The Dissipation Function for Unsaturated Soils

La répartition des charges dans le cas de sols non saturés

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Summary

The evaluation of saturated consolidation tests consists in "fitting" experimental time curves to the theoretical one based on the classical theory of one-dimensional consolidation. It should be noted that during this type of consolidation the soil sample remains fully saturated and, hence, its permeability is only affected by the change in grain structure (expressed, say, in terms of its porosity).

In the general case of unsaturated consolidation both air and water are expelled from the sample producing a change in the degree of saturation which strongly influences permeability.

Based on experimental permeability investigations on incompressible porous media, the author has developed a practical approach to the problem of "fitting" unsaturated test time curves.

An approximate expression is developed by means of which the saturated theoretical time curve may be corrected in order to furnish a more realistic basis for comparison. The correction depends on the possibility of measuring separately the amounts of air and water which leave the sample during its consolidation.

Sommaire

La valeur des essais de consolidation de sols saturés résulte de l'adaptation des courbes expérimentales de tassement en fonction du temps à la courbe théorique, basée sur la théorie classique de la consolidation à une dimension.

Il faut remarquer que, pendant ce genre de consolidation, l'échantillon de sol reste saturé ; donc sa perméabilité est influencée uniquement par le changement de la structure granulaire (exprimée p. e. en fonction de sa porosité).

Dans le cas général de la consolidation non-saturée, il y a élimination d'air ainsi que d'eau dans l'échantillon. Le changement du degré de saturation qui en résulte influence, à son tour, considérablement, la perméabilité.

Basé sur des recherches expérimentales sur la perméabilité des milieux pulvérulents incompressibles, le présent rapport tente d'analyser d'une manière pratique le problème de l'application des courbes de tassement de matériaux non-saturés.

L'auteur expose une formule approchée qui permet de corriger la courbe de tassement en fonction du temps théorique afin d'obtenir une base de comparaison plus pratique.

Cette correction dépend de la possibilité de mesurer séparément les quantités d'air et d'eau perdues par l'échantillon durant sa consolidation.

Introduction

In the classical, one-dimensional, theory of consolidation for saturated soils the progress of the consolidation process with time is expressed in terms of the dissipation of excess pore water pressure induced in the soil water by the application of a load.

The degree of dissipation (or load transfer to the soil grains skeleton) is most conveniently expressed by the function $U = f(T)$, where T is a dimensionless time factor related to actual time by the well-known expression :

$$T = \frac{c_v}{H^2} t \qquad (1)$$

Here c_v is a constant "coefficient of consolidation" and H is the length of the drainage path in the saturated soil Taylor, 1948, Ch. 10).

As applied to the conventional oedometer test, the degree of dissipation, U, refers to the average value of the pore water pressure distribution throughout the soil sample. If, however, the consolidation test is carried out in the triaxial apparatus, it is possible to measure the variation with time of the pore water pressure at the undrained base of the sample. In this case the drainage path, H, is the height of the tested sample and the dissipation function, corresponding to this type of test, is given in terms of pore pressure variation in a particular plane of the consolidating soil (Bishop and Henkel, 1957, Table 9, p. 135). In Fig. 1, curve II represents this type of dissipation function, whereas curve I corresponds to the previously mentioned average pore pressure condition. In the following only the dissipation function represented by curve II will be considered.

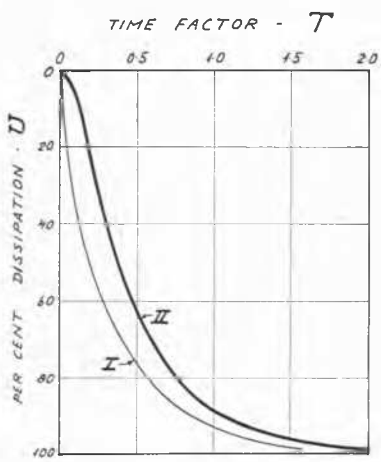

Fig. 1 Theorical dissipation Curves.
Courbes théoriques de dissipation de la surpression interstitielle.

Under the usual assumptions of the theory of one-dimensional saturated consolidation the constancy of c_v in Eq. 1 implies that the expression $\frac{k(1+e)}{a_v}$ is also a constant, where k is the saturated permeability coefficient (degree of saturation $S_w = 1$), e the average void ratio of the soil, and a_v its coefficient of compressibility (Taylor, 1948, Ch. X).

To evaluate c_v, the dissipation time curve is compared with the theoretical function and, after "fitting" them (f.e. by placing the experimental time curve, plotted on tracing paper, on top of the theoretical curve — the plots usually being in a semi-logarithmic coordinate system), the values of T and t corresponding to, say, 50 per cent dissipation are introduced into Eq. 1, H being taken as the average of the heights before and after consolidation of the tested sample.

In considering the consolidation of an unsaturated soil, i.e. a soil in which the pores are filled with a compressible fluid (air and water), it is reasonable to analyse the process in two stages : upon application of the load (the all-round cell pressure in our case) there is isotropic compression most of which occurs rapidly and which can be expressed in terms of Boyle's and Henry's laws and the compressibility of the grain structure (Skempton and Bishop, 1954); the second stage consists of the drainage of the pore fluid from the sample at atmospheric pressure until the gradient causing the flow ceases to operate.

The variation of pore pressure with time during the second stage represents the dissipation function of the unsaturated soil on which the analysis of its consolidation characteristics should be based. If one assumes that the relation $\dfrac{k(1+e)}{a_v} = \text{const.}$ also holds for this case then the conventional function $f(T)$ may be used for fitting the experimental time curves and thus evaluating c_v. The validity of this assumption is, however, doubtful and in the following an attempt is made to formulate the correction which should be applied to the saturated U-function in order to make it more representative of existing drainage conditions.

The approach is based on the experimental possibility of measuring the volumes of both air and water which have been expelled from the sample at any given instant during consolidation. A possible experimental arrangement to meet the case is presented in Fig. 2 (from Bishop and Henkel, 1957, p. 71).

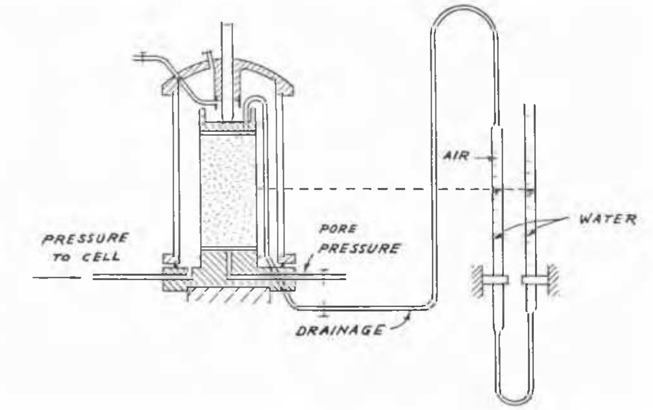

Fig. 2 Measurement of the volume of both air and water expelled from a partly saturated sample during consolidation (from : Bishop and Henkel, 1957).
Mesure des volumes d'air et d'eau expulsés d'un échantillon partiellement saturé pendant sa consolidation (d'après Bishop et Henkel, 1957).

Analysis

Investigations of multi phase flow in incompressible porous media have led, based on the fundamental work of J. Kozeny and P.C. Carman (Rose and Bruce, 1949, References), to the establishment of the following relationship :

$$K_w = S_w(A/A_{EW})^2 \tag{2}$$

where :

K_w = relative permeability = k/k_{sat}, $(0 < K_w \leqslant 1)$,
S_w = degree of saturation,
A = the surface area of soil particles per unit void space,
A_{EW} = the effective surface (also per unit void space), separating the wetting phase from all the other phases in the system; i.e. in our case separating the water from the air and the soil grains.

Denoting A_{AW} as the area separating the water from the air, it follows :

$$A_{EW} = A + A_{AW} \tag{3}$$

Eq. 3 implies that there is no contact area between soil grains and air. For high degrees of saturation, where the soil air most probably exists in the form of bubbles, this implication appears reasonable; at low degrees of saturation (air cavities and not bubbles) a fairly thick film of moisture may be assumed to cover the soil grains (Alpan, 1959). Eq. 3 appears, therefore, as justified.

For full saturation, A_{AW} is practically negligible, hence A_{EW} approximately equals A and, since $S_w = 1$, Eq. 2 furnishes, as expected, $K_w = 1$.

Consider now the various interfaces :

(a) For a volumetric strain of $\Delta V/V_0$, due to a volume change of ΔV of the consolidating sample (initial volume = V_0) the particle surface area A, as previously defined, changes and may be expressed as follows :

$$\frac{A}{A_0} = \frac{e_0}{e_0 + \Delta e} = \frac{1}{1 + \Delta e/e_0} \tag{4}$$

where A_0 is the initial area, e_0 the initial void ratio, A, the final area and $(e_0 + \Delta e)$ the final void ratio. Eq. 4 follows from a consideration of an elementary soil unit; denoting with $\bar{A}$ the surface of the unit volume of solids, which remains unchanged being incompressible, it follows from the definition of A : $A_0 = \bar{A}/e_0$ and $A = \bar{A}/(e_0 + \Delta e)$, hence Eq. 4.

In the usual notation, the initial porosity of the soil is given by $n_0 = e_0/(1 + e_0)$, hence

$$\frac{A}{A_0} = \frac{1}{1 + \dfrac{\Delta V}{n_0 V_0}} \tag{5}$$

(b) If $\overline{\Delta V_a}$ is the volume of air, measured at atmospheric pressure during a dissipation test after having been expelled from the sample, and ΔV_a the volume this air would occupy in the sample at pore pressure $p_{\text{abs.}}$ = measured pore pressure (u) + 1 atm., we have, on applying Boyle's law and remembering that we are measuring a loss in the air volume of the sample :

$$\overline{\Delta V_a} = -(1{\cdot}0/p)\Delta V_a$$

and hence

$$\frac{\Delta V_a}{V_0} = -\frac{\overline{\Delta V_a}}{p \cdot V_0} \tag{6}$$

Assuming isotropic contraction of the air space, we obtain, as a first approximation of the change of A_{AW} due to the loss of ΔV_a :

$$A_{AW} = A^\circ_{AW}\left[1 + \frac{2\,\Delta V}{3\,V_\circ}\right] = A^\circ_{AW}\left[1 - \frac{2\,\overline{\Delta V_a}}{3p\;V_0}\right] \tag{7}$$

where the superscript (°) denotes initial conditions, i.e. the state immediately prior to the start of dissipation, the sample having reached equilibrium under the ambient cell pressure.

It follows from Eqs. 5, 6 and 7 :

$$\frac{A_{AW}}{A} = \frac{A^\circ_{AW}}{A_\circ} \frac{\left[1 - \frac{2}{3p}\frac{\overline{\Delta V_a}}{V_\circ}\right]}{\left[\frac{1}{1 + \frac{\Delta V}{n_\circ V_\circ}}\right]} = \quad (8)$$

$$= \frac{A^\circ_{AW}}{A_\circ}\left[1 - \frac{2}{3p}\frac{\overline{\Delta V_a}}{V_0}\right]\left[1 + \frac{\Delta V}{n_0 V_0}\right]$$

It should be noted that $\overline{\Delta V_a}$ is always measured as positive whereas ΔV is negative if the sample decreases in volume. The technique of accurately measuring volume changes in triaxially tested samples is described in detail in the monograph by Bishop and Henkel (see References).

Introducing Eq. 3 into Eq. 2 :

$$K_w = S_w\left[\frac{A}{A + A_{AW}}\right]^2 = \frac{S_w}{\left[1 + \frac{A_{AW}}{A}\right]^2} \quad \ldots. \quad (9)$$

Using the notation :

$$m = \frac{A^\circ_{AW}}{A_0}$$

and $\ldots. \quad (10)$

$$\psi = \left[1 - \frac{2}{3p}\frac{\overline{\Delta V_a}}{V_0}\right]\left[1 + \frac{\Delta V}{n_0 V_0}\right]$$

the relative permeability can now be expressed thus :

$$K_w = \frac{S_w}{(1 + m\psi)^2} \quad (11)$$

Before the start of the dissipation process, $\psi = 1$ and $K_w = K_w^\circ$, the relative permeability of the material at a given porosity, n_0, and a given degree of saturation, S°_w, which may be expressed as :

$$K^\circ_w = \frac{S^\circ_W}{(1 + m)^2} \quad (12)$$

The correction to operate on the dissipation function should represent the way in which the initial permeability of the sample is affected by the changes of not only the porosity (accounted for by the classical theory) but specifically of the degree of saturation ; hence, it should equal the ratio of the relative permeability at any time during consolidation and the initial one.

Using Eqs. 11 and 12 :

$$\frac{K_w}{K^\circ_w} = \frac{S_w}{S^\circ_w}\left[\frac{1 + m}{1 + m\psi}\right]^2 \quad (13)$$

It appears from experimental data on rigid porous materials (Rose, 1949 ; Rose and Bruce, 1949) that, particularly for a saturation range of from 70 per cent upward, the following relation (based on the capillary tube model) represents an adequate approximation :

$$K_w = \frac{S_w}{(2 - S_w)^2} \quad (14)$$

It follows, using Eq. 12 :

$$K^\circ_w = \frac{S^\circ_w}{(1 + m)^2} = \frac{S^\circ_w}{(2 - S^\circ_w)^2}$$

whence :

$$m = 1 - S^\circ_w \quad (15)$$

It is now possible to give an approximate expression for the correction factor, ρ, as defined by Eq. 13, and using Eq. 15 :

$$\rho = \frac{K_w}{K^\circ_w} = \frac{S_w}{S^\circ_w}\left[\frac{2 - S^\circ_w}{1 + (1 - S^\circ w)\psi}\right]^2 \quad \ldots. \quad (16)$$

Before the start of dissipation, $S_w = S^\circ_w$, $\psi = 1$ and, hence, $\rho = 1$. Should the degree of saturation not change during the dissipation, the correction factor stays equal to unity and the saturated dissipation function can be used for time fitting. Similarly, at full saturation, $S^\circ_w = 1$ and $K_w = K^\circ_w = 1$, whence $\rho = 1$, as is to be expected.

The ranges of the various parameters and functions are :

$$0 < \psi \leqslant 1$$

$$1 \leqslant \frac{2 - S^\circ_w}{1 + (1 - S^\circ_w)\psi} < \text{finite} \quad (17)$$

$$0 < \rho \leqslant 1$$

Conclusions

The application of the foregoing analysis to partly saturated consolidation consists in modifying the saturated dissipation function by the appropriate introduction of the correction factor, ρ, as given by Eq. 16. It follows from Eq. 1 that the modified time factor is directly proportional to ρ, i.e. :

$$T_{\text{mod.}} = \rho \cdot T \quad \ldots. \quad (18)$$

Hence, if $U = f(T)$ is the saturated dissipation function,

$$U_{\text{mod.}} = f(\rho \cdot T) \quad (19)$$

Suppose now that the curve II of Fig. 1 can be approximated, for a certain range, by a function of the type :

$$U = a\sqrt{T} \quad (20)$$

For each reading of the pore water pressure u the corresponding percentage dissipation, the expelled air and water volumes and the degree of saturation can be found ; hence, the value of ρ can be computed, noting that $u = p - 1{\cdot}0$ atm.

Assuming Eq. 20 to be applicable, it follows from Eq. 19 :

$$U_{\text{mod.}} = U \cdot \sqrt{\rho} \quad (21)$$

Thus a modified dissipation curve, in terms of T, for partial saturation may be drawn to which the experimental time curve is fitted in order to permit the evaluation of the coefficient of consolidation, c_v.

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