in which the "effective stress" is presumed to be known. This approach holds its dangers since it may well be impossible to follow comparable stress histories in both cases and it certainly will be difficult to ensure comparable electro-chemical forces in two systems of different moisture regimes. Thus the value of the χ term so obtained may be highly dependent upon the technique adopted.

The difficulties and inherent errors involved in attempts to quantify the χ term suggest that the alternate course of action, in which all controls of χ are maintained at correct values during a test process, may be more feasible even though apparently fraught with complexities.

Turning now to the second non-traditional term in Eq (1), that of soil suction, it may be similarly observed that the apparently simple term $u_a - u_w$ (or the pressure deficiency in the soil water with respect to the soil air) may, in fact, need careful consideration. Soil suction may be considered as a free energy term with the two components of total suction generally defined as matrix suction and solute suction.

Some clear-cut relationships exist, in terms of basic thermodynamics, between the various physical and chemical manifestations of total, matrix, and solute suctions. *Total suction* (h) may be derived from the measurement of the partial pressure of water vapour in equilibrium with the soil water:

$$h = (-RT/g)\log_e(p/p_0).$$

Solute suction (h_s) may be derived from the measurement of the vapour pressure of the soil extract relative to the vapour pressure of pure water:

$$h_s = (-RT/g)\log_e(p/p_0),$$

where p = vapour pressure of soil water, p_e = vapour pressure of soil extract, p_0 = vapour pressure of pure free water.

Matrix suction (h_m) may be derived from the difference between total and solute suction,

$$h_m = h - h_s.$$

The accepted definitions of total and matrix suction are useful for soil mechanics purposes. *Total suction* (h) is defined as the negative gauge pressure relative to the external gas pressure on the soil water to which a pool of water must be subjected to be in equilibrium through a semi-permeable membrane with the soil water. $h = (u_a - u_w)$, semi-permeable membrane backed by pure water. *Matrix suction* (h_m) is defined as the negative gauge pressure relative to the external gas pressure on the soil water to which a solution identical in composition with the soil water must be subjected in order to be in equilibrium through a porous permeable wall with the soil water. $h_m = (u_a - u_w)$, permeable membrane backed by soil solution. It follows that the suction term $(u_a - u_w)$ may therefore have values between h and h_m depending on the technique adopted in the measurement of u_w. That is, $h \geqslant (u_a - u_w) \geqslant h_m$.

This uncertainty as to the value of $(u_a - u_w)$ which is being measured in a test may not be serious in many soils. However, a surprisingly large range of soils exhibit large values of solute suction (often much higher than the associated values of matrix suction) and in such circumstances it is most important that a proper technique should be adopted for pore water pressure measurements. (A true matrix suction value can be obtained by using in the pore pressure apparatus a solution identical in composition to the soil solution.)

However, it is not clear in what circumstances solute and matrix suctions can be equated physically. For example, it is well established that a clay with a fully developed diffuse double layer (e.g., a disperse sodium montmorillonite) will react, in terms of volume change, to changes in solute suctions as if these changes were changes in effective stress. But conversely it has been shown (Quirk and Aylmore, 1960) that a calcium illite exhibits no significant volume change for a large range of solute suctions (although the same clay changes volume in response to matrix suctions).

The situation with respect to the comparative influences of solute suctions and matrix suctions on soil strength is equally indeterminate since very few sets of complete data have been reported.* In this context it is important to note that significant solute suctions may exist in many soils. In a typical triaxial test sample these solute suctions may well be of a magnitude substantially greater than either the applied stress or the observed pressure difference between the pore water and the pore air. If due to the particular characteristics of the test procedure some part of this solute suction is reflected in the observed suction the test may well be incomprehensible. This source of error is readily eliminated if recognized but no records of attention to this point have been noted.

Probably the most important situation in which the recognition of the separate components of suction may be vital occurs in the comparison between observations of suction and measurements of suction-dependent properties. If suction measurements on samples are made in terms of total suction (as is common in the higher portion of the suction range) and if strength measurements are made against matrix suctions (as in current practice) any inferred strength values may be in error due to the neglect of the solute suction term. Again this source of error is readily eliminated by the recognition of the dual components of suction and by the establishment of both components at correct values in the test process.

The effective stress statement of Eq (1) thus is subjected to the dual complexity of firstly, a multiplicity of factors influencing the χ term and secondly, a composite suction term. However, since χ is itself suction dependent, it follows that a complete statement of the controls of χ must embrace the components of suction.

At the conclusion of these comments it might be pertinent to ask: where is the soil mechanics of unsaturated soils? Is it necessary to introduce these complexities of physics and chemistry and stress history and so destroy the elegant simplicity of traditional soil mechanics? The answers to these questions are, in the opinion of this panel member, quite straightforward. Firstly, there can be no adequate development of the soil mechanics of unsaturated soils without the proper recognition of all factors affecting soil properties. Secondly, this complexity does not represent a retrograde step; nor does it imply the abandonment of elegant and rigorous statements of soil behaviour. On the contrary it is suggested that the challenges involved in studying and stating the laws of behaviour of unsaturated soils must lead to more rational and more complete statements of general soil mechanics.

As an addendum to the preceding comments the General Reporter has requested me to discuss briefly a few aspects of work in Australia, particularly in so far as it may be relevant to the development of practices of current value in foundation engineering.

The necessity for study of unsaturated soils arose from an awareness of the aridity of much of Australia and of many Australian soils. As a first task therefore an attempt was made to obtain a quantitative expression of the moisture status of soils throughout the country. Soil suction was chosen as the variable to express the soil moisture status and it has been shown (Aitchison and Richards, 1965) that a useful set of relationships exists at a regional level between environmental controls (principally expressed in terms of a climatic index and the groundwater table level) and the suction in soils beneath structures.

The soils throughout more than 70 per cent of the area of Australia have been shown to be normally at suctions exceeding 10 atmospheres at all depths of customary engineering interest. In much of the remainder of the country many of the soils may be at substantial suctions for significant periods. Attention has therefore been directed towards the study of soil properties as a function of soil suctions of significant magnitude. The principal academic focus of these studies has been on the question of the validity of the effective stress concept, particularly in heavy clay soils, at such high suctions. This work is covered by the preceding discussion. As yet, however, this phase of the work remains largely academic—especially in relation to natural soils.

An interim practical approach to strength measurements on unsaturated soils has emerged on the basis of three developments:

1. Techniques for the measurement of soil suction on undisturbed samples have been improved, in terms of simplicity of operation, speed, and accuracy, so that apparatus of this type is now an acceptable engineering tool (Richards, 1965).
2. Procedures for the prediction of the soil suction pertaining to equilibrium in any situation have been improved to the point of cautious acceptance (Aitchison, *et al.*, 1965).

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*Yong and Warkentin (1965) have shown that there is in fact a strength decrease with increase of solute suction in certain circumstances.

3. The extremely low permeability of soils, particularly clay soils, at high suctions (Aitchison, *et al.*, 1965) has demonstrated the impracticability of measurements of pore water pressure using any normal devices (involving the flow of even very small quantities of water to or from the soil). The only practicable form of strength test in such soils, at such suctions, and with present apparatus, is an "undrained" test, although this term may be meaningless in unsaturated soils.

Strength measurements are therefore undertaken at both the initial suction conditions and the predicted final suction conditions. In many heavy clay soils the value of ϕ_u (?) so obtained is close to zero and consequently an assumption of $\phi = 0$ with a relevant value of c_u seems reasonable.

In more permeable soils, where pore water pressure measurements are practicable with present devices, strength tests may be carried out at both the initial and predicted final suction conditions maintaining constant suctions during the test (by control of pore air pressure) thereby producing parameters corresponding to the drained strength parameters of saturated soils. Naturally the values of $\phi_{D\text{ (constant suction)}}$ and $c_{D\text{ (constant suction)}}$ so obtained will be suction dependent due to variations of the χ term but the proper selection of a relevant set of parameters should permit a stability analysis of accuracy comparable to that in saturated soils.

It is obvious that the above-mentioned practical test procedures are limited in scope but it is considered that they at least offer an approximate indication of the effect of the suction variable on the strength of unsaturated soils. It is not improbable that the errors involved in such approximate strength statements may be much less serious than the errors involved in extrapolating concepts of stability through a wide range of unsaturation in soils.

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CHAIRMAN COOLING

Well, gentlemen, that concludes the panel discussion. We will have an intermission of fifteen minutes before we invite discussions from the floor.

(There occurred an intermission of fifteen minutes.)

CHAIRMAN COOLING

For the oral discussion I have chosen, as far as I can, people from different countries.

M. I. ESRIG and S. M. BEMBEN (U.S.A.)

Several papers in Division 2 (*2/6, 2/9, 2/10, 2/48*) are concerned with the effect of the intermediate principal stress on the shear strength of soils. Other papers in the Conference (*5/1*) and elsewhere (Rowe and Peaker, 1965) have suggested that studies need to be made, not only of the effects of the intermediate principal stress on the shear strength of soils but also of the effects of strains and imposed restraints on strains. An investigation has recently been completed at Cornell University using both solid cylindrical and hollow cylindrical triaxial specimens in which measurements have been made of the principal stresses and strains throughout the shearing process. Data for a particular sand are presented in this discussion that indicate the relationship among the stresses and strains at failure for a variety of straining conditions.

The sand used in this investigation was that described in detail by *Broms and Jamal* (*2/10*). Both solid cylindrical and hollow cylindrical samples were tested in triaxial compression and extension. The solid samples were 3 in. in diameter and six in. high; the hollow samples were 4 in. in outside diameter, 3 in. in inside diameter, and 8 in. high. Essentially complete saturation of all samples was achieved by boiling the sand for 30 minutes prior to fabricating the sample by a technique of underwater deposition.

During the shearing process the samples were maintained at a constant volume by adjusting the cell pressures so that no water was expelled from or drawn into the soil. By this means, the pore water pressures were maintained equal to atmospheric pressure and the problems associated with volume changes due to partial saturation and due to cavitation of the pressure measuring system were eliminated.

Axial stresses and strains were measured directly. The radial and circumferential stresses in the hollow specimen tests were calculated from a knowledge of the internal and external cell pressures, as described by *Broms and Casbarian* (*2/9*) and *Broms and Jamal* (*2/10*) except for the tests in which no axial deformation was permitted. For this latter condition the more appropriate expression given by Whitman and Luscher (1962) was utilized to determine directly the soil strength at failure. From a knowledge of the volume changes in the inside chamber of the hollow samples and because the volume of the soil remained constant throughout the shearing process, the average strains in the radial and circumferential directions were computed on the assumption that the soil deformed as a right cylinder.

In some of the tests, volume changes of the central chamber were not permitted during the shearing process; in some others, where the pressures of the inside and outside chambers were maintained constant, the volume changes that occurred during the shearing process were measured;

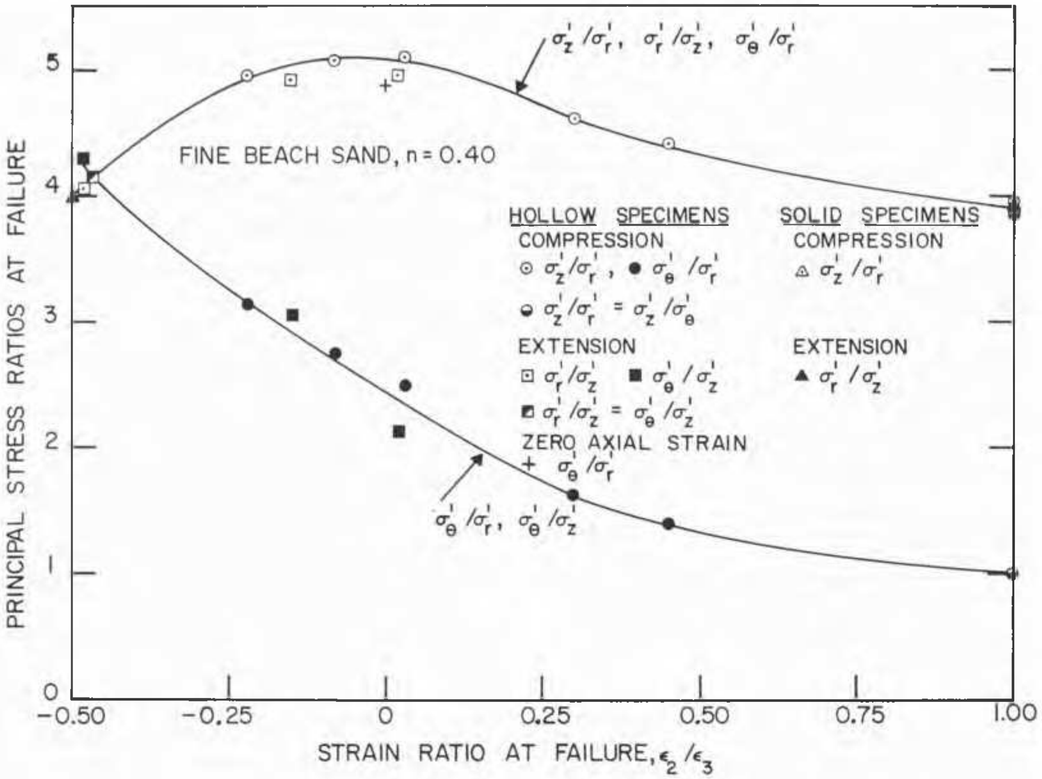

FIG. 23. Relationship between principal stress ratio at failure and strain ratio at failure for hollow and solid cylindrical sand specimens under triaxial compression and tension.

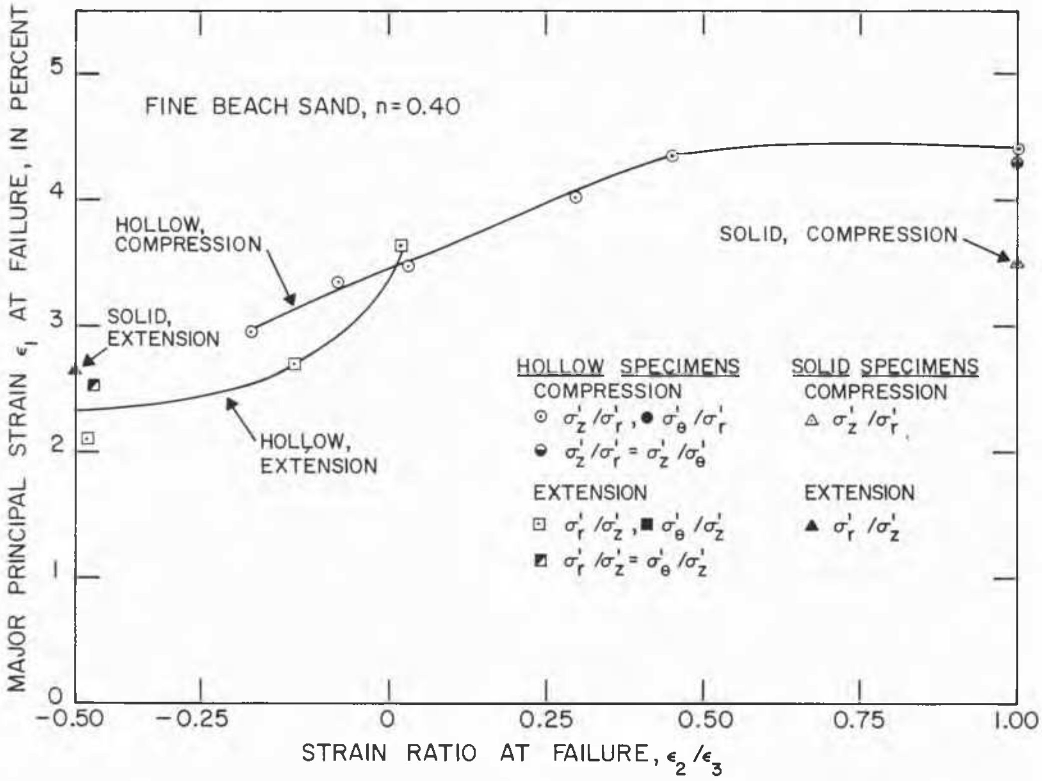

FIG. 24. Relationship between major principal strain at failure and strain ratio at failure for hollow and solid cylindrical sand specimens under triaxial compression and tension.

and in a third series the volume changes of the central chamber were strictly controlled throughout the shearing process. Control of the volume changes of the inner chamber was achieved by utilizing a specially designed apparatus to pump water in or out of the central chamber at a preselected rate. Variation of the rate of pumping produced variations in the strain ratios at failure.

In all tests, failure was considered to have occurred when the ratio of the major to the minor principal effective stresses first reached a maximum value. This ratio will be called the maximum stress ratio. Because no very loose samples were tested, the maximum stress ratio always occurred before the difference between the major principal and the minor principal stresses (the maximum stress difference) reached a maximum value. Limitations on the cell pressure which could be applied by the existing equipment prevented the continuation of most of the tests to the point where the stress difference reached a maximum value.

The results of tests on the particular sand used in this investigation when at an initial porosity $n = 0.40$ are presented in Fig. 23. The abscissa of this graph is the ratio of the intermediate principal strain to the minor principal strain, ϵ_2/ϵ_3, at failure. A strain ratio of unity corresponds to the strain conditions normally assumed in the conventional undrained triaxial compression test. A strain ratio of zero corresponds to plain strain conditions and a strain ratio of -0.5 corresponds to the straining conditions normally expected in the undrained triaxial extension test. The ordinate of this graph is the stress ratio at failure. The upper curve represents the relationship between the strain ratio and the ratio of the major to the minor principal effective stresses at failure, except in the neighbourhood of a ratio of -0.5 where the upper and lower curves cross. The lower curve represents the ratio of the intermediate principal effective stress to the minor principal effective stress when failure, as defined by the maximum stress ratio, occurred. The intersection of the curves occurs at a point that falls on both curves and was obtained from an axial extension test with the inside and outside chamber pressures equal. For this test the average of the radial and tangential strains was taken as the intermediate strain, ϵ_2.

It can be seen from Fig. 23 that the shear strength of the sand when tested in the conventional manner using solid cylindrical samples was approximately the same in compression and extension. However, the results of the hollow specimen extension test in which the volume of the central chamber was maintained constant suggests that the shear strength of the sand is higher in extension than in compression.

It can also be seen that the shear strength of the sand is highest when plane strain conditions are imposed on the soil and that there is a reasonably smooth transition to this maximum as the strain ratio approaches zero from either side. Moreover, as the strain ratio decreases from unity toward its minimum value of -0.5 the ratio of the intermediate principal stress to the minor principal stress increases. Hence, when the strength of the soil is determined in accordance with the Mohr-Coulomb failure criterion, it can be seen that the changes in strength are not proportional to the changes in the intermediate principal stress or to the ratio of the intermediate principal stress to the minor principal stress.

Further information about the strain behaviour of these samples is provided in Fig. 24. In this figure the major principal strain is shown as the ordinate against the strain ratio which is the abscissa. The curves indicate that for hollow specimens, as the ratio of the intermediate principal strain to the minor principal strain at failure decreases, the major principal strain at failure also decreases. It is of interest to note that the behaviour of the solid cylindrical samples when tested in extension and compression does not appear to be the same as the behaviour of the hollow samples. Although the behaviour of the solid samples, as shown on Fig. 24, appears to be anomalous, their behaviour was found to be entirely consistent with the behaviour of the hollow samples when strain energies at failure were considered. Energy considerations are beyond the scope of this discussion and will be treated fully elsewhere.

ACKNOWLEDGMENTS

The investigation summarized in this discussion was performed by Mr. Bemben and was supported, in part, by the National Science Foundation, Grant No. G-21833.

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K. Y. Lo (Canada)

The anisotropic shear strength properties of soils have recently attracted considerable attention. In addition to the paper under discussion (*2/9*) in which some data of the effect of rotation of principal stresses on the strength properties of a remoulded kaolinite clay are reported, two other papers (*2/1, 2/31*) presented to this Conference also deal briefly with this subject. It is important, therefore, to summarize the existing information at this early stage of development and differentiate between the types of anisotropy that have been encountered so far.

The results of unconfined compression tests on undisturbed samples reported by several investigators (Jakobson, 1953; Ward, *et al.*, 1959; Lo, 1965; Lo and Milligan, 1965), together with those contained in the paper, are shown in Fig. 25. In this polar diagram, the percentage strength C_i/C_1 is represented by the distance of the point from the origin, where C_i is the strength measured with the applied major principal stress inclined at an angle i to the vertical axis, and C_1 and C_2 are the principal strengths when $i = 0$ and 90 degrees, respectively.

It is obvious from the data collected in Fig. 25 that there are two principal types of anisotropy, *viz.*: $C_1 > C_i > C_2$ and $C_1 > C_i < C_2$, with $C_1 = C_i = C_2$ as a special case. It may be expected, however, that other types of anisotropy will be encountered as more investigations of this nature are carried out.

A method of taking the variation of undrained strength with direction of applied principal stresses into account in stability analyses, has been developed (Lo, 1965). It is also interesting to note that the results of the authors lie quite close to those obtained from block samples of a stratified clay taken from a shaft (Lo and Milligan, 1965), and that anisotropy is exhibited to such a marked degree even in a remoulded clay.

When the strength properties are considered in terms of effective stresses, the anisotropy must be attributed to the cohesion and friction components, such as the parameters C' and ϕ', since there is no physical reason why the pore pressure set up should be anisotropic in a homogeneous sample of soil. Triaxial consolidated-undrained stress-controlled

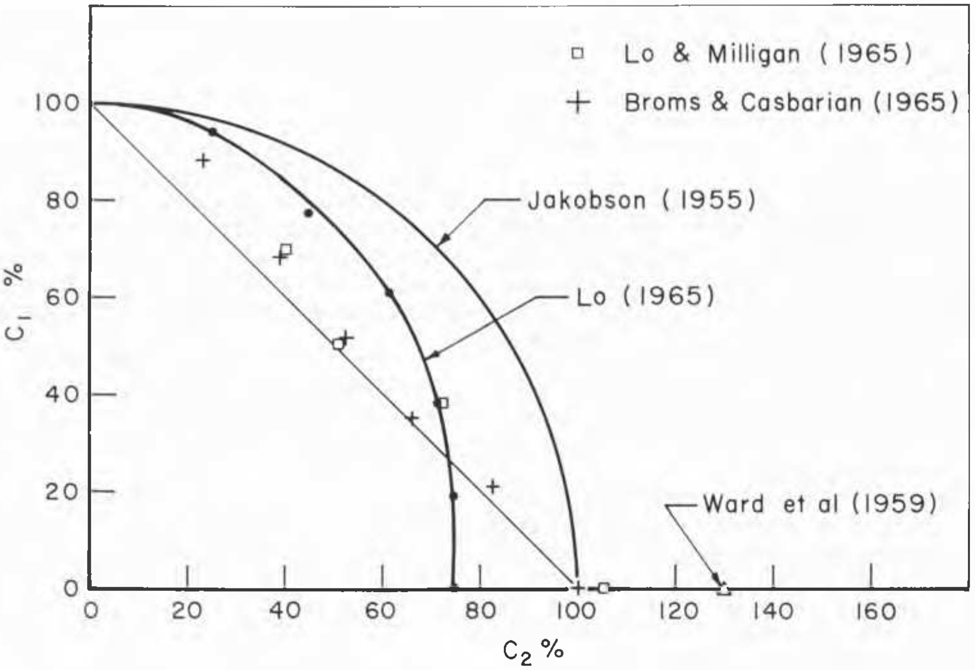

FIG. 25. Types of anisotropy of clays.

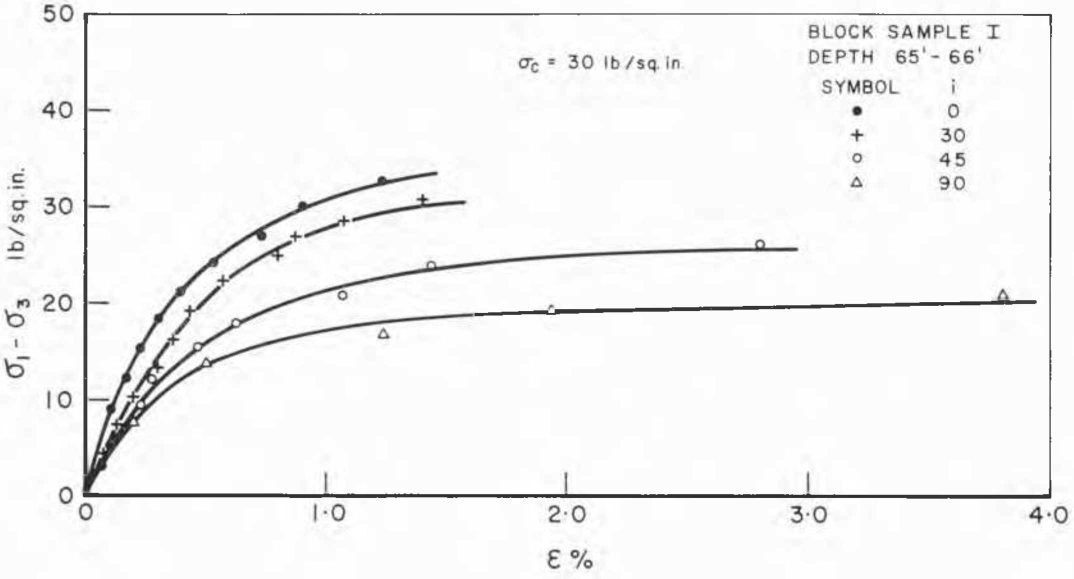

FIG. 26. Stress-strain relations of a clay tested at different values of *i*, where *i* is the angle between vertical axis of specimen and direction of major principal stress.

tests with pore pressure measurements have been carried out on specimens trimmed from undisturbed block samples of a clay $w_L = 39$ per cent, $w_P = 21$ per cent). These specimens were cut at different angles from the vertical axis and consolidated at a cell pressure of 30 lb/sq.in. Some typical results are shown in Figs. 26 and 27 and Table V.

It will be seen from Fig. 26 and Table V that as *i* increases from 0° to 90° (*a*) the stress-strain modulus decreases, (*b*) the maximum stress difference decreases, (*c*) the failure strain increases, and (*d*) the pore pressure parameter A_f increases. However, when the induced pore pressures from the different tests are plotted *versus* strain in Fig. 27, a unique curve is obtained, independent of the direction of the applied major principal stress. It is evident from Figs. 26 and 27 that the increase in A_f is due to the rise of pore pressure with strain and reduction of $(\sigma_1 - \sigma_3)_f$ with rotation of principal stress.

TABLE V. RESULTS OF C.I.U. TESTS

i (deg.)	W_i (%)	W_f (%)	$(\sigma_1 - \sigma_3)_f$ (lb/sq. in.)	ϵ_f (%)	A_f
0	32.6	31.0	36.0	1.3	0.23
30	28.6	28.6	30.8	1.4	0.27
45	30.3	29.6	26.0	2.6	0.44
90	31.8	31.0	19.9	4.0	0.60

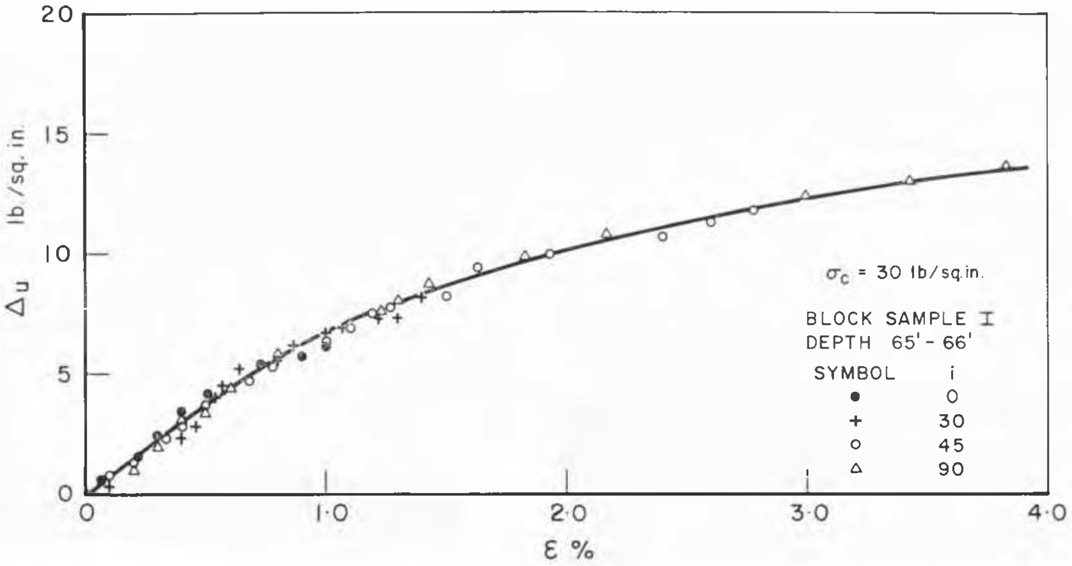

FIG. 27. Relationship of pore pressure and strain of a clay tested at different values of *i*.

A study of the results given in Table V of the paper will show that the pore pressures set up at each consolidation pressure vary within small limits. The apparent variation of A_f can, therefore, be accounted for by the dependence of the maximum stress difference on anisotropy.

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B. LADANYI (Canada)

When a virgin one-dimensional consolidation test is performed on a specimen of saturated clay, it is usually assumed that the ratio of the effective principal stresses remains constant throughout the test. This conclusion is based on the experimental fact that the points representing the values of the effective principal stresses at different loading levels in such a test, when plotted for example in a Rendulic stress plane, lie on a straight line passing through the origin of co-ordinates. One such example is the K_0 line in Fig. 4 in the paper by *Akai and Adachi* (*2/2*).

It may, however, be interesting to mention that the extrapolation of the K_0 line back to the origin may not always be justified. Such an extrapolation implies in fact, that even at very low normal stress levels or at very high water contents the shear strength of the clay is mobilized to a degree such that a state of stress different from a hydrostatic state can be maintained. This assumption, however, can only be close to reality if at the beginning of the test the water content of the clay is at the liquid limit or even lower.

If, on the contrary, it is assumed that in a one-dimensional consolidation test the clay particles are initially dispersed in an aqueous suspension it is evident that with increasing load, the hydrostatic state of stress will be maintained until the clay particles come close enough to one another for van der Waal's attractive forces to predominate over the Coulomb or repulsion forces, resulting in a shear strength measurable at low rates of strain. In addition, if lateral movement of the particles is prevented, they will tend to align themselves in a direction normal to the applied load and thus parallel to one another (Scott, 1963, p. 342).

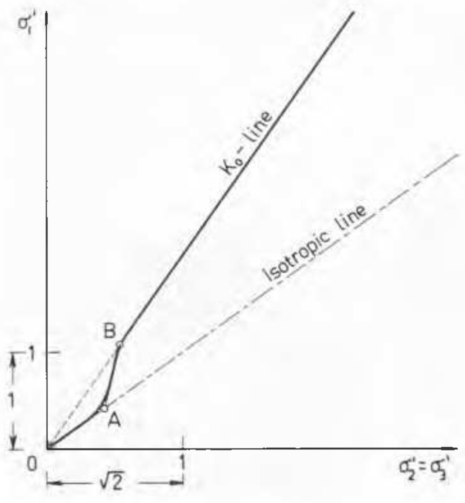

FIG. 28. Illustration of anticipated shape of K_0 line for clay in the transition (AB) between isotropic condition (fluid state) and consolidated condition.

In other words, if a one-dimensional consolidation test is performed on a clay specimen at very high initial water content, it can be expected that a transition region will be found in which the value of K_0 will decrease gradually from unity to a constant value reflecting the gradual mobilization of the shear strength of clay with decreasing interparticle spacing and increasing effect of particle orientation. The anticipated shape of the K_0 line in the transition region is shown schematically in Fig. 28 by the line OAB. If the shape of the K_0 line proves to be similar to that shown in Fig. 28, it could be argued that the true origin of that line should be placed at the point B, and not at the point 0 as it is usually assumed.

Tests designed for investigating the mentioned transition region are actually under way at Laval University.

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R. HAEFELI (Switzerland)

Kenney and Landva (2/28) of the Norwegian Geotechnical Institute have presented a substantial paper on the vane triaxial apparatus. For some years now a similar development has taken place in Switzerland in the Laboratory of Hydraulic Research and Soil Mechanics of the Swiss Federal Institute of Technology.

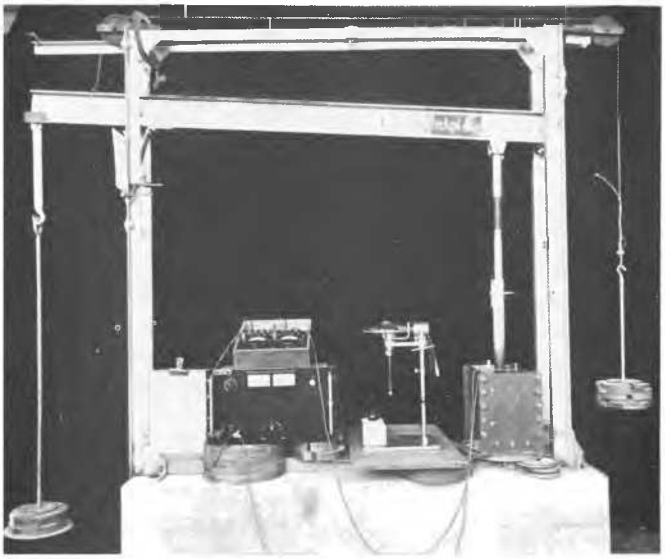

FIG. 29. Equipment consisting of a large consolidometer, electric apparatus for electro-osmosis and vane apparatus.

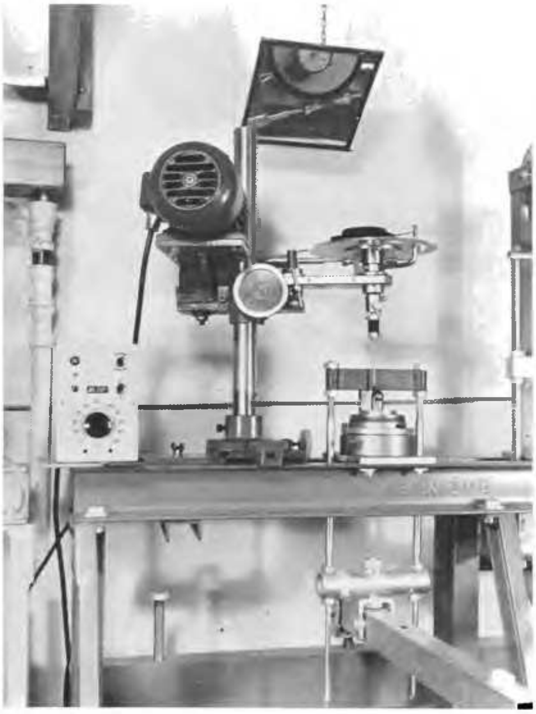

FIG. 30. Motorized vane.

In the soil mechanics section a diminutive vane apparatus, developed by J. Zeller, was used for the study of two different kinds of problems. The first one was the determination of the local change of shear strength due to the treatment of a clay by electro-osmosis. Fig. 29 shows the equipment, consisting of a large consolidometer, the electric apparatus for the electro-osmosis, and the vane apparatus. In Fig. 30 we see the motorized vane and in Fig. 31 the same vane in action.

Fig. 32 represents some results corresponding to the influence of the rate of shear on the shear strength of an undisturbed saturated clay sample. For low slip velocities this influence seems to be important; yet Professor Skempton has found that the shear strength of London clay is affected only to a slight degree by the rate of shear (drained tests).

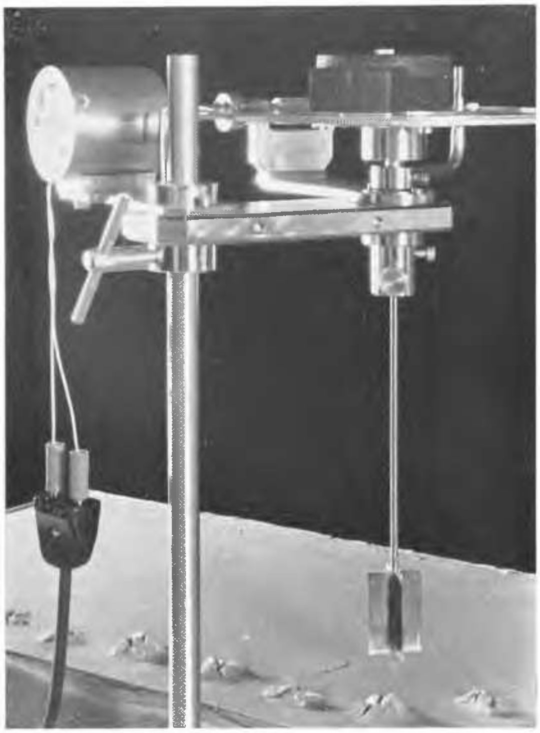

FIG. 31. Motorized vane—close up in operation.

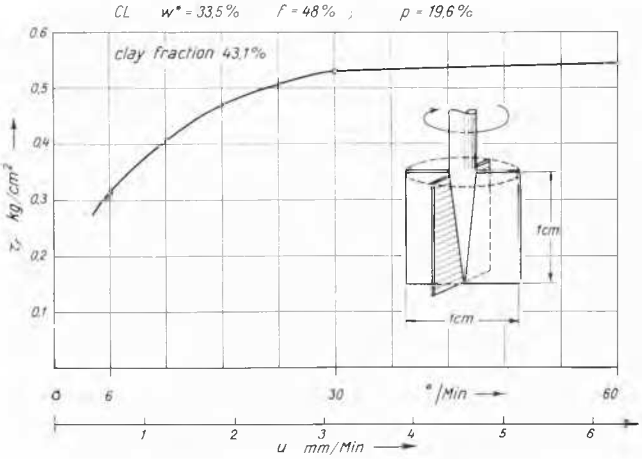

FIG. 32. Influence of rate of shear on shear strength of a saturated clay.

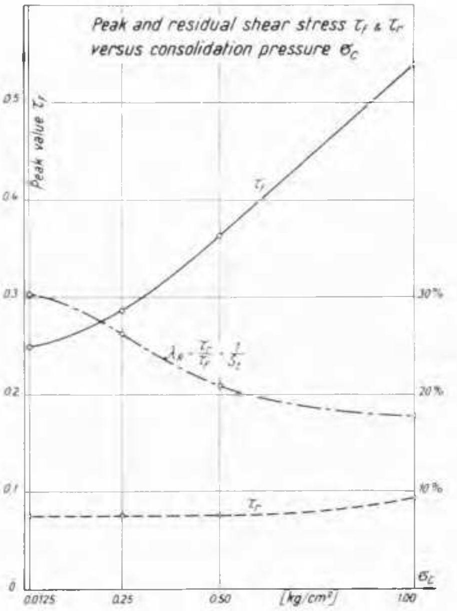

FIG. 33. Vane consolidometer test with undisturbed lake marl.

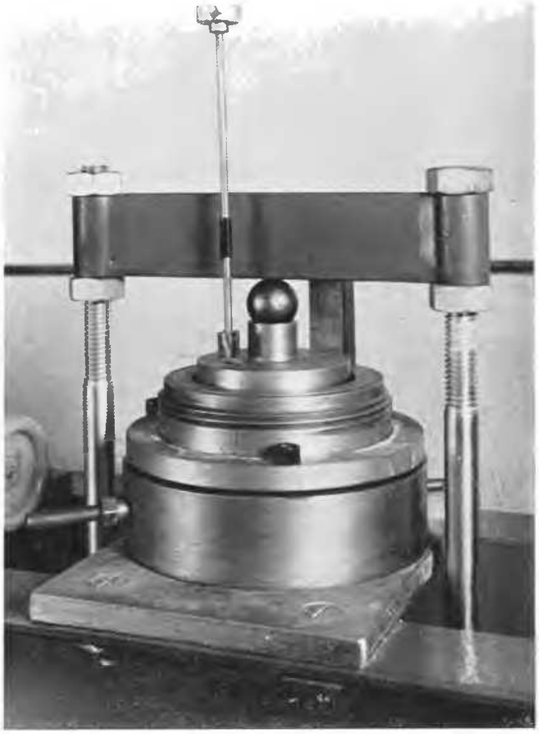

FIG. 34. Introduction of vane into consolidometer.

The second problem was the measurement of the residual ratio—corresponding to the inverse value of the sensitivity—of undisturbed samples of lake marl for different consolidation pressures. Here, the influence of the rate of shear on the peak value of the shear strength was not important. On the other hand a very pronounced dependence of the residual ratio on the consolidation pressure was found. In Fig. 33 the peak value and the residual value of the shear strength, as well as its ratio are plotted against the consolidation pressure. To get these results the little vane was introduced directly into the consolidometer after the consolidation of the sample. Fig. 34 shows the introduction of the vane in the consolidometer.

In conclusion, we think that the introduction of a diminutive, motorized vane in soil laboratories promises to be a valuable contribution in the field of shear strength and creep research. On the other hand we feel that in a fundamental investigation the colloidal chemical aspect of the phenomenon must be considered at least of the same importance as the mechanical and the rheological ones.

ACKNOWLEDGMENTS

I am indebted to Mr. Jaecklin and Mr. Fetz for the careful execution of the texts.

L. Bjerrum (Norway)

In a paper to this conference *Aas (2/1)* has described a series of vane tests carried out with vanes of different shape. These tests showed that the undrained shear strength is different in horizontal and vertical directions. Recent research on the highly sensitive Norwegian clays has further supported this finding, as it has clearly shown that the undrained shear strength is not a unique value, but depends on the way in which the clay is brought to failure and the direction of the failure plane.

The most interesting results were obtained by a recently developed direct simple-shear apparatus. In this apparatus a cylindrical sample 8 cm in diameter and 1 cm high is confined in a rubber membrane reinforced with a spiral winding

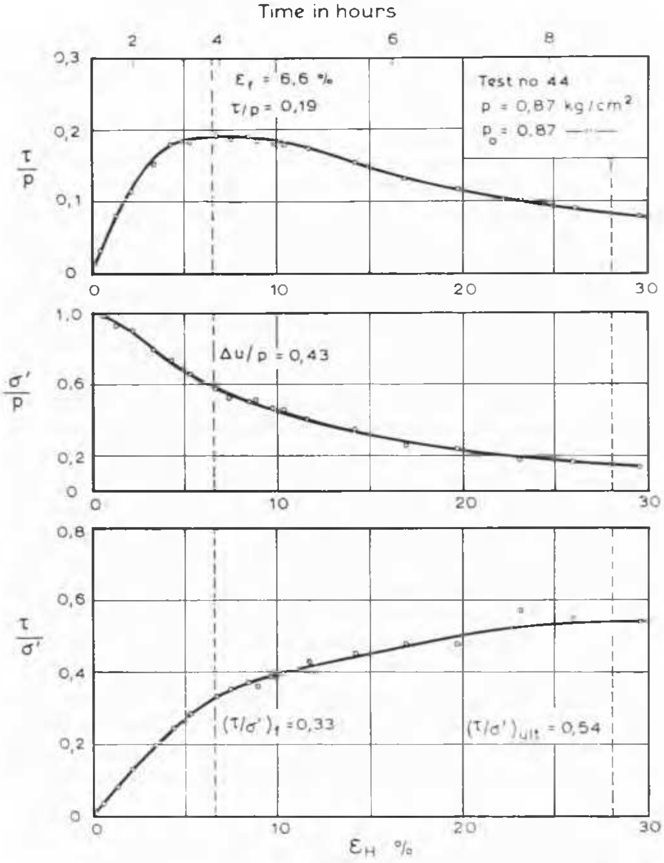

FIG. 35. Typical results of a constant volume, simple shear test on a specimen of quick clay consolidated at a pressure equal to the previous overburden.

of a wire. This membrane allows thickness changes of the specimen to occur with no change in diameter so allowing the specimen to strain in simple shear. The clay tested was a quick clay sample taken from the test area at Manglerud near Oslo. The water content of the clay is about 34–38 per cent compared with a liquid limit of about 27 per cent. The undrained shear strength varies from 0.6 to 1.6 tons/sq.m. This corresponds to a ratio of undrained shear strength to effective overburden pressure of 0.13. The samples used were taken with a 10-cm, thin-walled, fixed-piston sampler.

Fig. 35 shows a typical set of results of an undrained or, more correctly, constant volume simple-shear test on a specimen consolidated at a vertical pressure equal to the effective overburden pressure it carried in the field. In the upper diagram the horizontal shear stress applied to the sample is plotted, and in order to obtain a dimensionless diagram the shear stress is divided by the vertical pressure at which the sample was consolidated. As observed from the diagram the failure value of the undrained shear strength ratio is 0.19. The average of 14 tests was 0.18 with a maximum deviation of ±14 per cent.

In the second diagram the effective normal stress acting on the sample during the test is shown. To maintain a constant volume the vertical stress was varied during the test, and as the shear strain was applied slowly enough to prevent the rise of any pore pressures, the effective stress is equal to the applied total stress. The change in effective stress shown in the diagram is thus equivalent to the pore pressures in a corresponding undrained test. As observed from the diagram, the rise in pore pressure in initially small. At a certain critical shear stress the pore pressure increases very rapidly; this occurs shortly before failure. This observation indicates that the clay at small strains shows a structural rigidity which is progressively broken down while approaching failure.

The lower diagram shows the computed value of the ratio of the applied shear stress to effective normal stress. This ratio thus expresses the proportion of the structural strength of the clay which at any strain has been mobilized on horizontal planes. It is a characteristic feature of these tests that failure occurred at a strain where the effective structural strength is far from fully mobilized. On the average only 60 per cent of the structural strength was mobilized at failure.

As mentioned, the ratio of the undrained shear strength to the vertical consolidation pressure found in these tests is 0.18. It is obviously most relevant to compare this value with the horizontal shear strength of the clay in the field. At the site at Manglerud from which the samples were taken, tests were carried out with vanes of different shapes, as described by Aas in his paper. From these tests we can derive the vertical as well as the horizontal shear strengths. Table VI gives the various shear strength values determined on the Manglerud clay, the results being expressed by the ratio of the undrained shear strength to effective vertical pressure at which the clay was consolidated. A comparison shows that the constant volume, direct shear test gives results which are equal to the horizontal vane shear strength, both ratios being 0.18. The uncertainties involved in the determination of the field anisotropy are so great, however, that the only conclusion which can be drawn is that the order of magnitude is the same.

A comparison of the different values listed in the table shows that the values obtained on the constant volume, direct shear tests on specimens consolidated under stresses approaching the field conditions are considerably lower than the values measured in triaxial tests on parallel samples. The study of the undrained shear strength of the Manglerud clay has thus shown that it is not a unique value, but varies within wide limits, its magnitude being dependent on the way failure is brought about.

TABLE VI. RATIO OF UNDRAINED SHEAR STRENGTH TO CONSOLIDATION PRESSURE MEASURED BY VANE TESTS *in situ*, BY UNCONFINED COMPRESSION TESTS ON UNDISTURBED SAMPLES AND ON SAMPLES CONSOLIDATED AT PRESSURES EQUAL TO THE OVERBURDEN IN THE FIELD

Type of test	Average value of s_u/p	Range of variation
Standard vane tests	0.13	0.10–0.17
Anisotropy vane tests		
Horizontal	0.18	
Vertical	0.12	
Unconfined compression tests	(0.17)	0.16–0.20
Undrained triaxial tests		
Isotropic consolidation	0.29	
Anisotropic consolidation	0.28	
Constant volume, direct shear tests	0.18	0.16–0.21

A more complete description of the results of this study will be published soon in a paper by Mr. Arvid Landva and myself.

K. H. Roscoe (Great Britain)

In order to test soils under conditions of plane strain, which probably prevail more frequently in the field than the radially symmetric conditions of the triaxial test, a number of models of the simple shear apparatus have been developed at Cambridge. The nature of three of these models has been briefly indicated by Roscoe (1953, 1961, and 1963). A typical section of a sample, after shearing through an angle α in any of these models, is shown in Fig. 36.

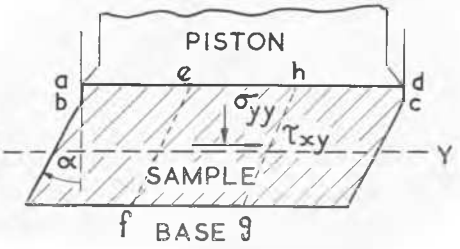

FIG. 36. Cross-section in the plane of strain of a sample in early models of the simple shear apparatus.

With any of the three early models of the apparatus it was necessary to assume that the normal stress σ_{yy} and the shear stress τ_{xy} on any horizontal plane YY are uniform and can be determined respectively from the piston load and the horizontal force applied to the apparatus. Three alternative methods were available to determine the average general stress system imposed on the sample at any stage of the test. One method is to assume that YY is a plane of maximum obliquity of stress thereby implying the validity of the Mohr-Coulomb criterion of failure. We have frequently preferred to assume that the stress τ_{xy} on the plane YY is the maximum shear stress. More recently Arthur, *et al.* (1964) have assumed that the axes of stress and strain coincide so that Mohr's circles of stress and strain are geometrically similar. Both circles can then be defined from the fact that the linear

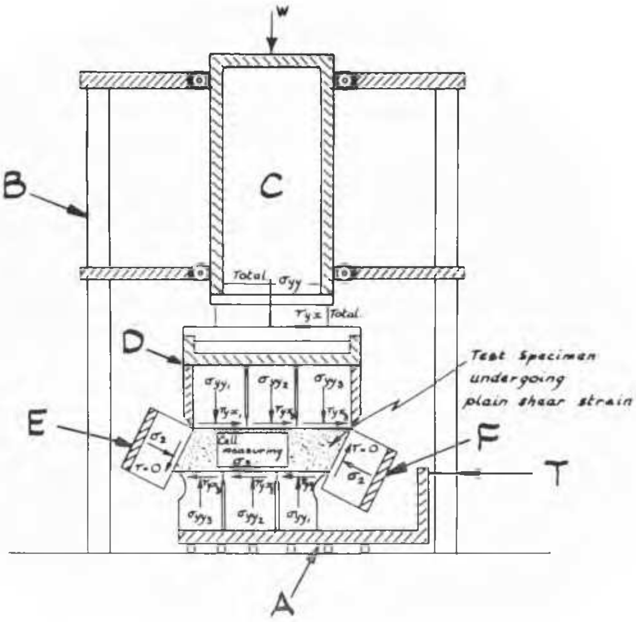

FIG. 37. Diagram of a later model of the simple shear apparatus showing load cells.

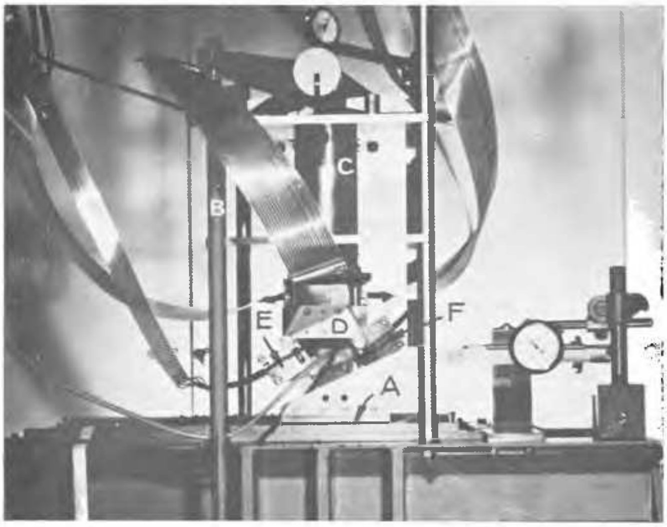

FIG. 38. Photograph of the later model of the simple shear apparatus shown diagramatically in Fig. 37.

strain in any horizontal plane YY is zero. None of the above methods can be considered to be entirely satisfactory.

A fourth model of the simple shear apparatus has now been made (see Figs. 37 and 38) in which it is not necessary to adopt any of the above methods to determine the relevant Mohr's circle of stress. The sample is bounded by ten load cells as indicated in Fig. 37. These cells are of similar design but different shape to those described by Arthur and Roscoe (1961). Each cell measures simultaneously the magnitude of the shear force and the magnitude and eccentricity of the normal force on their active faces. The sample is supported on three load cells attached to the base A which can be made to travel on roller tracks mounted on the load frame by the thrust T. Attached to the load frame is a guide frame B which permits the rest of the apparatus C, D, E, and F to move freely as a unit in a vertical direction. C is a cylindrical guide pillar for the top piston D which contains three load cells in contact with the top of the soil sample. Between C and D there is one large load cell to record the over-all vertical and horizontal forces on the sample. Freely hinged to D are the end flaps E and F which are themselves load cells and are always constrained to remain in contact with the outer top edges of the base A. The lateral stress (perpendicular to the plane of Fig. 37) can be measured by load cells as shown, or these cells can be replaced by perspex sides for the transmission of X-rays through the sample. All the load cells are extremely stiff and the data from them can be punched directly on to tape and analysed with the aid of the Cambridge Titan computer.

The theoretical analysis by Roscoe (1953) suggested that stress conditions within the middle third of the sample (efgh in Fig. 36) would be more uniform than in the remainder of the sample. X-ray photographs of the samples at various stages of strain (such as shown by Roscoe, 1961) also suggest that the strains are more uniform in this region than elsewhere provided the influence of the dead zone (abcd in Fig. 36) is ignored. Attention is therefore concentrated upon this middle third region in the apparatus shown in Fig. 37. The normal and shear forces on the top and bottom faces of this region are determined directly from the central load cells in the piston and base respectively. The forces recorded by the three load cells round each end of the sample are then compounded by simple statistics to give the normal and shear forces on both sides (such as ef and gh in Fig. 36) of the middle third region. It is then assumed that the stresses on each face of the middle third are uniform and consequently the complete stress system within this region is known.

A special feature of this model of the simple shear apparatus is that it eliminates the dead zone (abcd in Fig. 36) of earlier models. By careful selection of the original size of the soil sample it is possible to ensure that the size of the dead zone is very small compared to that of the remainder of the sample. Early models of the simple shear apparatus gave excellent results for samples of randomly packed steel balls or normally consolidated, saturated, remoulded clays. With such soils the voids ratio or excess pore water pressure remained constant for shear strains beyond the critical state. However, this was not the case when testing dense samples of (18 > B.S. sieve > 25) Leighton Buzzard sand. Typical curves for a drained test on this sand have been published by Roscoe (1961). These appeared to show that the sample was continuing to dilate despite having been sheared through angles exceeding 60°. X-ray photographs (see Roscoe, 1961, Fig. 6) showed that this continued dilatation was due to the gradual diminution, with further shear, in the size of the dead zone. From the data obtained on these early models of the simple shear apparatus for this sand it was not possible to locate accurately the critical state line for this sand. However, it has been possible to obtain very satisfactory data for this sand from the new model of the simple shear apparatus as shown by the curves in Fig. 39 relating τ/σ and $\tan\alpha$, and also the change of voids ratio and $\tan\alpha$, during five tests on samples of this sand with widely different initial void ratios. These curves were obtained by R. H. Bassett who made the detailed design of the equipment.

Bassett's work has shown a number of important features in connection with the setting up and testing of samples of this sand in the simple shear apparatus. Only two will be mentioned. The first is that the use of a rubber lining backed by silicone grease leads to considerable friction between the

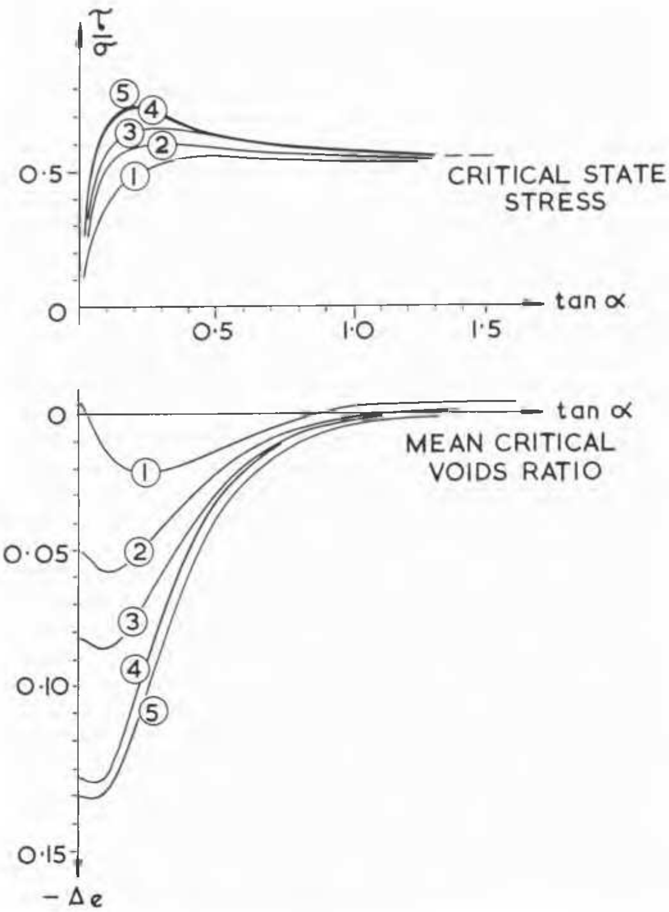

FIG. 39. Data from drained tests with the later model of the simple shear apparatus on samples of Leighton Buzzard sand (18 > B.S. sieve > 25) with five different initial voids ratios.

sand sample and the inside walls of the apparatus. This was found to be due to the coarse grains bedding into the rubber lining causing direct contact between the rubber and the walls of the apparatus. For the data presented in Fig. 39 the inside walls of the apparatus were made of hard glass, thereby dispensing with the rubber lining and silicone grease. The second feature was that it was not possible to obtain uniform readings of the vertical load cells when applying a vertical load to a sample prior to the application of any shear. This suggests that the pouring methods of preparing the sample and the techniques of obtaining a level top surface for the sample are still not entirely satisfactory. This fact, together with data obtained by K. Z. Sirwan (1965) for the variation of strain at different radii in triaxial samples, indicates that further research is required before samples of sand with real initial uniformity can be set up in laboratory test apparatus.

Finally it should be mentioned that the simple shear apparatus is being further developed by E. R. L. Cole. To ensure skew symmetry he makes one of the end flaps hinge about a bottom corner of the sample in Fig. 37 while the other hinges about the diagonally opposite, top corner.

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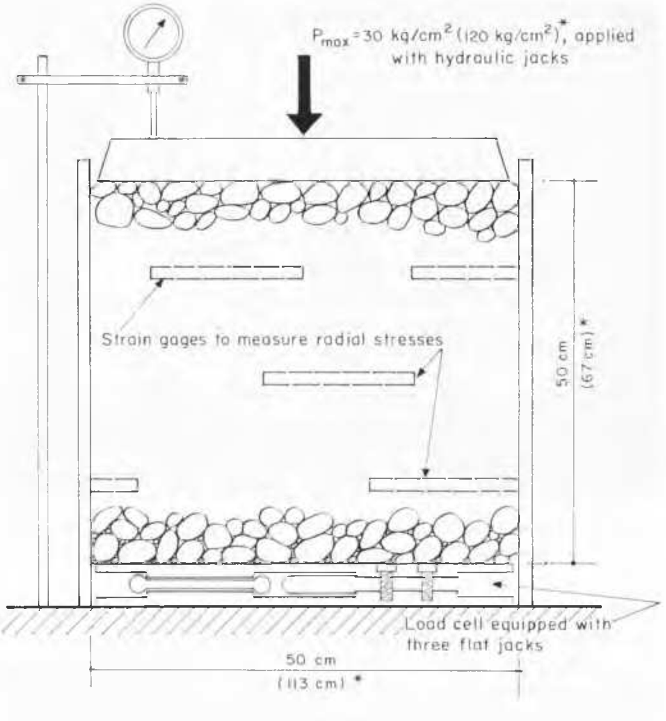
L. Ramírez de Arellano (Mexico)

As a part of a comprehensive research programme on the behaviour of rockfill materials, both in the field and the laboratory, the Comisión Federal de Electricidad (C.F.E.), in co-operation with the Institute of Engineering, National University of Mexico (U.N.A.M.), has performed one-dimensional compression tests with various rockfill materials, among them those obtained from the shoulders of two major rockfill dams in Mexico. The main characteristics of the two devices used so far to study the compressibility of rockfills in the laboratory are schematically presented in Fig. 40.


P_{max} = 30 kg/cm² (120 kg/cm²)*, applied with hydraulic jacks
Strain gages to measure radial stresses
50 cm
(67 cm)*
Load cell equipped with three flat jacks
50 cm
(113 cm)*

* Data corresponding to new device

FIG. 40. Diagram showing principal dimensions and features of one-dimensional compression test apparatus for rockfill.

The first set of data corresponds to an oedometer operated at the Institute of Engineering. In this apparatus, vertical pressures can be measured at both ends of the specimen, thus permitting an estimate of average pressures. Settlements of the loading plate are measured by means of dial gauges, and horizontal radial pressures exerted by the specimen against the confining steel cylinder are detected with 12 electrical strain gauges, located at 3 different elevations. The second device has recently started to operate at the El Infiernillo site rockfill laboratory; it is of the floating ring type.

In Fig. 41, a comparison is made of vertical strains measured in specimens taken from the shoulders of the dams and tested at the laboratory, in both dense and loose states, with actual field data obtained with a modified version of the cross-arm developed by the U.S. Bureau of Reclamation. As construction of the rockfill progresses, the pipe sections of the instrument are installed, thus enabling continuous measurement of vertical strains within the rockfill mass starting shortly after each layer is placed. In each of the graphs, laboratory and field curves are plotted together to facilitate comparison. Field curves were obtained by averaging observations made at four different points of each dam. Vertical pressures acting on the layer being considered were computed in this case by multiplying the height of rockfill above the middle plane of the layer under consideration by the average unit weight of the rockfill determined in large scale field density tests.

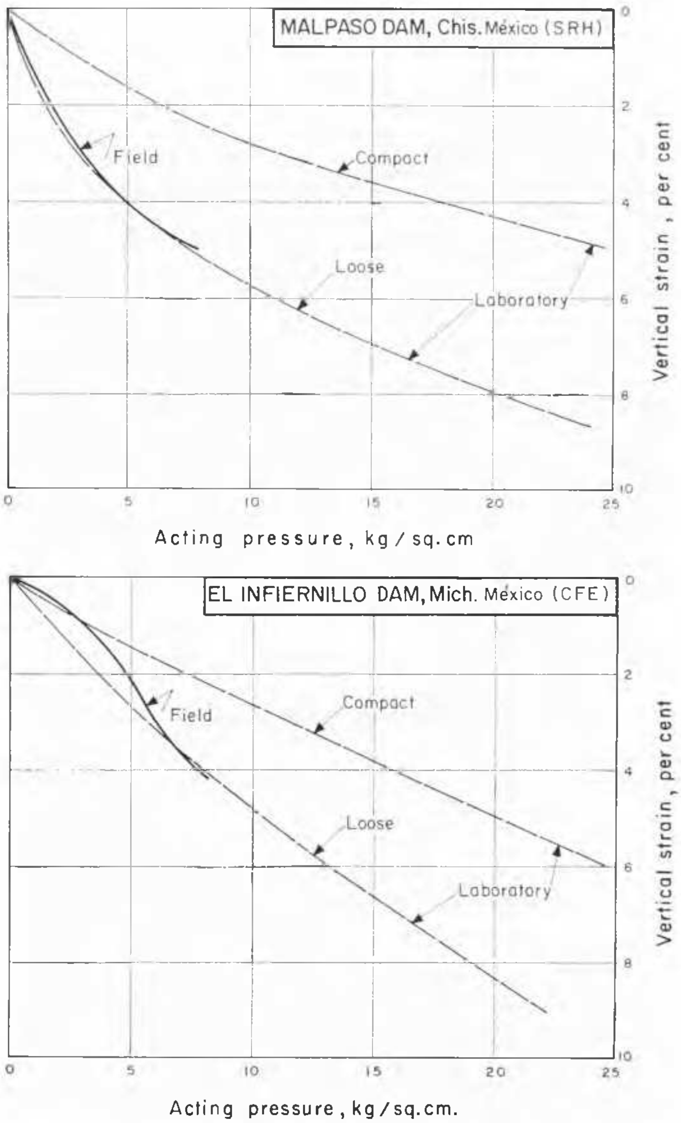


FIG. 41. Comparisons between compression of rockfill measured in the field and in laboratory tests.

The Malpaso Dam rockfill is a poorly cemented conglomerate which breaks into a well-graded sand and gravel upon spreading. In this case, field compression data closely follow the laboratory curve for the loose condition. It is not yet possible to arrive at definite conclusions, since there might be differences between the gradation of the laboratory specimen and the material in the field, but it should be pointed out that in this case the material was placed in layers up to one meter thick, compaction being effected only by traffic of the hauling equipment and passage of tractors used to spread the rockfill in layers.

Laboratory and field data for El Infiernillo Dam are presented in the graph at the bottom of Fig. 41. In this case, an interesting phenomenon can be appreciated in the field curve, also obtained by averaging values corresponding to four different points in the embankment. The first portion of the curve is concave downwards, suggesting that the mass compressibility of the rockfill increases with the applied pressure. Similar trends have been observed in the laboratory in the case of coarse, uniformly graded specimens, due probably to the high contact forces between particles, with the consequent point breakage. However, in this case it could be due to an initial adjustment between the hand-placed rockfill in the immediate vicinity of the instrument and the crawler-tractor compacted rockfill in the surrounding area. Here, the maximum thickness of the layers was also one meter, and the material involved is an excellent quarry-blasted silicified conglomerate with angular particles. The final portion of the compression curve is concave upwards as one would expect it to be, and practically coincides with the laboratory loose condition, also suggesting the possibility of differences in gradation between the field and the laboratory, and the low efficiency of light compactive effort in the field.

In this preliminary investigation, the depth of the field installations was rather limited, since at the time placement of instruments was started in 1962 there was no precedent in the literature for this type of field measurement in clean, coarse rockfill materials. In spite of the above-mentioned limitations one can confidently predict that in the near future it will be possible to estimate in advance settlements within the rockfill shoulders of high dams by means of laboratory investigations. This aspect is of primary importance, especially in high dams with thin impervious cores where the stress-strain characteristics of the core and rockfill shoulders might be so different as to make the use of certain types of materials incompatible.

B. WACK (France)

Je voudrais donner rapidement les résultats obtenus à l'appareil triaxial avec une mesure fine de la densité. Notre but était de mesurer la densité des matériaux pulvérulents lorsqu'ils se trouvaient en plasticité et de vérifier l'hypothèse suivant laquelle la densité d'un matériau pulvérulent reste constante aux grandes déformations, toutes choses étant égales par ailleurs.

Habituellement, on mesure la variation du volume total de l'échantillon; mais, du fait de l'hétérogénéité des déformations on ne peut alors avoir de renseignements précis sur la variation de volume des zones en plasticité. Pour contourner cette difficulté, nous avons utilisé une méthode de mesure plus fine, basée sur la transmission des rayons radioactifs gamma. Nous avons pu ainsi localiser nos mesures de densité à la seule zone de l'échantillon où se produisent les plus grandes déformations.

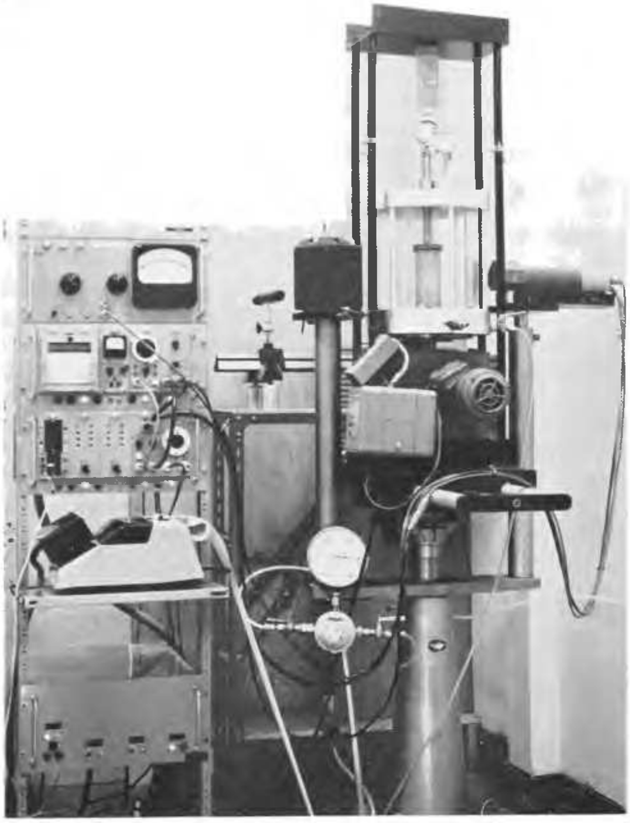


FIG. 42. Montage expérimental.

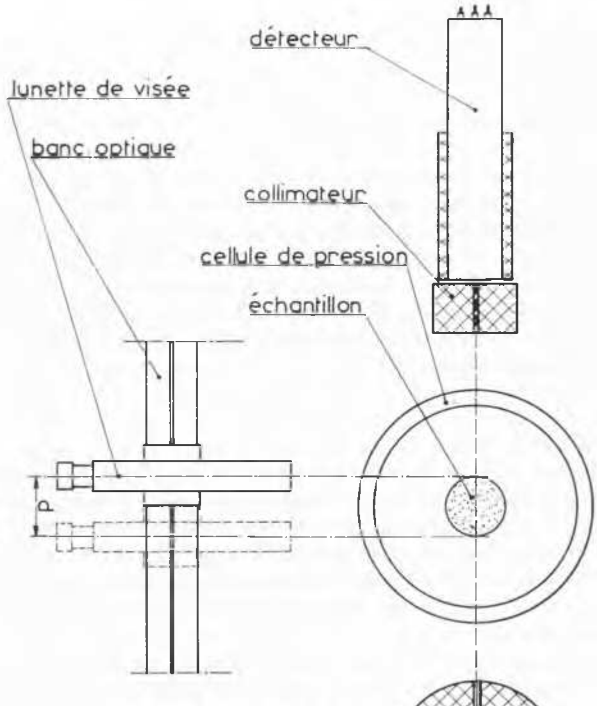


FIG. 43. Schéma de la mesure radioactive et optique.

Le montage expérimental est représenté à la fig. 42 : il s'agit d'un appareil triaxial classique, monté sur une plate-forme mobile verticalement, ce qui permet de présenter n'importe quelle section horizontale de l'échantillon dans le plan de mesure. Celui-ci est schématisé à la fig. 43 : la mesure de l'absorption du rayonnement donne la valeur de la quantité de matière interceptée par le faisceau radioactif le long d'un diamètre de l'échantillon. Connaissant le diamètre mesuré optiquement, on calcule aisément la valeur moyenne de la densité dans la partie du matériau traversée par le faisceau radioactif.

Dans la première série d'essais exécutés (Wack, 1965), tous les échantillons ont été fabriqués à partir du même sable et déformés sous une pression latérale de 1 kg/cm.ca. Seule la densité initiale est différente d'un échantillon à un autre. Les résultats sont représentés à la fig. 44 où l'on a porté la variation de densité (fig. 44a) et la variation de contrainte axiale de superposition (fig. 44b) en fonction du pourcentage de déformation axiale de l'échantillon. La valeur de la densité est celle de la valeur moyenne le long du diamètre de la section la plus grande de l'échantillon.

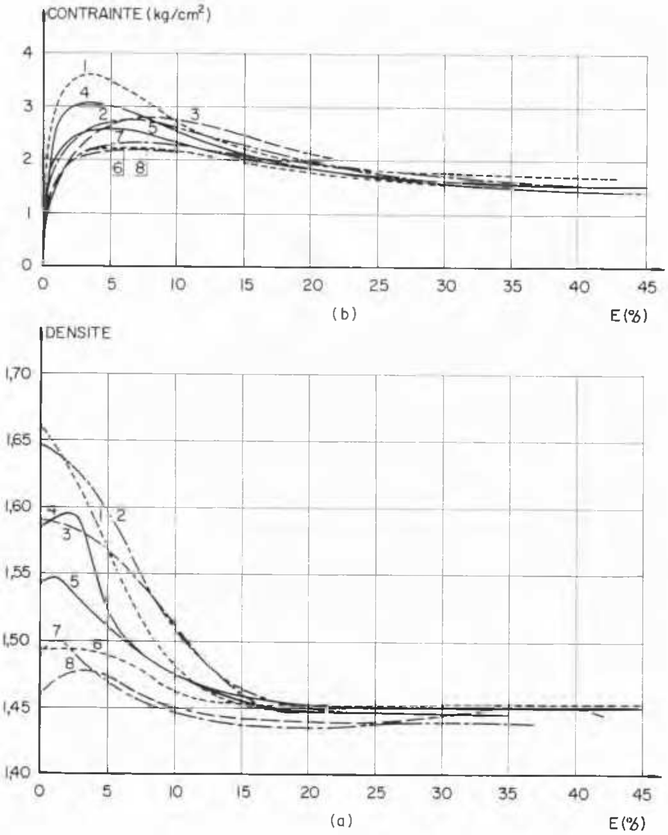


FIG. 44. Variation (a) de densité et (b) de contrainte axiale en fonction du pourcentage de déformation axiale de l'échantillon.

Ces résultats confirment l'existence d'une densité limite qui apparaît après une déformation de 20 pour cent et que l'on peut lier à la valeur de l'état de contrainte limite. On constate que la densité limite est indépendante de la densité initiale de l'échantillon : elle est définie dans nos essais avec une précision relative de ±0.5 pour-cent, alors que la précision relative sur la mesure de la densité est de ±0.4 pour-cent.

Les résultats d'une deuxième série d'essais ont été dépouillés et seront publiés dans les *Comptes rendus* des Séances de l'Académie des sciences de Paris (sept.–oct., 1965) : l'influence de la valeur de la pression latérale sur la valeur de la densité limite y est étudiée (Wack, 1965).

Les essais dont nous venons de donner les résultats ici représentent le début d'un programme de recherches entrepris par le Centre de recherches et d'essais de Chatou d'Electricité de France sur l'utilisation d'une mesure fine de la densité dans les échantillons de sol soumis aux essais triaxiaux. Les premiers résultats obtenus mettent en évidence les bonnes performances de la méthode de mesure et laissent prévoir des informations supplémentaires très intéressantes sur l'essai triaxial.

RÉFÉRENCE

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CHAIRMAN COOLING

If there is no objection, may I ask the General Reporter to summarize briefly the ideas discussed in this session.

(*Dr. Moretto's oral closure to Session 2 is presented on pp. 378–9.*)

CHAIRMAN COOLING

Ladies and Gentlemen, I am sure you will wish me to thank the General Reporter for all the work he has put in not only while he was preparing his report, but also for his oral summary and remarks during this session. I am also sure you will wish me to thank the panel members for their contributions to this discussion, and finally I would like to thank all the oral discussers. With that we will close the session.

(*The remarks of the General Reporter for Session 2 presented to the Closing Session appear on pp. 592–3.*)

WRITTEN CONTRIBUTIONS CONTRIBUTIONS ÉCRITES

K. AKAI (Japan)

In accordance with the suggestion of the General Reporter I would like to supplement our paper *(2/2)*.

The proportion of the components of vertical strain during the one-dimensional consolidation can be determined by combining Figs. 4, 5, and 6. Fig. 6A shows that the vertical strain ρ_c due to the mean effective normal stress σ'_m is equal to about 30 per cent of the total vertical strain ρ_c. Fig. 6B is drawn from Fig. 6A by transforming the abscissa into $(\sigma_1 - \sigma_3)$ using Figs. 4 and 5. From this figure one can see that the vertical distortion ρ_d due to the dilatancy effect during the one-dimensional consolidation is about 15 per cent of the total vertical distortion $(\rho_\gamma + \rho_d)$, which is equal to about 70 per cent of the total vertical strain ρ. Therefore, the proportions of the components of vertical strain, ρ_c, ρ_γ, and ρ_d become about 30 per cent, 60 per cent and 10 per cent, respectively.

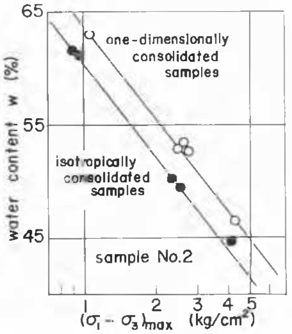

FIG. 45. Correlation between water content and difference between maximum principal stresses.

With regard to the discussion on the test data of the shear strength of one-dimensionally consolidated samples, the two accompanying figures may be helpful. Fig. 45 shows the correlation between the water content w and the maximum principal difference $(\sigma_1 - \sigma_3)_{max}$. This figure shows that the anisotropic stress history during consolidation causes an increase in strength for given water content and that a linear relation between the water content and strength exists on the semi-logarithmic paper. The result that, for a given water content, anisotropic consolidation induces a higher undrained strength than isotropic consolidation, must be related to difference in the length of time that the samples have been subjected to shear stress. The samples with one-dimensional consolidation have been subjected to shear stress for many hours, whereas those with isotropic consolidation have experienced shear only during the few hours of the undrained shear test. It may be seen that some sort of thixotropic hardening developed during the consolidation process.

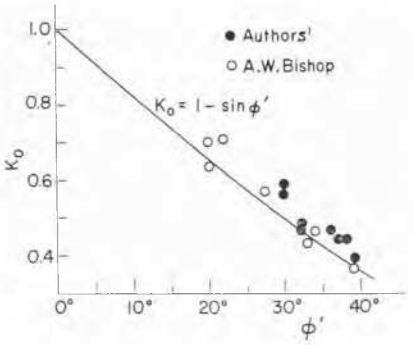

FIG. 46. Correlation between K_0 and ϕ'.

Applying Jáky's equation (1948), Bishop (1958) proposed the following expression for the correlation between the K_0 value and the angle of shearing resistance in terms of effective stress ϕ':

$$K_0 = 1 - \sin\phi' \qquad (1)$$

Both Bishop's and the authors' test results of the correlation between K_0 and ϕ' are plotted in Fig. 46. This figure indicates that Eq (1) is essentially satisfied by these results. Although the physical meaning of the above expression is unknown, it may be useful as an experimental equation for obtaining the coefficient of earth pressure at rest.

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T. K. CHAPLIN (Great Britain)

Dr. Golder commented on the need for information about the reduction in the angle of shearing resistance at higher pressures in granular materials. For a long time it has been realized that ϕ is lower in sands, typically by about 10 degrees, at high normal stresses compared with low stresses. In his paper on this subject *de Beer (2/6)* quotes the results of graphs in which cot ϕ' (ϕ' merely represents the secant value of ϕ) is plotted against $\sqrt{(\sigma_m/E_q)}$, where $E_q = E$ of the grain material, although he appears in fact to take a nominal value of 1 kg/sq.cm for E_q. The value of ϕ for a sand is stated to be the intercept on the cot ϕ axis for $\sigma_m = E_q$, and most unfortunately no results are quoted. It would greatly enhance the value of his paper if at least some of those graphs of ϕ *versus* σ_m could be reproduced elsewhere, together with a description of the particle shape grading and mineral character of the sands concerned.

Since the writer had himself plotted ϕ against the square root of stress in the past, and that presentation appeared to give fair results, it seems instructive to examine the influence of crushing on the variation of ϕ with pressure.

Fig. 47 shows a graph of ϕ against $\sqrt{\sigma_1}$ at failure for the results of Hirschfeld and and Poulos (1963) on a sand and a silt, which gave very different results. It appears that the graphs for both the silt and the sand might conceivably be flattening out as ϕ approaches ϕ_e, that is as the rate of volume change at failure approaches zero. Though both the sand and the silt have virtually identical values of ϕ when the major principal stress is less than about 100 to 200 lb/sq.in., at higher pressures the values for silt fall far more rapidly than those for the sand. It is perhaps of interest that the value of 100 to 200 lb/sq.in. coincides with the order of stress at which a typical sand appears to first undergo minor crushing when tested at zero lateral strain in a fairly dense state.

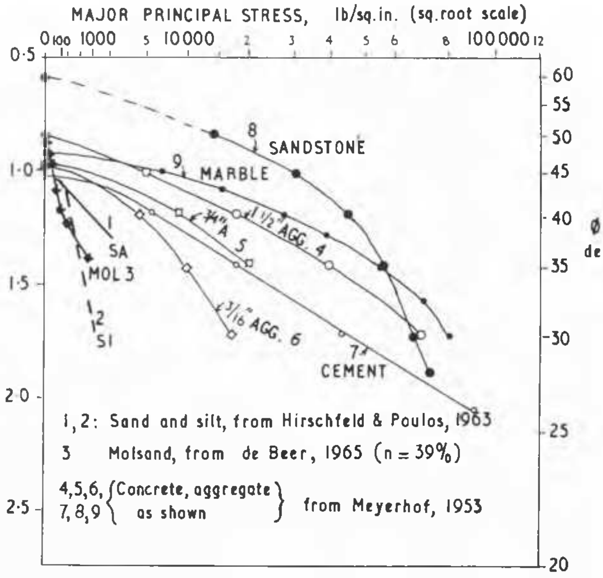

FIG. 47. Influence of pressure on ϕ in crushing region.

Fig. 48 shows the result of plotting cot ϕ (as used by de Beer) against the square root of the major principal stress. Both the sand and the silt graphs appear to become closely linear when the major principal stress exceeds about 2000 lb/sq.in., suggesting that this form of plot might be suitable for general analysis. Clearly the slope of the silt graph is much steeper than that of the sand; the simplest explanation is that, though the sand is undergoing a very small amount of crushing, the particle shape of the silt grains is such that they undergo much more severe crushing. One may suspect that the materials with a large difference between ϕ and ϕ_e at low pressures will drop their values of ϕ more readily at higher pressures, although this might merely be a reflection of the different mineral character.

When values of cot ϕ are plotted to an arithmetic scale the resulting scale of ϕ is almost exactly plotted to a logarithmic scale. This means that one would obtain equivalent results by plotting ϕ on a logarithmic scale. Since for values of ϕ between 40 and 45° a linear scale of ϕ corresponds very closely to a logarithmic scale of the principal stress ratio σ_1/σ_3, one could equally well plot the principal stress ratio

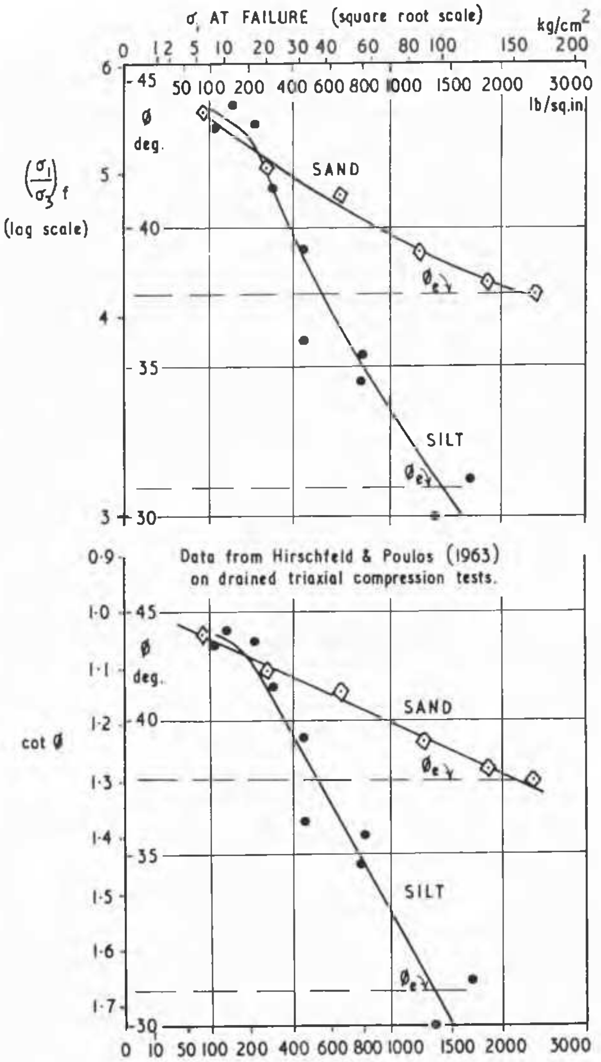

FIG. 48. Influence of high pressure on ϕ near major crushing region.

at failure on a double logarithmic scale if one wished to get a result similar to plotting cot ϕ to an arithmetic scale.

The fundamental significance of plotting ϕ on some scale against the square root of a stress is that over a stress range when the geometry of the particle contacts is reasonably constant, the square root of the pressure is a fair measure of the average contact dimension and the flattening of a typical grain.

To compare the reduction in ϕ for sands and silt with the naturally or artificially cemented materials with relatively small voids, Fig. 48 compares the results from Fig. 47 and an additional test from de Beer's paper (2/6) with those quoted by Meyerhof (1953) for concrete, marble, and sandstone. It is apparent that the materials with a low void ratio are not giving excessive values of ϕ at low pressure (with the exception of the sandstone) but have values of ϕ about the same as the sands. However, the variation of ϕ with pressure is much more rapid in the two sands and the silt than in the cemented materials. Within the cemented materials, the variation of ϕ with pressure is interesting. The concrete with larger aggregate shows much greater resistance to crushing at higher pressures than the concrete made with much smaller aggregate. One may interpret this as an indication that though the cement is holding the concrete at nearly constant volume during shearing, the grain-to-grain contacts in the less broadly graded aggregate are crushing much more severely. One would expect that the concrete made with the smaller maximum size of aggregate had a larger quantity of cement filling the voids, and one would expect that at large stresses approaching failure the void ratios of the coarse and fine aggregates matter very much.

From Fig. 48 the results of the sandstone, marble, and coarsest aggregate concrete suggest that for minerals there is a limiting curve which corresponds to a much more severe crushing, roughly corresponding for quartz to a major principal stress at failure of around 5000 to 8000 lb/sq.in, that is about 2.2 to 3.5 tons/sq.in. This feature suggests that, however small we are able to make the voids of a material, there is very severe crushing when the average stresses become comparable with the crushing strength of the aggregate particles.

Here one may point out the apparent error in Terzaghi's book, *Erdbaumechanik*, regarding the resistance to shearing of particle contacts. Although he stated (Bjerrum, *et al.*, 1960) that the frictional resistance was equal to the particle shearing strength developed over the contact area, in fact, because of the partial mobilization of "bearing capacity" at a contact under higher loads, the actual resistance to shearing could be appreciably lower than the product of particle shearing strength and contact area. At small loads, the interlocking of fine asperities may give a resistance considerably larger than that product.

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K. Drozd (Czechoslovakia)

I would like to tell you about the results of some dynamic penetration tests, carried out on dry, moist, and water-saturated sands to establish the influence of the untrue cohesive strength and the pore pressure.

The influence of variations in moisture content in sand on the number of blows recorded in the dynamic pentration test was investigated partly by field penetration tests and partly in the laboratory by means of model tests. The research has been mainly directed towards clarifying the reasons for differences observed for sand of a certain density in a dry, moist, or water-saturated condition. For the same conditions (the penetrometers and the testing methods being identical, and the tests being carried out at the same depth) penetration depends on the value of the effective stress between the grains and the value of the internal friction. As the angle of the internal friction in sand does not change too much with changes of moisture the differences in the number of blows must be attributed to the differences in the effective stress. The greatest influences on these changes are the acting capillary forces and the changes of pore pressure, which are affected either by a shock with a short vibration effect caused by the drop weight and spread in the sand or by the changes of the porosity in the neighbourhood of the driven apparatus.

MEASUREMENT OF DYNAMIC EFFECTS

Field penetration tests were carried out on the surface of a sand layer on which the dynamic effects were measured. The horizontal and vertical components of displacement were measured at various distances from the place of the penetration test using two types of vibrographs as recording equipment (the two-component Czechoslovak-made mechanico-optical vibrograph, Somet, working in the frequency range 1–50 Hz, and a Cambridge one-component mechanical vibrograph). An accelerometer, Brüel and Kjaer, was also used. The vibrographs recorded only the displacement of the surface and the velocity of this motion; subsequently they oscillated only in their natural frequency. The accelerometer recorded only small vibrations (frequency about 1000 Hz) which were evidently caused by the oscillation of the guide rod for the drop weight and were spread in the sand. The vibrographs employed have not recorded any conspicuous frequency which accompanies a shock. That means that dynamic effects cannot influence the changes of effective stress.

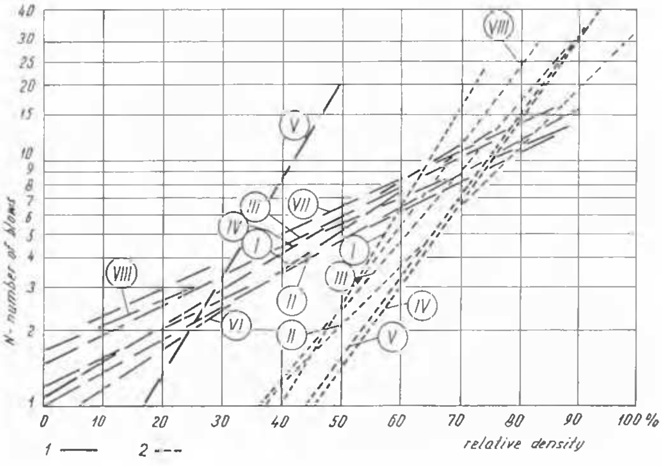


FIG. 49. Relation between the relative density and the number of blows in model penetration tests. (1, dry samples; 2, water-saturated samples.) (The results can be used for standard penetration field tests carried out on the open surface of a sand layer or at the bottom of open trenches by dividing the number by 7.7.)

MEASUREMENT OF PORE PRESSURE

Several piezometric tubes were installed in a model vessel filled with saturated sand of a certain density. The changes of water levels in the tubes (when the penetrometer dropped into the sand by its own weight, when it was driven by a blow, and when it was drawn out from the sand) were recorded by a film camera. In the saturated loose sand an impressive rise of the water level in the piezometric tubes was recorded, whereas in the saturated compacted sand a fall of the water level was recorded. Minimal changes were recorded in the sample at a relative density of about 45 per cent. Nearly the same changes in water levels were observed when the model penetrometer dropped or was drawn out as occurred when no dynamic effects could arise.

MODEL PENETRATION TESTS ON DRY AND SATURATED SANDS

Eight samples of sands were used for model tests. (The diameter of particles was 0.1–5.0 mm; there were well graded or uniformly graded samples, only sample V having 30 per cent of the particles smaller than 0.1 mm and 10 per cent of the particles smaller than 0.005 mm). All results are shown in Fig. 49.

The results were similar regardless of the grain size distribution, with the exception of sample V—sand in dry condition. In saturated loose sand compaction of the sand took place; therefore the pore pressure at the moment of penetration increased and the effective stress and the number of blows decreased. In the saturated compacted sand loosening of the sand took place, and therefore the pore pressure decreased and the effective stress and the number of blows increased .

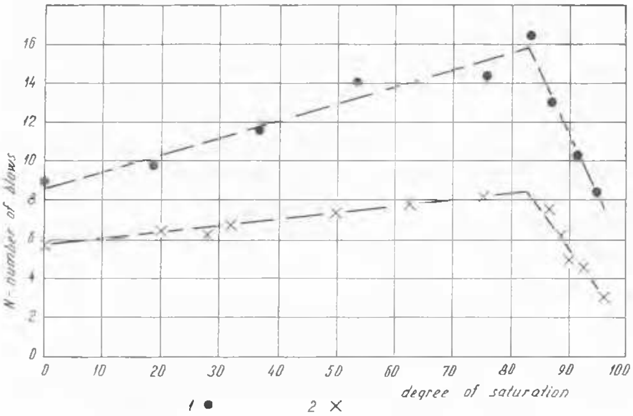


FIG. 50. Relation between the number of blows in the model dynamic penetration test and the degree of saturation (sample II). (1, tests at a relative density of sand of about 75 per cent; 2, tests at a relative density of about 57 per cent.)

Equal numbers of blows in dry and water-saturated sand were observed at relative densities of 65–85 per cent (Fig. 49). This phenomenon took place for sand II at a relative density of 85 per cent, but minimal changes of pore pressure in water-saturated sand II occurred at a relative density of about 45 per cent. To understand this it is necessary to consider the uplift. The equal number of blows evidently occurred when the negative pore pressure eliminated the decrease of the weight of sand particles owing to the uplift.

The increase or decrease in the number of blows could be also caused by the overpressure or underpressure of air in the voids as it was observed for the dry sample V. Owing to the larger quantity of small grains, this sample was less "permeable" to air than other samples and therefore behaved like a water-saturated sample.

MODEL PENETRATION TESTS IN SAND OF VARIABLE MOISTURE CONTENT

These model tests were carried out on sample II at two different densities and at various degrees of saturation. The arrangement of homogeneous samples in the vessel was very difficult and the practice and description of the model tests cannot be described in this short report. It was observed that the number of blows increased linearly, owing to the rise of capillary forces, up to a certain value (about 83 per cent) of the degree of saturation; then the hydrodynamic phenomena started to act (Fig. 50). The number of blows in the loose and medium density sand decreased for degrees of saturation higher than 83 per cent, but further increase was observed in the compacted saturated sand.

REFERENCE

Drozd, K. (1964). The influence of variable moisture content in sand on the results of dynamic penetration test. *Sborník geologických věd*, HIG, No. 1.

M. J. Hvorslev (U.S.A.)

The paper by *K. V. Helenelund (2/22)* contains a very interesting description of *in-situ* determination of the shear strength characteristics of soils by means of annular plates which are rotated under various normal loads. Equipment of the same type but with different details has been developed by several organizations and has been used in mobility investigations by the U.S. Army Engineer Waterways Experiment Station. A diagrammatic sketch of this apparatus,

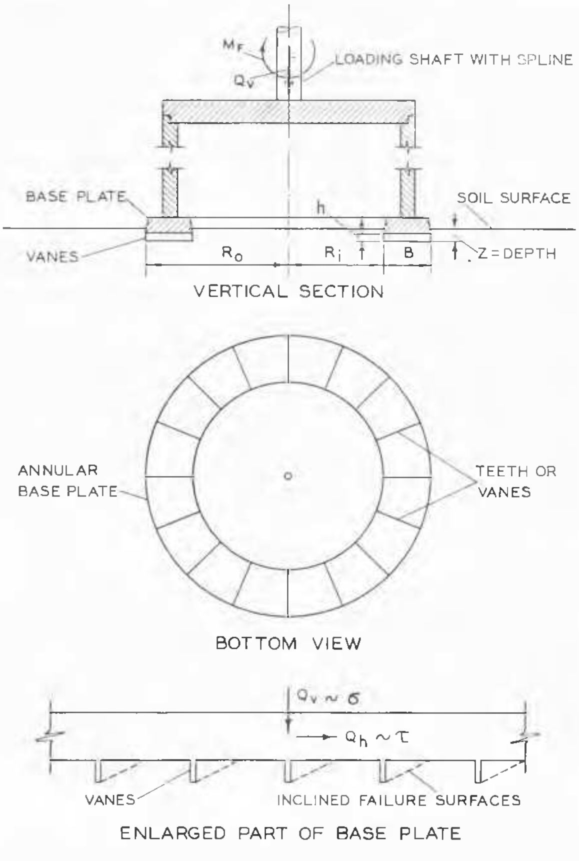
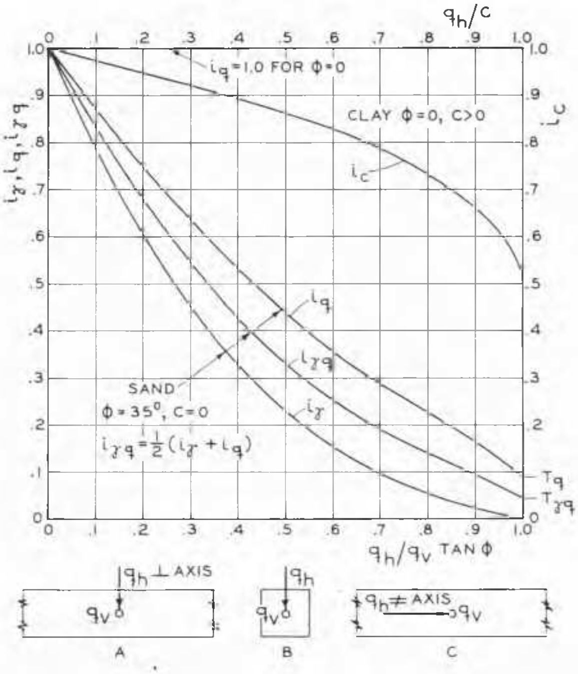


FIG. 51. Annular plate shear test.

originally developed by the Land Locomotion Laboratory of the U.S. Army Tank and Automotive Center, is shown in Fig. 51. The results obtained were at times inconclusive and difficult to evaluate. Some sources of error have been eliminated in more recently developed equipment, but two basic problems remain unsolved.

The first problem is that of transferring shear forces from the plate to firm clays without excessive disturbance of the soil structure. Sliding between plate and soil is usually prevented by thin radial vanes or teeth, but inclined failure surfaces may be formed in tests on firm clays under low normal loads, as indicated by a slight rise of the plate during a test. This phenomenon is also observed in direct shear box tests. Evaluation of the test data on the basis of horizontal instead of inclined failure surfaces yields too low shear strengths and a misleading inclination of the shear strength line. Theoretical considerations indicate that the inclination of the failure surface decreases with increasing normal load. The failure surface becomes horizontal when the normal stress exceeds a value which is a function of the shear strength, the width of the plate, and the height and spacing of the vanes. Helenelund and others have provided circumferential covers for the vanes which prevent radial flow of soil during the test. A relatively large height-spacing ratio of the vanes decreases the danger of formation of inclined failure surfaces, but it also increases initial disturbance of brittle soils.

The second problem is that failure in bearing capacity may occur before the horizontal shear strength is attained. *In-situ* plate shear tests are in fact plate loading tests with inclined loads. Inclination of the load causes a decrease of the bearing capacity, which can be taken into account by multiplying the $N\gamma$, N_q, and N_c terms of the bearing capacity formula with appropriate inclination factors $i\gamma$, i_q, and i_c. The inclination factors shown in Fig. 52 apply to purely cohesive and cohesionless soils and are obtained from the theories and diagrams by Meyerhof, Brinch Hansen, and de Beer. It should be noted that these factors apply only to strip foundations with the horizontal load acting perpendicularly to the axis of the strip.



q_v = VERTICAL. q_h = HORIZONTAL. DIAGRAMS APPLY TO CASE A. VALUES OF THE INCLINATION FACTORS ARE LARGER FOR CASES B AND C.

FIG. 52. Inclination factors for $\phi = 0$ and $\phi = 35°$.

The bearing capacity for normal loads is designated by q_f, whereas q_v is the vertical component and q_h the horizontal component of the bearing capacity under inclined loads. Critical values of q_h/c or q_h/q_f *versus* q_v/q_f are easily obtained from the inclination factors in Fig. 52.

Derivation of equations and results obtained for purely cohesive soils are shown in Fig. 53. The diagram applies to loads at the surface and at shallow depths when the N_q term is negligible. It is seen that $q_h/c = 1.0$ for values of q_v/q_f smaller than 0.5; that is, the full horizontal shear strength can be developed when the normal stress is less than half the bearing capacity for normal loads. The critical value of q_h/c decreases rapidly when q_v/q_f exceeds 0.5 and becomes zero for $q_v = q_f$.

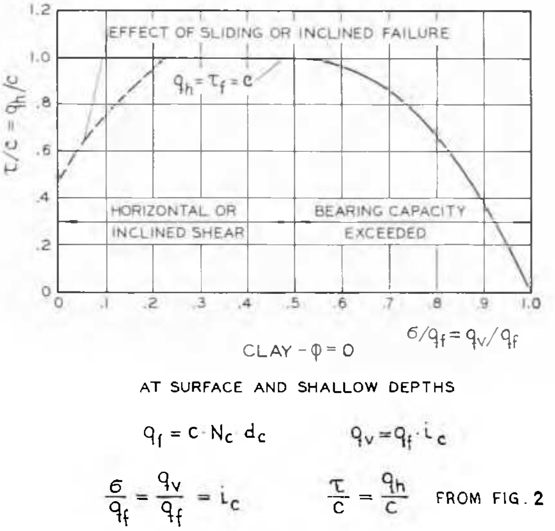


FIG. 53. Critical horizontal stress ratios for clay strip load, $\phi = 0$.

Critical values of q_h/q_f at the surface of sands with $\phi = 35°$ and $c = 0$ are shown by the lower curve in Fig. 54, which is derived from the i_γ factors. The upper solid curve is based on the i_q factors and applies to depth ratios, z/B, greater than 5, in which case the N_γ term is negligible in comparison with the N_q term. The intermediate dashed curve is obtained from the $i_{\gamma q}$ factors in Fig. 52 and corresponds to $z/B = 0.52$ at which the N_γ term is equal to the N_q term. The straight line represents the available horizontal shear strength; and it is seen that, excepting low values of q_v/q_f, only a small part of the available horizontal shear strength can be attained without causing failure in bearing capacity and consequent sinkage. Sinkage increases q_f and the attainable values of q_h, but the total torque is also increased by sidewall friction. The influence of the sidewall friction may be quite large and the test data may become unreliable unless arrangements are made to measure the torque exerted by the base plate alone. The sidewall friction may be eliminated by excavation but this decreases q_f and q_h, and it is difficult to perform the excavation during rapid sinkage of the plate.

It is again emphasized that the diagrams in Figs. 52, 53, and 54 apply only to strip foundations with the horizontal load component acting perpendicularly to the axis of the strip. The horizontal load component in annular plate shear

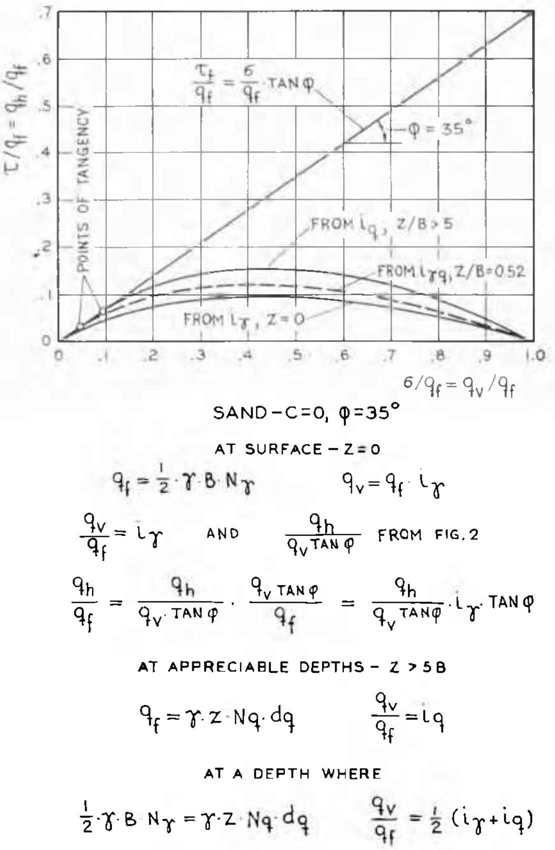


FIG. 54. Critical horizontal stress ratios for sand strip load, $c = 0$.

tests is parallel to the axis of the contact surface; the inclination factors for this case are not known, but they are undoubtedly larger than those shown in Fig. 52, and the critical values of the horizontal load component are greater than those indicated in Figs. 53 and 54. The data presented by Helenelund show good agreement between results of laboratory tests and those of *in-situ* tests with annular plates. The tests were performed on dense glacial till and sandy silt with some cohesion, and it is probable that the bearing capacity was not exceeded appreciably and had no or little influence on the test results. However, during tests on sand at the Waterways Experiment Station, large and sudden sinkages of the annular plate, Fig. 51, often occurred before the peak shear strength was attained. It appears that *in-situ* plate shear tests cannot yield reliable values of the shear strength when the normal load exceeds certain limiting values which are in need of further experimental investigations.

L. KEINONEN (Finland)

In connection with the treatment of triaxial test results no attention is generally paid to the effective normal stress on the failure plane, because the calculation of this stress is not usually necessary for the interpretation of the test results. However, we can show that the effective normal stress on the failure plane has a very strong correlation with the consolidation pressure. This discovery has great significance.

The author has computed the magnitudes of the effective normal stress σ_f' from the results of triaxial tests presented in the literature and compared them to the consolidation pressure p_c. It was found that, with normally consolidated samples,

$$\sigma_f' = \lambda p_c, \tag{1}$$

and with over-consolidated samples,

$$\sigma_f' = a + \lambda p_c, \tag{2}$$

as seen in Fig. 55. The constants λ and a in Eqs (1) and (2) are specific to the soil tested. In the cases where Eq (1) or (2) does not appear to be valid, the test results, even when presented by means of Mohr's circles, do not agree.

By applying Eq (1) and the known relation $c = \mathfrak{S} p_c$, Coulomb-Terzaghi's equation can be written in the form

$$\tau_f = c + (\sigma - u) \tan \phi = \mathfrak{S}\, p_c + \lambda p_c \tan \phi. \tag{3}$$

This kind of correlation between shear strength and consolidation pressure was observed by Hvorslev in Vienna clay, but according to the present author Eq (3) seems to apply to saturated clays in general.

By means of the new strength parameters λ and a it is possible to estimate the effect of consolidation on the shear strength of clay. The effective normal stress on the failure plane can be computed from the triaxial test results with great accuracy by means of the equation:

$$\sigma_f' = \sigma_3' + 0.4(\sigma_1 - \sigma_3). \tag{4}$$

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SIMONS, N. E. (1961). Comprehensive investigations of the shear strength of an undisturbed Drammen Clay. *Proc. Research Conference Shear Strength of Cohesive Soils* (Boulder, Colorado), pp. 727–45.

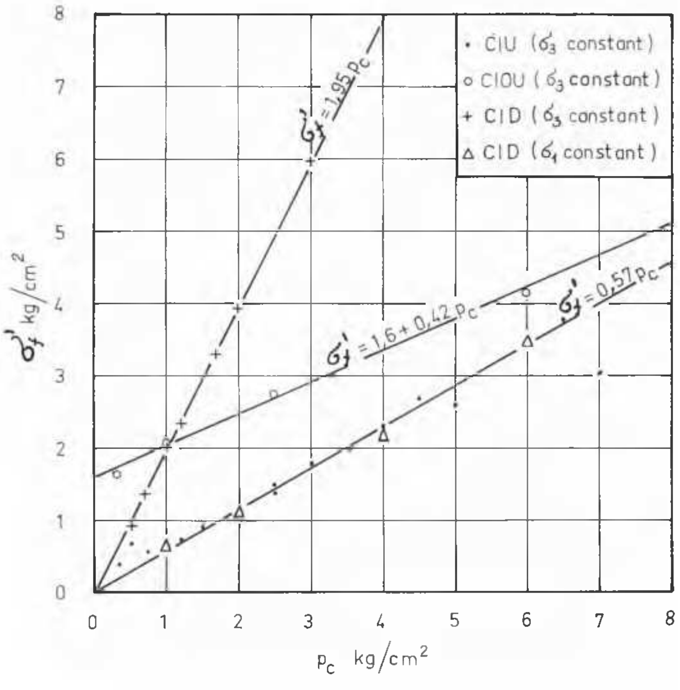

FIG. 55. Effective normal stress on failure plane σ_1' as a function of consolidation pressure p_c. Calculations based on the triaxial tests made by Simons (1961). Symbols: CIU = an isotropically consolidated undrained test; CIOU = an undrained test made with an overconsolidated sample; CID = an isotropically consolidated drained test.

B. LADANYI (Canada)

I would like to make some comments concerning the differences in the stress-strain behaviour of clays in undrained shear under plane strain and axially symmetric conditions respectively.

When the results of consolidated undrained tests performed with a saturated clay are plotted in the ordinary way, that is principal stress difference ($\sigma'_1 - \sigma'_3$) *versus* axial strain, ϵ_1, it is generally found that representative stress-strain curves for specimens subjected to plane strain conditions have a higher peak principal stress difference at a lower axial strain than the corresponding curves for specimens tested with axial symmetry. The results of a comprehensive study of the shear behaviour of the Weald clay carried out by Sowa (1963) and Wade (1963) have led to the same conclusion.

On the other hand, by using the same test results, Wade (1963) found that the octahedral stress paths prior to failure showed reasonable agreement in the two types of test and concluded therefrom that the initial yield of the specimens was controlled by the octahedral stresses. If the finding is accepted to be true, a next logical step would be to investigate whether the corresponding specimens in both types of test will have also similar octahedral strain paths, or, more generally, whether, at a given rate of strain, the stress-strain behaviour of a given saturated clay during shear under combined stresses can be represented, at least approximately, by a common relationship

$$\tau_{oct} = f(\gamma_{oct}), \qquad (1)$$

where τ_{oct} and γ_{oct} are octahedral shear stress and octahedral shear strain respectively.

For the yielding of ductile metals under combined stresses, Nadai (1950) quotes a number of investigations in which Eq (1) was found to be common for tests carried out under various stress conditions. For soils, because of their particulate character and their anisotropy, it may hardly be expected that a common relationship such as Eq (1) will ever be found to be generally valid. It seems, nevertheless, advantageous, at least for saturated clays which appear to be yielding according to the von Mises criterion, to use the relationship of Eq (1), rather than the usual one between the principal stress difference and the axial strain, when comparing the results of tests obtained under different stress conditions. It may, in fact, help in eliminating some of the minor discrepancies that have been observed, as it was suggested by Newmark (1960).

It is interesting to note that, if a common relationship (Eq 1) is assumed for tests carried out under plane strain and axially symmetric conditions respectively, and if Poisson's ratio is taken equal to one-half, it is found that, in a standard plot, at any given strain, the stress difference in a plane strain test would be about 15 per cent higher, and the axial strain about 15 per cent lower than in a corresponding test with axial symmetry. This may account for at least part of the difference observed in the two types of test (Ladanyi, 1965).

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SOWA, V. A. (1963). A comparison of the effects of isotropic and anisotropic consolidation on the shear behaviour of a clay. Ph.D. thesis, University of London.

WADE, N. H. (1963). Plane strain failure characteristics of a saturated clay. Ph.D. thesis, University of London.

B. LADANYI (Canada)

The interesting paper by Bishop, *et al. (2/7)* shows that if a virgin one-dimensional consolidation test is performed on sand, up to relatively high pressures, the measured value of the coefficient of earth pressure at rest, K_0, has a tendency to increase with increasing consolidation pressure. The same phenomenon has been observed by Hendron (1963) in his tests on several sands performed in a special type of oedometer. Hendron concludes that "the phenomenon is probably due to the angle of internal friction decreasing with pressure and is also a consequence of crushing of the grains." These investigations suggest that a definite relationship may exist between the slope of the failure envelope and the corresponding value of K_0 at a given effective mean normal pressure.

For the region of pressures where the failure envelope is a straight line, all investigations to date seem to agree that the observed value of K_0 can be estimated with a very good approximation by the empirical relation:

$$K_0 = (1 - \sin \Phi'), \qquad (1)$$

where Φ' is the angle of shearing resistance in terms of effective stresses. On the other hand, when for sand a curved failure envelope is found at higher pressures, as is the case in Fig. 4 of paper 2/7, the questions arise of how the coefficient K_0 should be defined in that case and whether Eq (1) can still be applied.

Since, in a one-dimensional consolidation test, the behaviour of sand is continually changing with increasing pressure, due to the increase in density as well as the rearrangement and crushing of the grains, it seems logical to assume that, at a given mean normal pressure, the slope of the (σ'_1 *versus* σ'_3) line observed in such a test will be related to the slope of the failure envelope by a relation similar to Eq (1):

$$\Delta\sigma'_3 \,/\, \Delta\sigma'_1 = K_{0.t} = (1 - \sin \Phi'_t), \qquad (2)$$

where $K_{0.t}$ is a tangent value of the coefficient of earth pressure at rest, and Φ'_t is the slope angle of the tangent to the failure envelope at the considered mean normal pressure. Alternately, for any effective mean normal pressure, a secant value of K_0 could be defined as

$$\sigma'_3 \,/\, \sigma'_1 = K_{0,s}. \qquad (3)$$

If the whole (σ'_1 *versus* σ'_3) curve has been determined in a virgin one-dimensional consolidation test, the relationship between the above two values of K_0 can easily be established. It is evident that both definitions will give the same value for K_0 only if the (σ'_1 *versus* σ'_3) plot is a straight line, but will furnish different values for the coefficient if the plot is a curve.

While in Fig. 5 of paper 2/7, the first-mentioned definition of K_0 is used, it seems that the second definition has been used for calculating the numerical values of K_0 observed for Ham River sand in test No. 11 of Fig. 8.

It would be of interest for further research in this field, if a single definition of K_0 could be adopted also for the case when a curved (σ'_1 *versus* σ'_3) line is obtained in a one-dimensional consolidation test.

REFERENCE

HENDRON, A. J. (1963). The behavior of sand in one-dimensional compression. Ph.D. thesis, Department of Civil Engineering, University of Illinois.

H. LORENZ (Germany)

Due to restrictions on the length of papers in the *Proceedings*, paper *2/34* had to be confined essentially to a description of the tests. The accompanying explanations, reasonings, and comparisons were so scanty that we wish to clarify them with the following.

First, it must be stressed that the tests did not aim at the shear strength, but at the deformation behaviour of sand before rupture. It is a well-known fact that many problems of soil mechanics, especially of bearing capacity, cannot be solved, if only the shear strength parameters are known. The constitutive law needed for the solution of deformation problems cannot be drawn from standard tests. Therefore the new device was built first of all for the plane strains that frequently occur in practice. In the meantime we developed integration methods for the calculation of plane deformations, based on the test results. The case of a continuous footing is included.

SOME REMARKS ON THE GENERAL REPORT

In the triaxial cell the sample is in a state of axial symmetric deformation, a state which occurs rarely in practice. We do not know of any triaxial cell suitable for homogeneous, plane parallel deformations. (We would be grateful to the General Reporter for elaboration of the statement, ". . . using a prismatic specimen encased in a triaxial cell, as has been done elsewhere," that appears in his General Report on page 224.) Concerning the deformation characteristics, we therefore can renounce comparative tests. Still the comparison of shear strengths is of interest and will be reiterated.

The angle ϕ_p in relation to the maximum of ϵ_v is by no means identical with the angle of internal friction ϕ, but only a parameter of deformation. Thus the independence of ϕ_p does not contradict the dependence of ϕ. Only the modulus E_p depends considerably on the porosity according to Fig. 4; however, the derivation $\partial E_p \,/\, \partial(\sigma_1 - \sigma_3)$ does not. This was not made sufficiently clear in the text. In comparing ϵ_v with triaxial results we committed an error; indeed there is no essential difference in the deviatoric region. We are grateful to the General Reporter for drawing attention to this.

D. K. MAJUMDAR and G. G. LAFRANCE (U.S.A.)

Hanrahan and Walsh (2/19) focus attention on one of the most important elastic properties of materials, Poisson's ratio (μ). They have calculated values of μ from the relationship between the shear modulus and the modulus of elasticity: $G = E/2(1 + \mu)$. The values of shear modulus G were determined from the stress-strain curves of vane tests, while the modulus of elasticity has been computed from the stress-strain curves of the triaxial tests.

An analysis of the authors' Fig. 2, which relates vane shear stress to vane sheer strain, indicates that the elastic behaviour of peat soil is limited to the range of shear stress values where the shear stress does not exceed approximately 50 per cent of the ultimate shear strength. Beyond this range the stress-strain curve is clearly non-linear and includes the effects of plastic deformation. Therefore values of G computed using the non-linear portions of the curve are not believed to be within the limits of the elastic behaviour of the soil.

Similarly, an inspection of the authors' Fig. 3, which relates the deviator stress, $\sigma_1 - \sigma_3$, to vertical compression, shows that some initial yielding of the soil occurs at a vertical compression of approximately 0.2 in. It is believed that the modulus of elasticity E should be computed from the portion of the curve where the vertical compression does not exceed 0.2 in. In this range the stress-strain relationship is linear and the effects of plastic deformation can be considered minimal. The validity of the values $\mu = 1.83$ and $\mu = 6.10$, shown in the authors' Table I, appears to be questionable since they are based upon the non-elastic behaviour of the soil.

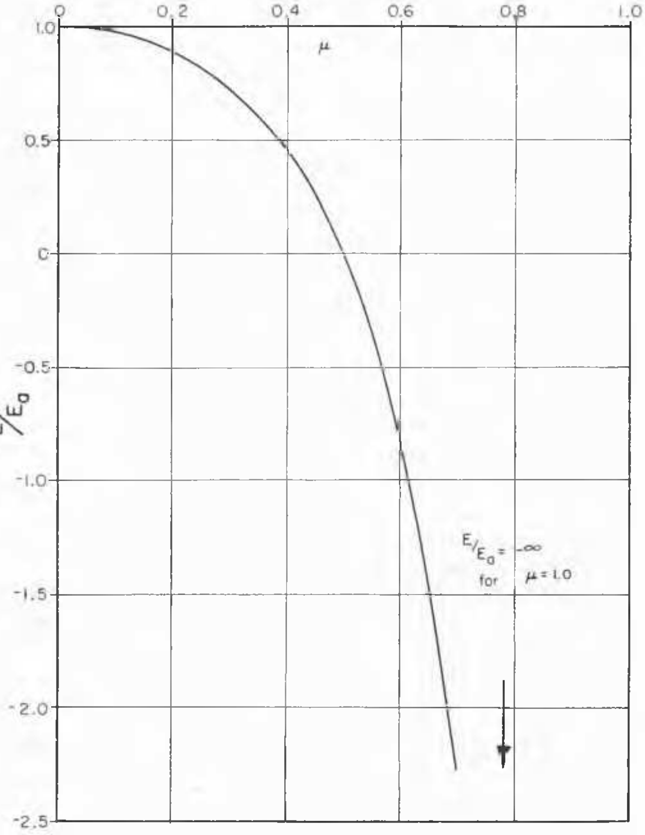


FIG. 56. Relationship between ratio of true modulus of elasticity to apparent modulus of elasticity and Poisson's ratio.

In their presentation of the confined loading case, the authors' derivation of Eqs 5 and 6 is not what one expects from the basic equations of elasticity. As a result, the values of e_{max} computed from Eq 6 and shown in Table II are believed to be erroneous. In fact, for $\mu = 0.5$, which represents a case of incompressibility in a confined load test, e_{max} cannot be 0.14, but must tend to zero, and also by assumption, $e_{min} = 0$.

An analysis of a confined loading case in which only one-dimensional compression can occur, such as exists in a standard consolidation test, reveals the following characteristics: a_v = coefficient of compressibility derived from a linear plot of void ratio *versus* pressure; μ = Poisson's ratio; E = modulus of elasticity; K = ratio of minor to major principal stress; e_0 = initial void ratio of the soil; Δe = change in void ratio due to the applied load; e_1 = unit vertical strain = $\Delta e/(1 + e_0)$; $\Delta\sigma_1$ = change in vertical stress; E_a = apparent modulus of elasticity = $(1 + e_0)/a_v$. From the basic equations of elasticity,

$$e_1 = 1/E[\sigma_1 - \mu(\sigma_2 + \sigma_3)] \qquad \text{(A)}$$

$$e_3 = 1/E[\sigma_3 - \mu(\sigma_1 + \sigma_2)]. \qquad \text{(B)}$$

For one-dimensional compression in a confined loading test, $\sigma_2 = \sigma_3$ and $e_2 = e_3 = 0$. Then,

$$e_3 = 1/E[\sigma_3 - \mu(\sigma_1 + \sigma_3) = 0,$$

or

$$\sigma_3 = \mu(\sigma_1 + \sigma_3) \qquad \text{(C)}$$

and

$$K = \sigma_3/\sigma_1 = \mu/(1 - \mu).$$

Substituting in Eq (A),

$$\begin{aligned} e_1 &= 1/E[\sigma_1 - 2\mu k\sigma_1] \\ &= \sigma_1/E[1 - 2\mu^2/(1 - \mu)]. \qquad \text{(D)} \end{aligned}$$

For one-dimensional compression, the measured unit vertical strain is equal to

$$e_1 = \Delta c/(1 + e_0) = a_v\Delta\sigma_1/(1 + e_0) = \Delta\sigma_1/(1 + e_0/a_v)$$

For $\Delta\sigma_1$ from 0 to σ_1

$$e_1 = \sigma_1/(1 + e_0/a_v). \qquad \text{(E)}$$

If $1 + e_0/a_v$ is defined as the apparent modulus of elasticity E_a measured from the test results, Eq (E) becomes

$$e_1 = \sigma_1/E_a. \qquad \text{(F)}$$

Substituting in Eq (D), the relationship between the apparent modulus of elasticity E_a and the true modulus of elasticity E is as follows:

$$E = E_a[1 - 2\mu^2/(1 - \mu)]$$

or

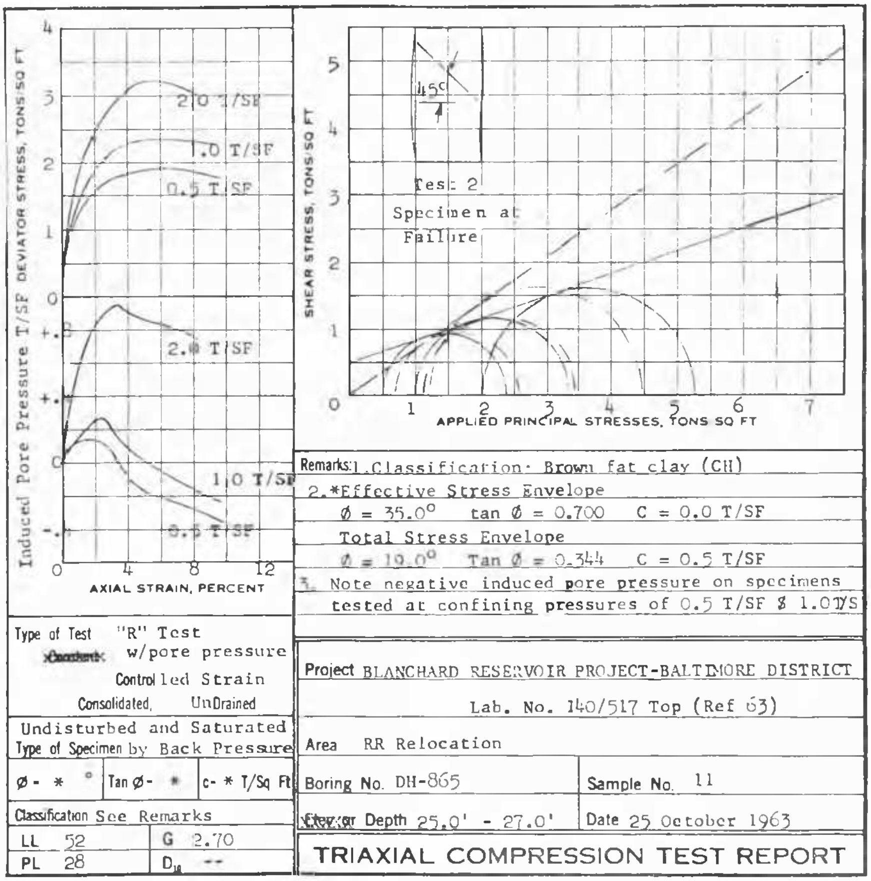
$$E/E_a = 1 - 2\mu^2/(1 - \mu) = f(\mu). \qquad \text{(G)}$$

The ratio of the true modulus of elasticity to the apparent modulus of elasticity is presented in Fig. 56, showing its variation with different values of μ.

D. K. MAJUMDAR, G. G. LAFRANCE, and C. A. PICKERING (U.S.A.)

Denissov (*2/15*) has made a noteworthy contribution by his discussion of the mechanics of underconsolidated clay soils. While the author's presentation is primarily concerned with highly sensitive clays having natural water contents greater than their liquid limits, his analysis of the problem provides a general insight into the different aspects of the pore pressure and strength of all cohesive materials.

The author's Eq 7, $u = \alpha\beta(\sigma_c - \sigma_{01})$, can be observed to indicate the strength increase of a soil-cement mixture. As the cementation bond of the soil-cement increases with time, the values of β and pore pressure are correspondingly reduced, thereby resulting in an increase of strength.



Remarks: 1. Classification: Brown fat clay (CH)

2. *Effective Stress Envelope

Ø = 35.0° tan Ø = 0.700 C = 0.0 T/SF

Total Stress Envelope

Ø = 19.0° Tan Ø = 0.344 C = 0.5 T/SF

3. Note negative induced pore pressure on specimens tested at confining pressures of 0.5 T/SF $ 1.0T/S

Type of Test "R" Test ~~Constant~~ w/pore pressure Controlled Strain Consolidated, UnDrained

Undisturbed and Saturated Type of Specimen by Back Pressure

Ø - * °	Tan Ø - *	c- * T/Sq Ft

Classification See Remarks

LL 52	G 2.70
PL 28	D_{10} --

Project BLANCHARD RESERVOIR PROJECT-BALTIMORE DISTRICT

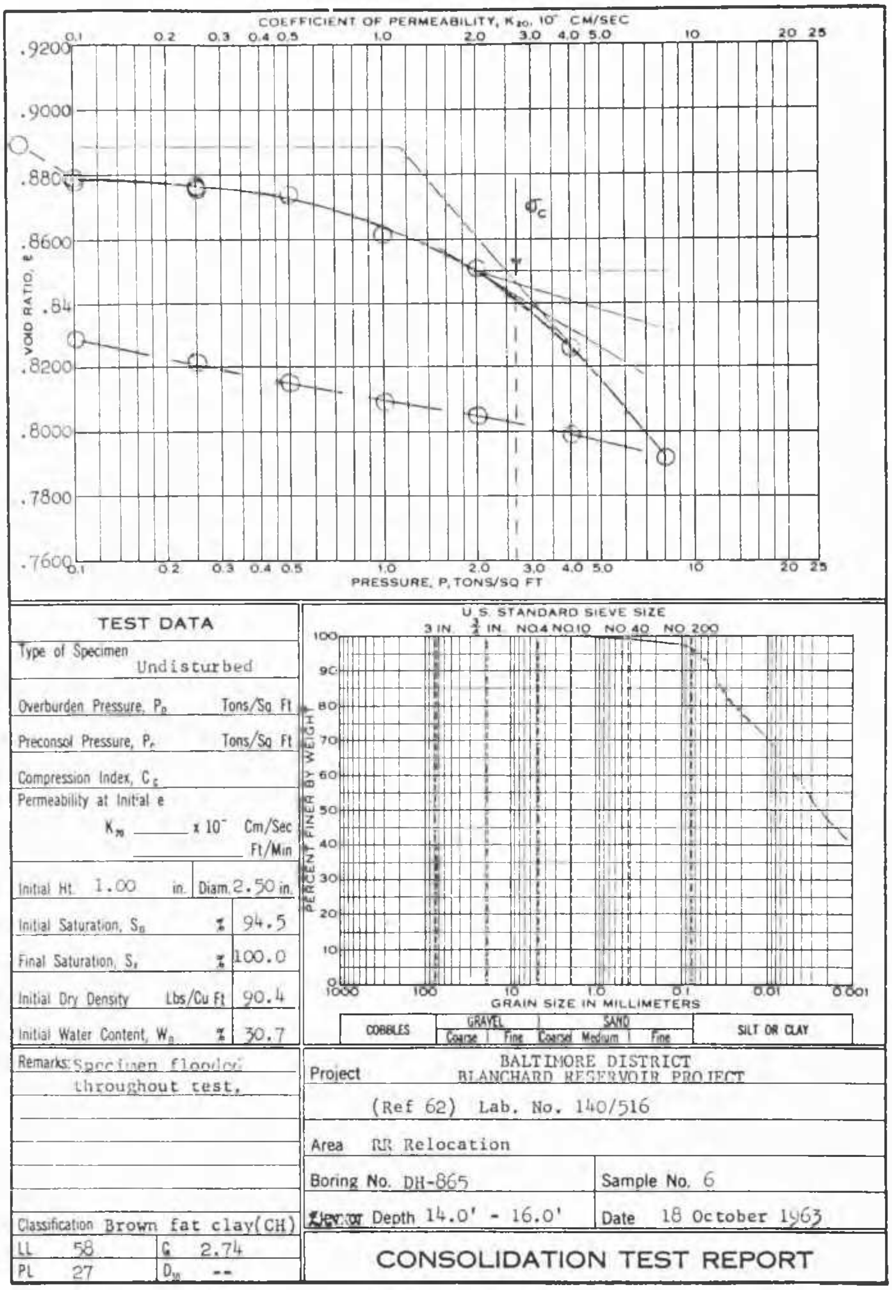
Lab. No. 140/517 Top (Ref 63)

Area RR Relocation

Boring No. DH-865	Sample No. 11
~~Elev. or~~ Depth 25.0' - 27.0'	Date 25 October 1963

TRIAXIAL COMPRESSION TEST REPORT

FIG. 57. Results of three consolidated-undrained triaxial shear tests on an undisturbed residual clay of high plasticity.



TEST DATA

Type of Specimen Undisturbed

Overburden Pressure, P_o Tons/Sq Ft

Preconsol Pressure, P_c Tons/Sq Ft

Compression Index, C_c

Permeability at Initial e

K_{20} ______ x 10^{-} Cm/Sec

______ Ft/Min

Initial Ht. 1.00 in. Diam. 2.50 in.

Initial Saturation, S_o % 94.5

Final Saturation, S_f % 100.0

Initial Dry Density Lbs/Cu Ft 90.4

Initial Water Content, W_o % 30.7

Remarks: Specimen flooded throughout test.

Classification Brown fat clay(CH)

LL 58 G_s 2.74

PL 27 D_{10} --

Project BALTIMORE DISTRICT
BLANCHARD RESERVOIR PROJECT

(Ref 62) Lab. No. 140/516

Area RR Relocation

Boring No. DH-865 Sample No. 6

~~Elev. or~~ Depth 14.0' - 16.0' Date 18 October 1963

CONSOLIDATION TEST REPORT

FIG. 58. Consolidation test results for the same soil for which triaxial test results are shown on Fig. 57.

Denissov has altered Skempton's formula for computing the pore pressure parameter, A_f, to include the effect of the pressure corresponding to the density of the soil under normal consolidation. His Eq 9, $A_f = (\sigma_1 - \sigma_0)/(\sigma_1 - \sigma_3)$, adequately explains the effect of pore pressure on the strength of the soil under different consolidation pressures. When the soil is underconsolidated, $\sigma_0 < \sigma_1$, A_f may be any value within the interval 0 to 1. When it is normally consolidated, $\sigma_0 = \sigma_1$ and $A_f = 0$ (the state of critical density). When the soil is overconsolidated, $\sigma_0 > \sigma_1$ and $A_f < 0$.

This concept implies the development of negative pore pressures in overconsolidated clays during an undrained shear test for the range of σ_1 values less than σ_0. Shown in Fig. 57 are the results of three consolidated-undrained triaxial shear tests on an undisturbed residual clay of high plasticity. The consolidation characteristics of the same soil are presented in Fig. 58. It can be noted from a comparison of Figs. 57 and 58 that negative pore pressures were developed when σ_1 did not exceed the preconsolidation pressure of the soil, σ_c, and positive pore pressures occurred when $\sigma_1 > \sigma_c$. Since these tests were performed on a clay of low sensitivity for which σ_0 is nearly equal to σ_c, the results substantiate the ideas advanced by Denissov.

The author has given his analysis for a plane stress condition in which $\sigma_2 = \sigma_3$. One may ask whether, for a three-dimensional case where $\sigma_1 > \sigma_2 > \sigma_3$, his Eq 7 can be combined with Henkel's stress invariants or octahedral stresses, that is,

$$\Delta u = \Delta p + A_f\sqrt{[(\Delta\sigma_1 - \Delta\sigma_2)^2 + (\Delta\sigma_2 - \Delta\sigma_3)^2 + (\Delta\sigma_3 - \Delta\sigma_2]}$$

where, $\Delta p = 1/3(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3)$ (Henkel, 1960).

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V. MENCL (Czechoslovakia)

I would like to discuss the problem of shear strength of rocks. With clay the peak strength is generally taken as the basis although other conceptions by Tiedeman (1937) and recently by Coates, Burn, and McRostie (1963) with sensitive clays exist.

With rock two approaches are encountered. Some take the peak value as a basis, in combination with a rather large safety factor. Others very often take yield stress as a basic value. I believe that the latter is based on the reasonable idea that progressive damage of the material occurs in the course of strain-hardening. The accepted safety factor is then about 1.1.

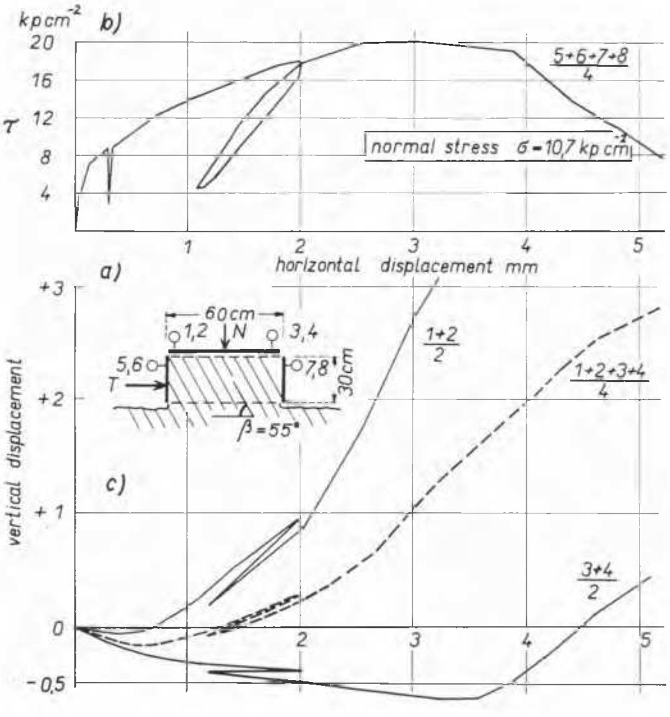

FIG. 59. Direct shear test on *in-situ* rock: (*a*) diagrammatic arrangement of test showing dimensions, loads, and measuring points; (*b*) relationship between shear stress and horizontal displacement; (*c*) relationship between vertical and horizontal displacements during test.

Let me suggest another concept. I believe that the state of strain at the outset of dilatancy corresponds to the beginning of disintegration of the material (in Fig. 59c the lowest point of the strain-vertical deformation curve). It may be argued that it does not matter which limit is taken as a basis, because by choosing suitable safety factors no difference occurs in the final value. This is not true. Mohr's curve at the outset of dilatancy (τ_d in Fig. 60) departs from the general trend of both the peak value curve τ_f, and the yield stress curve τ_y. It means that by accepting τ_y as a basis a large safety margin is reached with small normal stress σ whereas it is rather small with a large σ. I hope that anyone who works with the foundations of large concrete dams will find something of his own thoughts in this approach.

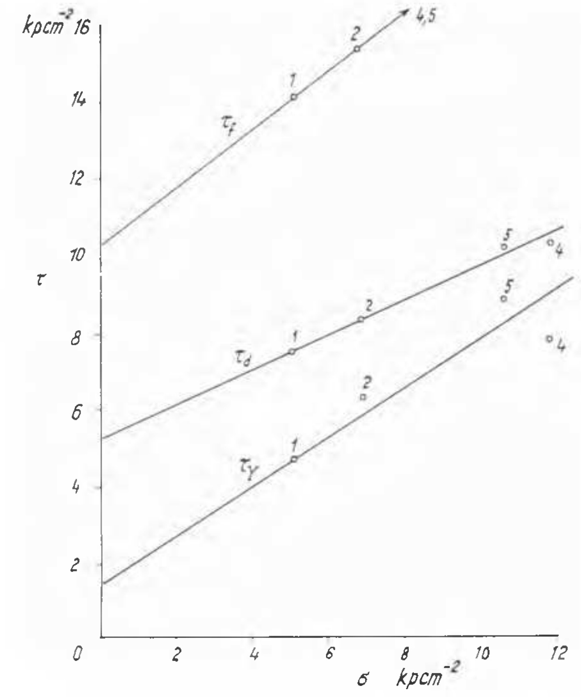

FIG. 60. Relationship between shear stress τ and normal stress σ for direct shear test on *in-situ* rock. τ_f—relationship using peak values as failure criterion; τ_d—relationship using value corresponding to beginning of dilatancy as failure criterion; τ_y—relationship using yield stress as failure criterion.

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COATES, D. F., K. N. BURN, and G. C. MCROSTIE (1963). *Strain-time-strength-relationship in a marine clay.* Research Paper 206, National Research Council of Canada, Division of Building Research, Ottawa.

S. MURAYAMA and N. YAGI (Japan)

Professor *de Beer* in his paper (*2/6*) has made an interesting and valuable contribution on the influence on the shear strength of the mean principal stress initially applied to a sand sample.

The shearing resistance obtained in direct shear or triaxial tests is considered to be caused by the surface friction between sand grains and the resistance against the volumetric change of sand. The former resistance may be assumed to be independent of the stress condition, but the latter one depends on the mean effective principal stress σ_m. Generally, the total specific volumetric change of sand, $\Delta V/V$, caused by external stresses is composed of two parts, the change in the mean effective principal stress $(\Delta V/V)_m$ and the dilatancy associated with shearing deformation $(\Delta V/V)_d$. Near the failure state of sand, variation of the mean principal stress $\Delta\sigma_m$ applied on the sand seems to be very small as compared with variation of the volume. Therefore, in addition to the mean principal stress initially applied, the influence of the mean principal stress on the shearing strength of sand may involve a problem to be investigated in relation to the volumetric change during the test due to dilatancy. If $(\Delta V/V)_m$ and $(\Delta V/V)_d$ are assumed to be independent of each other, the following relations may be obtained:

$$\left.\begin{aligned} \Delta V/V &= (\Delta V/V)_m + (\Delta V/V)_d \\ (\Delta V/V)_m &= \text{function of } \Delta\sigma_m \\ (\Delta V/V)_d &= \text{function of } (\sigma_1 - \sigma_3) \end{aligned}\right\} \qquad (1)$$

where $(\sigma_1 - \sigma_3)$ is the deviatoric stress.

As to the character of the functions expressed in Eq (1), the writers would like to present the following relations obtained in their experiments. In order to find the deformation character of sand, a series of drained triaxial compression tests was performed using samples of clean sand with nearly the same void ratio under three types of loading conditions. The clean sand used in the tests is a typical Japanese testing sand whose effective grain size and coefficient of uniformity are $d_{10} = 0.12$ mm and $d_{60}/d_{10} = 0.21/0.12 = 1.75$, respectively.

TEST I. ISOTROPIC COMPRESSION TESTS

Under the isotropic condition, that is $\sigma_1 = \sigma_2 = \sigma_3$, the compressive stress σ_m was increased in steps from 0.2 kg/sq.cm. to 12 kg/sq.cm. and then decreased gradually to 0.2 kg/sq.cm. Such stress cycles were repeated. At every stress cycle the relation between the specific volumetric change $(\Delta V/V)_m$ and σ_m was measured and is shown in Figs. 61 and 62. In the measurement, the effect of sleeve misfit or sleeve penetration on the volume was considered to be very small. The two curves drawn in Fig. 61 show the relation between $(\Delta V/V)_m$ and σ_m for the initial compression and for the final compression of the steady state.

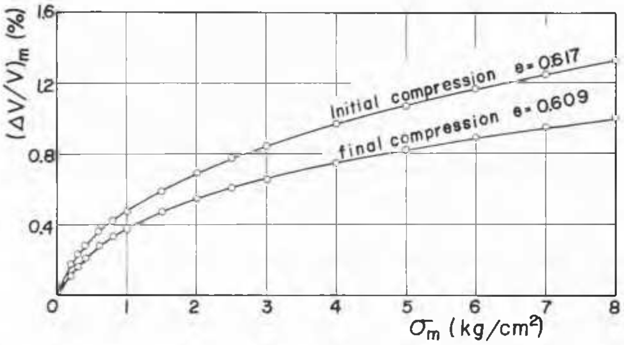

FIG. 61. Relation between volumetric change $(\Delta V/V)_m$ and mean principal stress σ_m obtained in repetitional isotropic compression test.

The relation between log $(\Delta V/V)_m$ and log σ_m is not always linear, as reported by other researchers, but tends to lose the linearity as the sand becomes denser. From these test results, $(\Delta V/V)_m$ may be expressed as follows:

$$(\Delta V/V)_m = C \cdot \Delta\sigma_m, \qquad (2)$$

where C is a factor which depends on σ_m, the initial void ratio, and the kind of sand.

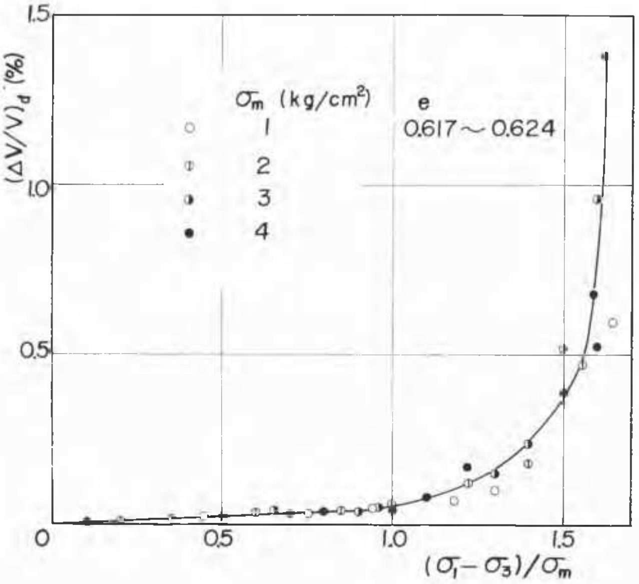

FIG. 62. Relation between volumetric change $(\Delta V/V)_d$ due to dilatancy and stress ratio $(\sigma_1 - \sigma_3)/\sigma_m$ obtained in a triaxial test whose mean principal stress is kept constant.

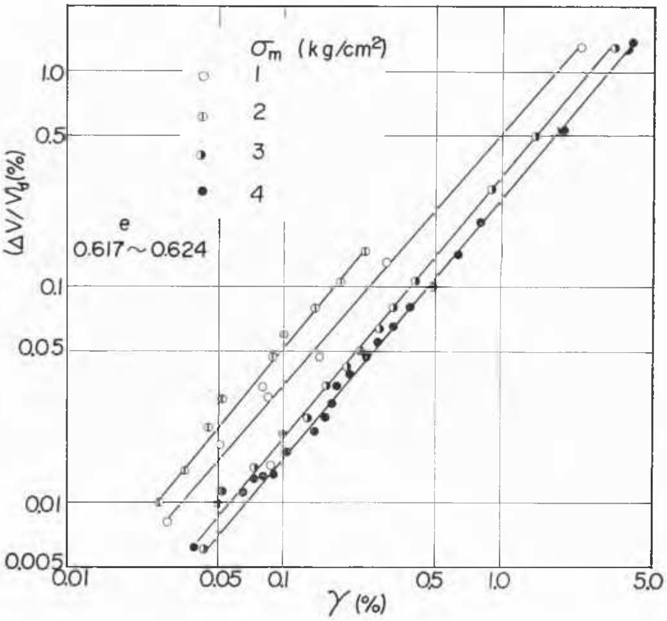

FIG. 63. Relation between volumetric change $(\Delta V/V)_d$ due to dilatancy and shearing strain γ obtained in a triaxial test whose mean principal stress is kept constant.

TEST II. TRIAXIAL TESTS UNDER CONSTANT σ_m

The shearing strain γ, the axial compressive strain (or major principal strain) ϵ_{1d}, and the volumetric increase $(\Delta V/V)_d$ associated with shearing deformation were investigated in a triaxial test, with σ_m remaining constant and $(\sigma_1 - \sigma_3)$ increasing at a constant rate.

When $(\Delta V/V)_d$ is plotted as a function of $(\sigma_1 - \sigma_3)/\sigma_m$, the test results lie on a single curve as shown in Fig. 62. Therefore $(\Delta V/V)_d$ due to dilatancy can be expressed for any values of σ_m by the following relation:

$$(\Delta V/V)_d = D[(\sigma_1 - \sigma_3)/\sigma_m], \qquad (3)$$

where D is a factor depending on the initial void ratio

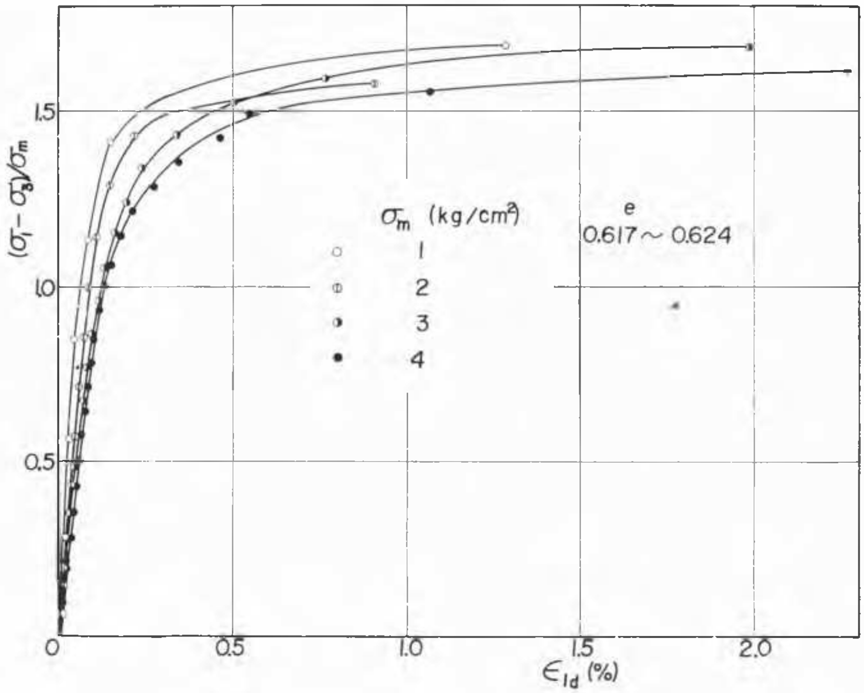

FIG. 64. Relation between major principal strain ϵ_{1d} and stress ratio $(\sigma_1 - \sigma_3)/\sigma_m$ obtained in a triaxial test whose mean principal stress is kept constant.

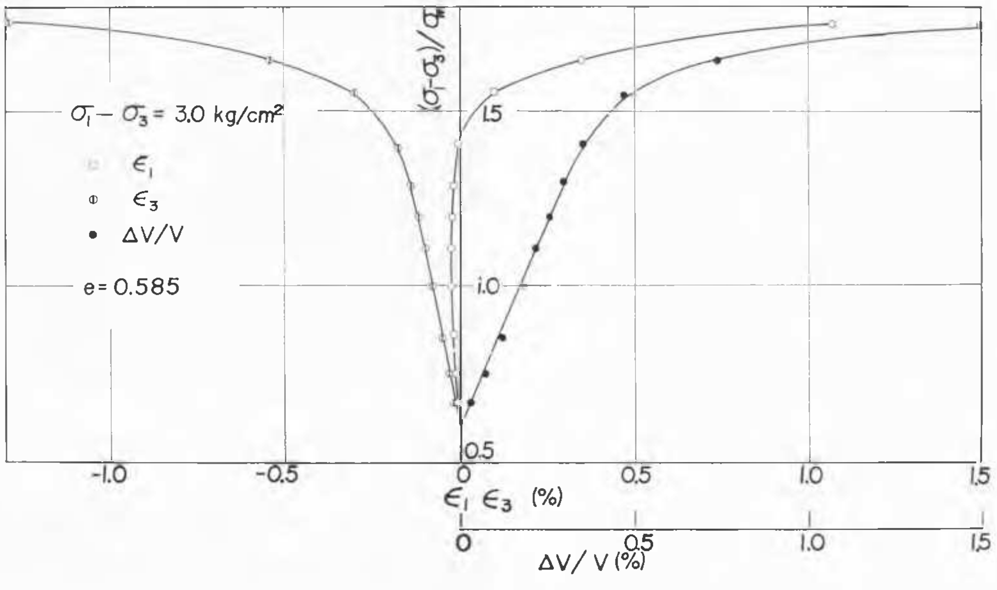

FIG. 65. Relation between major principal compressive strain ϵ_1, minor principal compressive strain ϵ_3, volumetric increment $\Delta V/V$, and stress ratio $(\sigma_1 - \sigma_3)/\sigma_m$ obtained in a triaxial test whose deviatoric stress is kept constant.

and the kind of sand, but not on σ_m. Furthermore, D seems to be a constant within a certain range of $(\sigma_1 - \sigma_3)/\sigma_m [0 < (\sigma_1 - \sigma_3)/\sigma_m \leqslant 1]$.

As shown in Fig. 63, the relation between $\log(\Delta V/V)_d$ and $\log \gamma$ can be represented as a straight line. Since the inclination of the straight line is approximately equal to 45°, the relation between $(\Delta V/V)_d$ and γ may be expressed linearly as

$$\gamma = a(\Delta V/V)_d. \qquad (4)$$

From Eqs (3) and (4), we get

$$\gamma = aD[(\sigma_1 - \sigma_3)/\sigma_m] \equiv A[(\sigma_1 - \sigma_3)/\sigma_m], \qquad (5)$$

where a and $A\,(=aD)$ are factors dependent on the void ratio.

The curves in Fig. 64 show the relations between the axial strain ϵ_{1d} and $(\sigma_1 - \sigma_3)/\sigma_m$ obtained under a constant σ_m. Therefore ϵ_{1d} can be expressed in a similar form to Eq (5) as follows:

$$\epsilon_{1d} = bD[(\sigma_1 - \sigma_3)/\sigma_m] \equiv B[(\sigma_1 - \sigma_3)/\sigma_m] \qquad (6)$$

where b and $B(=bD)$ are factors depending on the void ratio.

TABLE VII. COMPARISON OF COMPUTED AND OBSERVED VALUES OF MAJOR PRINCIPAL COMPRESSIVE STRAIN INCREMENT ($\Delta\epsilon_1$)

σ_m (kg/sq. cm.)	$(\sigma_1 - \sigma_3)$ (kg/sq. cm.)	$\Delta\sigma_m = \sigma_m - 5$ (kg/sq. cm.)	$\Delta\left(\frac{\sigma_1 - \sigma_3}{\sigma_m}\right)$	C (kg/sq.cm.)	B	$\Delta\epsilon_1$ computed	$\Delta\epsilon_1$ observed
4	3	−1	0.15	-0.85×10^{-3}	1.2×10^{-3}	−0.00010	−0.00011
3	3	−2	0.40	-1.0×10^{-3}	1.1×10^{-3}	−0.00022	−0.00025

TEST III. TRIAXIAL TESTS UNDER CONSTANT $(\sigma_1 - \sigma_3)$

The axial compressive strain ϵ_1, the lateral compressive strain ϵ_3, and the specific volumetric increase $\Delta V/V$ were measured in the tests performed under the condition that $(\sigma_1 - \sigma_3)$ remained constant equal to 3 kg/sq.cm. while σ_m decreased in steps from initial value of $\sigma_m = 5$ kg/sq.cm. As shown in Fig. 65, it is noteworthy that as $(\sigma_1 - \sigma_3)/\sigma_m$ increases (or σ_m decreases), the axial strain ϵ_1 first increases in magnitude with negative sign (expansive) and then turns to positive direction. This tendency of ϵ_1 to show expansion at an early stage may be explained in the following manner.

The increment $\Delta\epsilon_1$ of the major principal compressive strain in these tests can be divided into the increment $\Delta\epsilon_{1m}$ of the major principal compressive strain caused by the change $\Delta\sigma_m$ of the mean principal stress and the increment $\Delta\epsilon_{1d}$ due to the deviatoric stress. Hence, $\Delta\epsilon_1$ may be expressed as

$$\Delta\epsilon_1 = \Delta\epsilon_{1m} + \Delta\epsilon_{1d}. \tag{7}$$

As $\Delta\epsilon_{1m}$ can be obtained from Eq (2) and $\Delta\epsilon_{1d}$ from Eq (6), $\Delta\epsilon_1$ can be expressed as follows:

$$\Delta\epsilon_1 = -(C/3)\Delta\sigma_m + B\Delta[(\sigma_1 - \sigma_3)/\sigma_m^2]. \tag{8}$$

Using the stress conditions in these tests and factors C and B determined in Tests I and II, values of $\Delta\epsilon_1$ are computed and listed in Column 7 of Table VII. The values of $\Delta\epsilon_1$ measured in Test III are listed in Column 8, and a general agreement between the two columns is evident. From these considerations, we see that the axial expansive strain developed at earlier stages of the test is caused mainly by the decrement in σ_m. But in the region where $\Delta[(\sigma_1 - \sigma_3)/\sigma_m]$ is large, $\Delta\epsilon_1$ turns to positive direction (compressive strain) because the absolute value of the second term in the right hand side of Eq (8) becomes larger than that of the first term.

To summarize, the principal strain or the specific volumetric change of sand during triaxial tests can be divided into two kinds of strain, that is the strains caused by σ_m and by $(\sigma_1 - \sigma_3)/\sigma_m$. From these considerations the writers are interested to know the effects of the mean principal stress not only initially applied on the sand but also acting on it during the failure state.

Y. NISHIDA (Japan)

In paper *2/3 Bamert, et al.* present an interesting test in soil dynamics. They measured the dynamic stress but not the corresponding strain, and so the elastic modulus in the dynamic state by its definition cannot be discussed. Of course, *E* may be obtained by measuring the frontal velocity through the theory of elasticity, but I would like to ask them how the dynamic strain itself may be measured directly.

It is very interesting to read Paper *2/38* presented by *Mitchell and McConnell.* The test seems to be carried out on the sample with constant water content. I think the recoverable strain of soil is much affected by its water content. The recovering capacity of the strained soil seems to be different for water contents above and below the optimum value.

It is very instructive to find the paper presented by *Olson and Kane (2/41).* I would like to present additional information from a similar test on the strength of soil subject to impact by a falling weight. For confining stresses ($\sigma_3 = 0.5 \sim 4$ kg/sq.cm.) lower than in their report and for times to failure in the range of 5 to 10 milliseconds, the total stress failure envelopes are almost parallel to those in the static loading in each case for several water contents. It may be concluded that the angle of shearing resistance of soil in dynamic loading is almost equal to that in static loading.

C. SCHAERER (Switzerland)

The efficiency of compaction engines is usually controlled by measuring the dry density. This technique requires either the use of a nuclear testing apparatus or the taking of a sample. The nuclear device has been shown to be inappropriate for coarse materials containing boulders of 15 cm or more, since a very large sample must be taken if it is to be representative.

The encouraging results obtained using the technique described in paper *2/3* led my colleague, Mr. E. Bamert, to try a kind of "micro-impact-seismic," that is measurement of the behaviour of the material during compaction when submitted to a dynamic shock.

Initial laboratory tests were carried out in a cylindrical container 2.5 m in diameter using gravel compacted with a vibrating tamper. The impact was produced by a falling weight which strikes a dynamic pressure cell placed on the surface. A second dynamic pressure cell was buried before the test at a depth corresponding to the layer thickness. The pressure-time curves are registered by the two cells and may be interpreted as follows:

If the layer under test undergoes only elastic strain, the pressure maxima remain unchanged after each passage of the

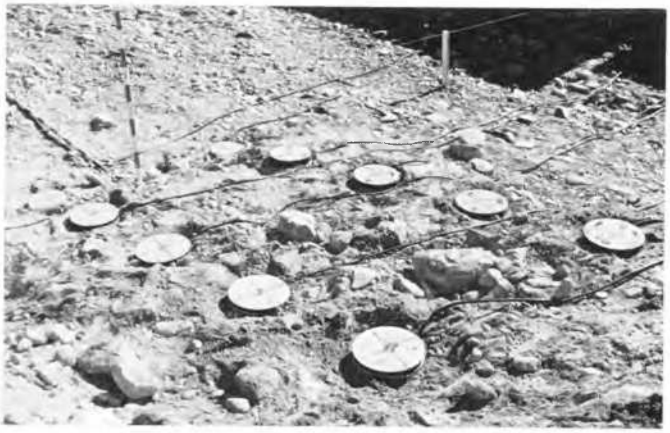

FIG. 66. Falling weight test—pressure cells buried at different depths.

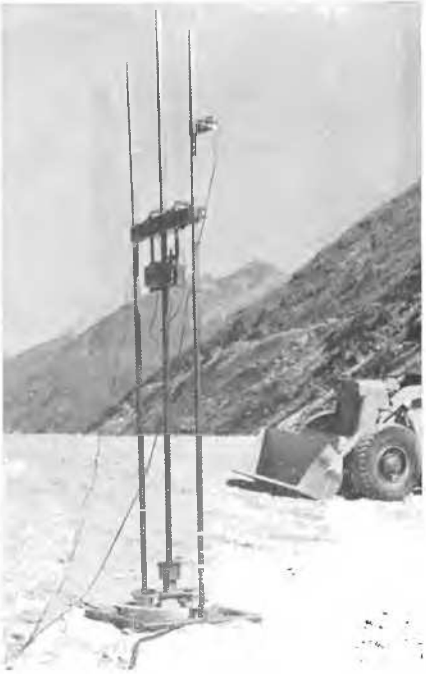

FIG. 67. Falling weight apparatus for compaction control.

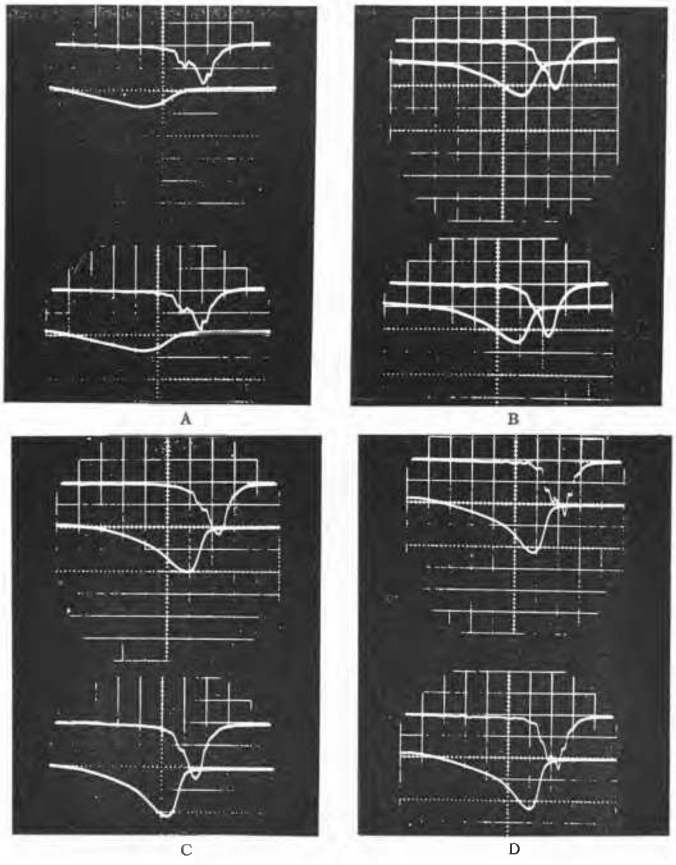

FIG. 68. Falling weight test—pressure-time curves photographed in a cathode-ray oscilloscope. Mattmark Dam—layer thickness, 70 cm. Upper curves registered by the upper cell (scale: □ width 1/500 sec, □ height 1.25 kg/sq.cm.). Lower curves registered by the lower cell (scale: □ width 1/500 sec, □ height 0.10 kg/sq. cm.). A, 0 passes; B, 2 passes; C, 4 passes; D, 6 passes.

compacting engine. If some plastic deformation is caused by the blow, the pressure maximum registered by the buried cell is smaller than that registered by the upper cell since energy is absorbed and the impulse intensity ∫p dt thus reduced.

As long as further passages of the compacting engine cause an increase in density, the pressure maxima registered by the dynamic control devices will alter, even if the deformation is purely elastic.

Two field tests have already been carried out. The first was on the core material of the Mattmark Dam, glacial drift with boulders up to 15 cm, which was compacted by an 80-ton rubber-tired roller. The equipment and results of tests are

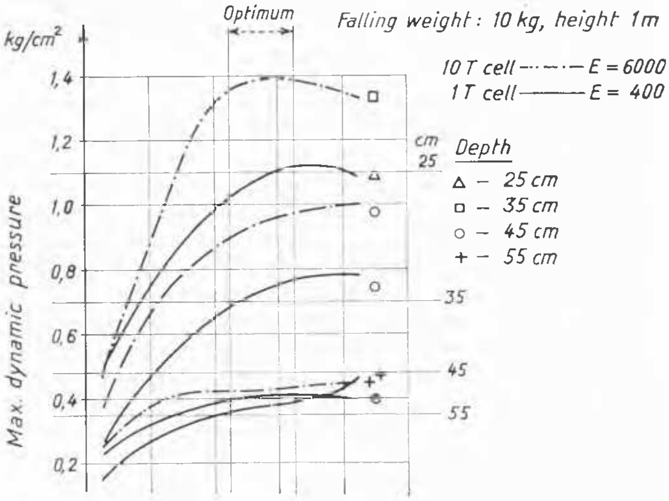

FIG. 69. Determination of the optimum number of passes for the core material of the Mattmark Dam. Compaction with an 80-ton tire roller.

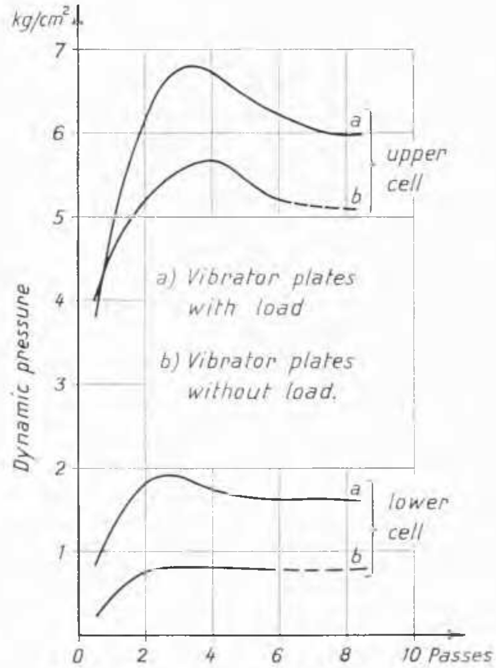

FIG. 70. Determination of the optimum number of passes. Thickness of the layer, 35–40 cm; soil, sandy gravel.

shown in Figs. 66 to 69. The buried pressure cells, diam. = 25 cm, surface area = 500 sq.cm., were placed in two rows at four different depths (Fig. 66). For one row, 10-ton cells with deformation modulus $E = 8000$ kg/sq.cm. were used and for the other, 1-ton cells with $E = 500$ kg/sq.cm. The impact device was a mass of 10 kg falling from a height of 1 m (Fig. 67). The registered pressure-time curves for various degrees of compaction, that is after 0, 2, 4, and 6 passes, show that a maximum value for the dynamic pressure was reached after 5 to 6 passes (Fig. 68). Fig. 69 shows a plot of the maximum pressure against the number of passes and indicates the optimum number. After running these tests for a week it was possible to fix the necessary number of passes and the adequate thickness for the layers. The same results would very probably have required a longer period of testing if conventional methods had been used.

In the second field test gravel was compacted in layers of 30 cm using a multiplate vibrator. It is interesting because it allows the well-known prejudicial effect of compaction loss with too much vibration to be measured, as shown in Fig. 70.

The major inconvenience of the method described lies in the fact that the buried cell must be retrieved after the test.

Theoretically the pressure measurements of the upper cell alone should provide sufficient information about the compaction obtained. However, the falling weight device is inadequate for this purpose since it produces secondary mass vibrations and therefore gives unsuitable pressure-time diagrams. We are now experimenting in the laboratory with a shock tube and small explosive blasts. Impacts produced in this way should allow measurements to be made using only one cell placed on the surface. Two diagrams will be registered, namely pressure-time and dynamic settlement-time.

We hope this method will allow rapid control of the compaction of coarse materials and even of rockfills.

E. P. SHUSHERINA and S. S. VYALOV (U.S.S.R.)

The experimental investigation of the triaxial compression strength of frozen soil in the creep range was carried out through the permafrost study chair of Moscow University (Vyalov and Shusherina, 1964).

The experimental results obtained were considered on the basis of *Vyalov's* theory *(2/58)*. Several series of tests were made on a frozen sandy clay of massive structure, having a moisture content of 26 per cent, a unit weight of 1.84 grams/cu. cm. and a temperature of −10 C, each series with varying values of mean normal stress $\sigma = 15$, 25, 40 and 50 kg/sq. cm. A few tests were made at $\sigma = 20$ and 100 kg/sq. cm. In each series the specimens were tested at different intensities of tangential stresses σ_i (Sokolovsky, 1950). During the whole experiment both σ and σ_i were kept constant for all the specimens.

The test results show that the creep curves of frozen soil under triaxial compression have in general the same character as in the case of other deformations (uniaxial compression and tension, shear). For example, the set of creep curves showing the change of shear deformation intensity ϵ_i (Sokolovsky, 1950) with time t for one value of σ is given in Fig. 71. Similar curves were also obtained for other σ values.

The correlation of the creep curves, obtained with the same σ_i, but with varying σ, shows that these curves differ essentially depending on the σ value: with the increase of σ the deformation process gets slower and the deformation becomes smaller. If, using the experimental data, the curves of the relation between σ_i and ϵ_i are drawn for different moments of time, t (Fig. 72A), we obtain separate curves $\sigma_i - \epsilon_i$ for every σ value. This fact indicates that the relation between σ and σ_i changes with mean pressure σ values. The influence of σ is still more evident in Fig. 72B obtained by the corresponding reconstruction of Fig. 72A. As shown in Fig. 72B, the dependence of σ_i on σ can be expressed by the equation

$$\sigma_i = \tau(\epsilon_i t) + \sigma \tan \Psi(\epsilon_i t). \qquad (1)$$

Here the first term represents pure shear and the second one the increase of shear strength due to σ influence, Ψ denoting the angle of stress deviation on the octohedron area. Values τ and $\tan \Psi$ are the functions of deformation ϵ_i and of time t. Let us consider the form of the functions $\tau\ (\epsilon_i t)$ and $\tan \Psi\ (\epsilon_i t)$. The dependencies of τ on ϵ_i and of $\tan \Psi$ on ϵ_i for different t are shown in Figs. 73A and 73B obtained by the reconstruction of Fig. 72B. From the experimental data analysis the function $\tau(\epsilon_i t)$ can be expressed by the equation

$$\tau = A\,(t)\epsilon_i^{\,m}. \qquad (2)$$

Neglecting the initial deformation, we obtain

$$A\,(t) = \xi t^{-\lambda}. \qquad (3)$$

The dependence of $\tan \Psi$ on ϵ_i (Fig. 73B) in the reported experiments proves to be practically the same for different values of t (which may not yet be considered as a general law) and according to the analysis of the experimental data is represented by the equation

$$\tan \Psi = B\epsilon_i^{\,n}. \qquad (4)$$

Putting Eqs (2), (3) and (4) into Eq (1), we obtain

$$\sigma_i = \xi t^{-\lambda}\epsilon_i^{\,m} + \sigma B\epsilon_i^{\,n}, \qquad (5)$$

the fundamental equation of frozen soil deformation in the case of complex stress and creep conditions. It combines stress, strain, and time and takes into account the influence of mean normal stress. Parameters ξ, λ, m, B, and n depend on the type of soil, ξ and B also varying with temperature.

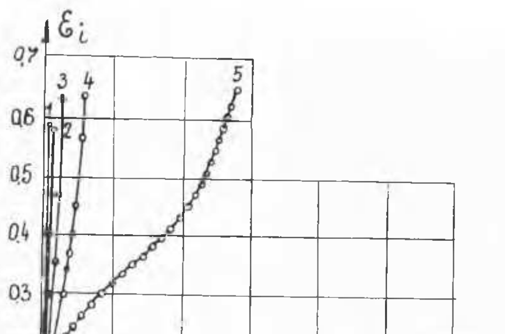

FIG. 71. The creep curves of frozen sandy clay under triaxial compression. $\sigma = 25$ kg/sq. cm. $\sigma_i = 43.1$ (1); 42.5 (2); 40.2 (3); 34.8 (4); 30.1 (5); 29.3 (6); 27.8 (7); 24.8 (8) kg/sq. cm.

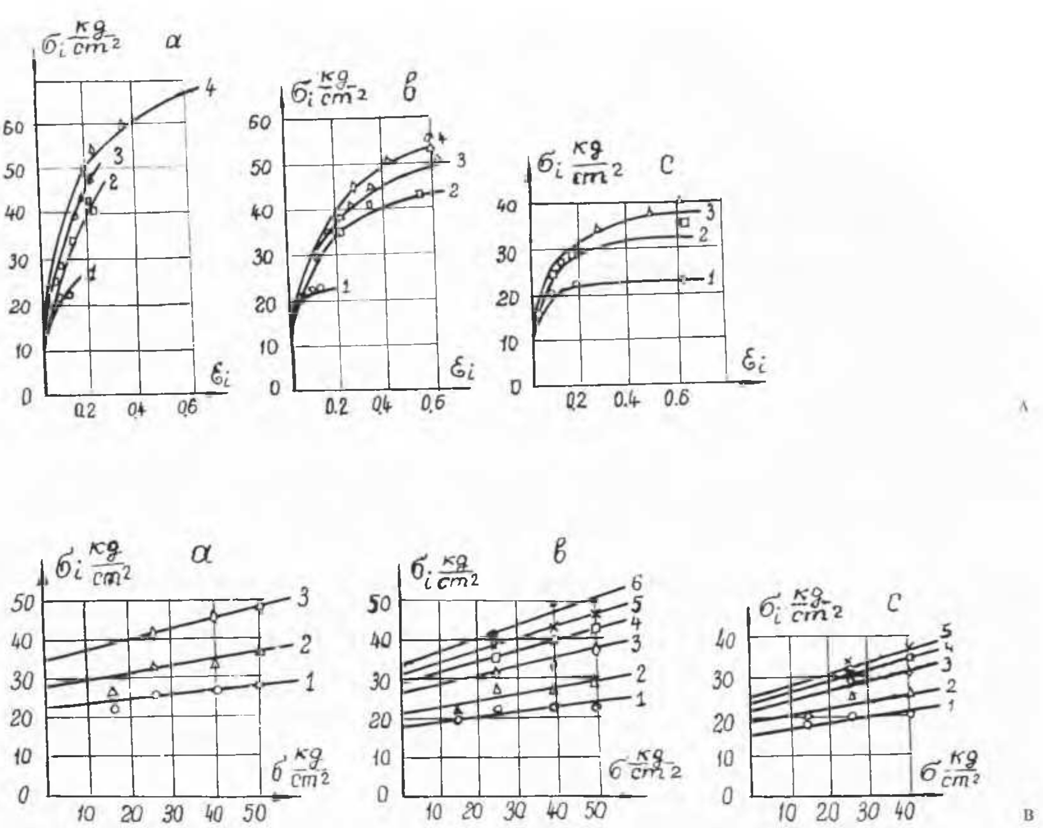

FIG. 72. The influence of mean normal stress on the creep of frozen soil, $t = 5$ min (a), 30 min, (b), 8 hr (c). A, the relation between σ_i and ϵ_i. $\sigma = 15$ (1), 25 (2), 40 (3), 50 (4) kg/sq. cm. B, the relation between σ_i and σ. $\epsilon_i = 0.048$ (1); 0.096 (2); 0.192 (3); 0.288 (4); 0.384 (5); 0.576 (6).

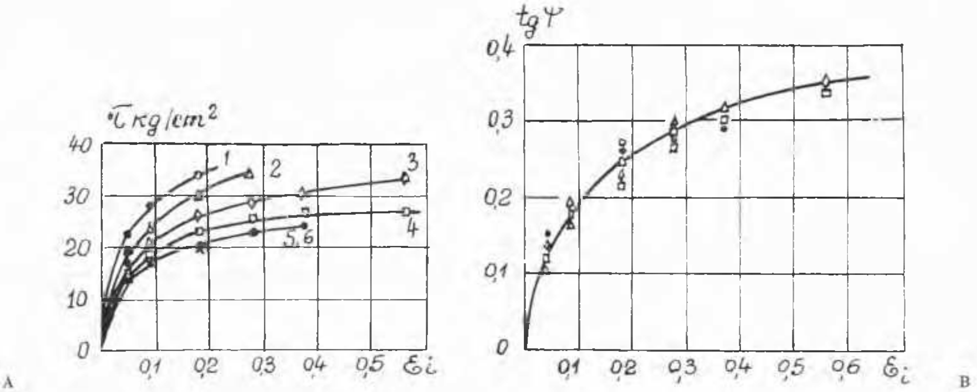

FIG. 73. The dependence of τ on ϵ_i (A) and of tan Ψ on ϵ_i (B). $t = 5$ min (1), 10 min (2), 30 min (3), 2 hr (4), 8 hr (5), 16 hr (6).

Table VIII compares the creep parameters, obtained by means of triaxial compression tests with those determined by S. E. Gorodetsky (Vyalov, *et al.*, 1962) by uniaxial compression (for the same soil and temperature). In the cases of uniaxial and triaxial compression the parameters m and λ must be theoretically equal, and the parameters ξ_1 and ξ are related by the formula $\xi_1 3 - (m + 1)/2 = \xi$. The above data show a good agreement between the frozen soil creep parameters obtained by means of simple and complex stress conditions.

TABLE VIII. COMPARISON OF CREEP PARAMETERS OBTAINED BY MEANS OF TRIAXIAL COMPRESSION TESTS WITH THOSE OBTAINED BY UNIAXIAL COMPRESSION TESTS

	m	λ	ξ(kg/sq. cm.-hr)	$\xi_1 3 - (m + 1)/2$
Uniaxial	0.27	0.11	78	39
Triaxial	0.26	0.08	40.5	—

Let us determine on the obtained creep curves (Fig. 71) the time t_i, when the stage of progressive flow begins

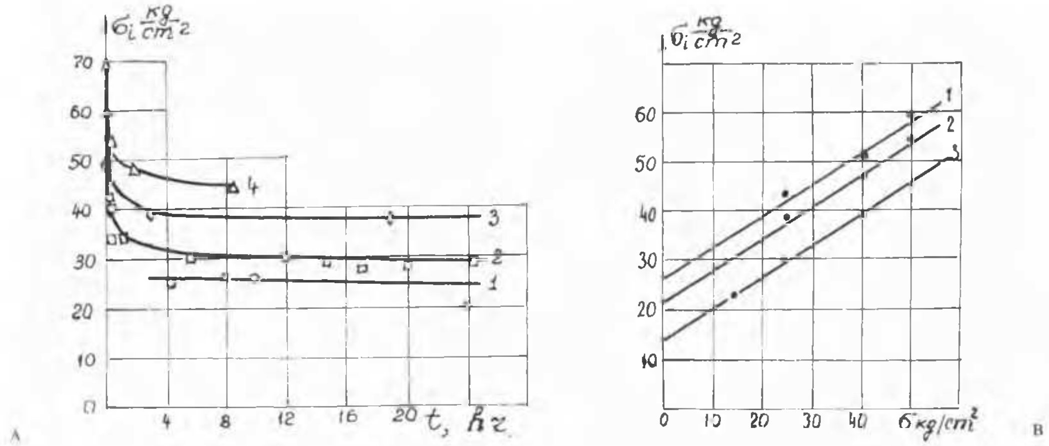

FIG. 74. The continuous strength curves of frozen soil under triaxial compression (A) and their reconstruction to co-ordinates, $\sigma_i - \sigma$ (B). $\sigma = 15$ (1), 25 (2), 40 (3), 50 (4) kg/sq. cm. $t = 10$ min (1), 30 min (2), 24 hr (3).

(assuming these conditions as the limiting ones), and let us draw the intensity change of tangential stress σ_i with t. Doing the same for the creep curves obtained at the other σ values, we obtain the set of continuous strength curves for the case of complex stress conditions, each curve corresponding to its value of the mean normal stress σ (Fig. 74A). The obtained curves are analogous to the continuous strength curves at uniaxial compression and shear (Vyalov, *et al.*, 1962).

The dependence of σ_1 on σ for the limit condition is well illustrated in Fig. 74B, and this dependence can be expressed by the equation

$$\sigma_i = \tau_s(t) + \sigma \tan \Psi_{0(s)}, \qquad (6)$$

where τ_s is the limit strength at pure shear, and $\Psi_{0(s)}$ is the friction angle on the octohedron area. Eq (6) is the Measis-Schleucher equation of the limit state, but it also takes into consideration the strength decrease with time, because τ_s is a function of t. The latter, as it follows from the analysis of the experimental data, is described by the equation suggested in the earlier work (Vyalov, *et al.*, 1962); $\Psi_{0(s)}$ for the soil in question does not depend on t. This means that the decrease of strength is caused solely by the cohesion; friction does not change.

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A. S. VESIĆ (U.S.A.)

This discussion refers to the second subject proposed by the General Reporter: behaviour of soils under high confining pressures. In particular, I should like to discuss the topic of strength and deformation characteristics of sands at pressures up to about 700 kg/sq.cm. or 10,000 lb/sq.in.

At the A.S.T.M. Shear Strength Conference in Ottawa in 1963 we presented some preliminary data on this subject, based on experiments with small samples of dense sand. Since that time these experiments have been continued by G. W. Clough, working on a master's thesis under my direction at the Georgia Institute of Technology. Triaxial tests have been made on larger, 2-in. diam. samples of the same sand at two relative densities, 78 per cent and 20 per cent. The results, which we intend to report in detail at the A.S.C.E. meeting at Miami Beach early in 1966, confirm, in general, our earlier findings and may be summarized as follows.

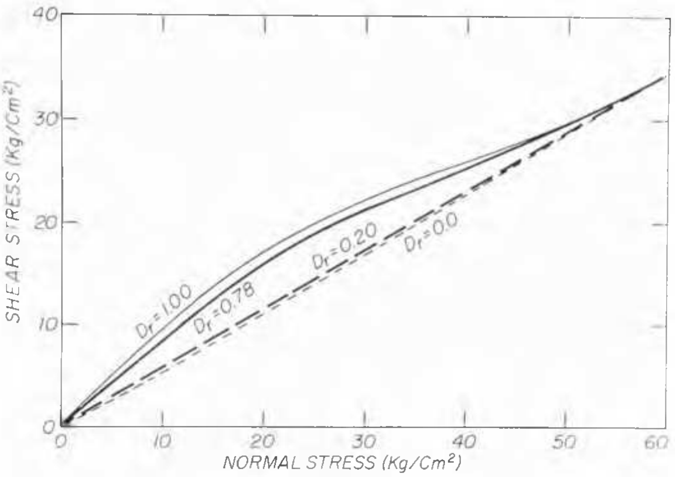

FIG. 75. Strength envelopes for different initial void ratios in the low pressure range.

The strength envelope of that sand for pressures beyond about 60 kg/sq.cm. and up to about 650 kg/sq.cm. can be represented by a straight line passing through the origin. In the mentioned pressure range this line appears to be independent of the initial void ratio of the soil. Its slope corresponds to the final angle ϕ_f obtained in the low pressure tests.

In the low pressure range, between 0 and 60 kg/sq.cm., the strength envelopes depend on the initial void ratio of the soil and may have very pronounced curvatures if the sand is initially very dense (Fig. 75). A picture of variation of the so-called secant angle of internal friction with the mean normal stress is shown in Fig. 76. For comparison we have included data on three other sands discussed by *de Beer (2/6),* all reduced to the same relative density of 78 per cent. It appears that the change of ϕ can be attributed to the

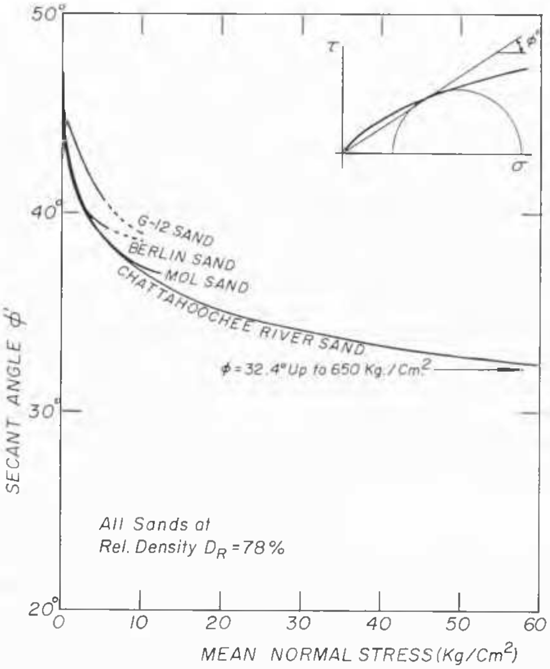

FIG. 76. Variation of the secant angle of internal friction with mean normal stress.

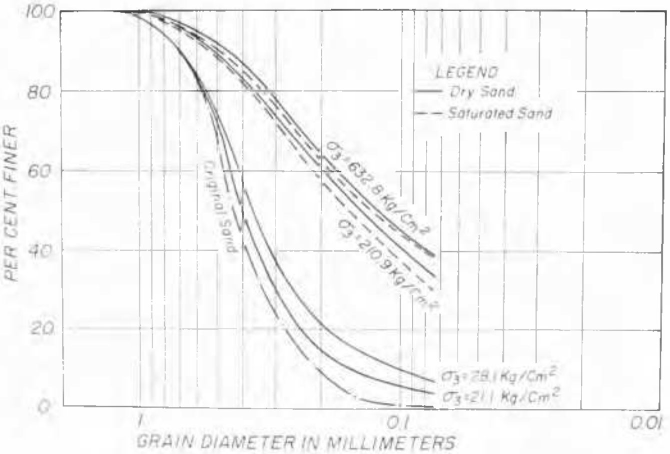

FIG. 77. Changes in particle size distribution caused by shear at high confining pressures.

reduction in the dilatancy component of the shear strength, coming predominantly from increased particle crushing in the respective range of confining pressure. The extent of this crushing can be seen from Fig. 77.

Typical volume change-strain and stress-strain curves obtained in this investigation are shown in Fig. 78. In contrast to similar curves presented by *Bishop, et al. (2/7)*, definite stress peaks were observed in most cases, generally at strains exceeding 20 per cent, however.

One important conclusion that was reached in this as well as in our previous investigations is that the well-known phenomenon of dilatancy disappears even in the most densely packed samples as soon as the mean normal stress becomes sufficiently high. Fig. 79 shows the final volume changes observed in constant mean normal stress tests. It can be seen in this figure that, at a relative density of 78 per cent, volume changes turn from negative to positive as soon as the mean normal stress exceeds about 35 kg/sq.cm. In other words,

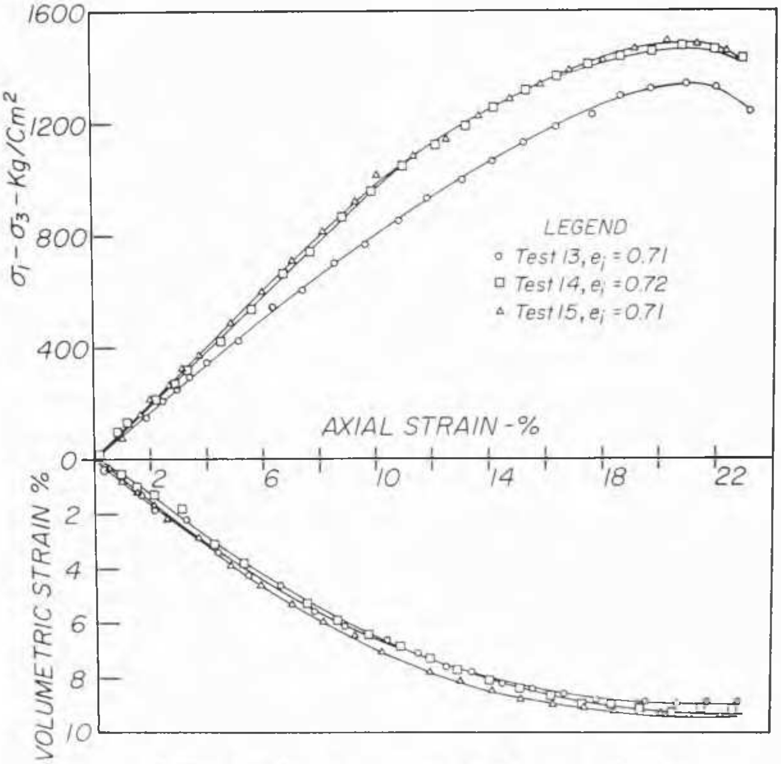

FIG. 78. Typical volume change-strain and stress-strain curves.

there is a critical pressure for any initial void ratio at which dilatancy disappears.

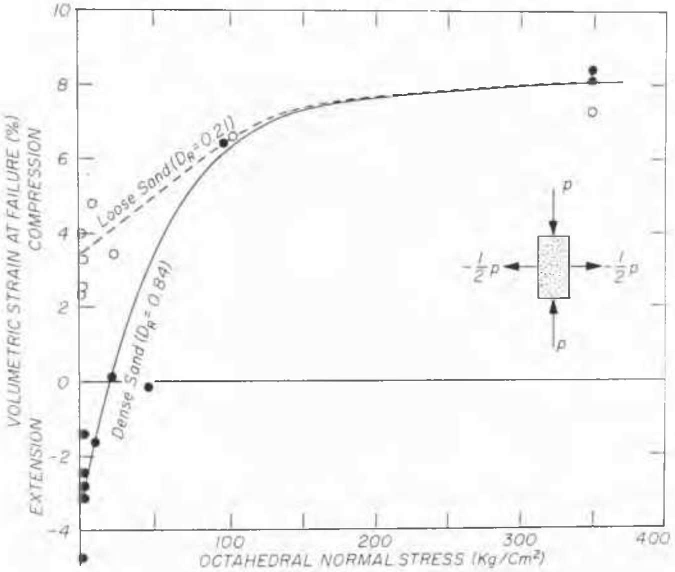

FIG. 79. Final volume changes observed in constant mean normal stress tests.

A study of stress-strain curves reveals that the initial modulus of deformation of the tested sand at low pressures increases as the 0.5 to 0.6 power of the confining pressure, a figure comparable to that obtained by earlier dynamic and low-pressure static tests on other sands (cf., for example, Richart, *et al.*, 1962). However, there are definite indications that the mentioned exponent varies with pressure range. This is another example pointing out to the very limited value of some recent idealizations of stress-strain behaviour of cohesionless soils mentioned in paper 2/7, as well as in the General Report of this Division.

The fact that, at very high pressures, the increase of the modulus of deformation of sand lags behind the increase of shear strength implies that the relative compressibility of sand increases with confining pressure. This finding has most interesting consequences when the bearing capacity of foundations is concerned (see Vesić, 1964). It offers an additional explanation of scale effects discussed by *de Beer (3/3)*.

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