The Mechanical Properties of Soils

Les propriétés mécaniques des sols

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Summary

It is known that the stress-strain relationships of soils are different for loading and unloading. An attempt is made by the author to separate theoretically this influence in order to be able to judge the effects of other influences, and particularly anisotropy in test results. By means of constant volume shear tests with the ring shear apparatus, the stress-strain relationships of a fine sand and a silty clay were investigated. Some test results are given.

Sommaire

Comme on le sait, les relations contrainte-déformation des sols sont différentes selon la charge et la décharge. L'auteur essaie d'analyser théoriquement ces phénomènes pour en étudier d'autres, en particulier l'anisotropie constatée lors des essais. Au moyen des essais de cisaillement à volume constant exécutés à l'aide d'un appareil de cisaillement par rotation, les relations contrainte-déformation d'un sable fin et d'une argile sableuse ont été examinées. Quelques résultats en sont donnés.

In the field of foundation and earthwork engineering we are concerned with the problem of expressing the mechanical properties of a soil by coefficients which can be used in theoretical computations. The determination of these coefficients is more difficult than in any other field of technical science because they are not generally constant and may vary under different conditions.

There are two main problems in soil mechanics : the stress-strain relationship of soil in an elastic state and the shear strength in a plastic state. It is customary to assume as coefficients for the elastic state the modulus of elasticity E and Poisson's ratio ν, substituting for the natural soil an ideal elastic material, and as coefficients for the plastic state the angle of internal friction φ, the cohesion c replacing the soil by an ideal plastic material.

Compression of a natural soil is caused mainly by a decrease of the porosity and not by a reduction of the grain size. For this reason, soil, even in its elastic state, behaves quite differently under loading and unloading. Only part of a strain caused by a certain load is reversible by removing the load. This has nothing to do with the plastic state of soil in the sense used above.

Since they are caused by a decrease in porosity, volume changes in natural soil are much greater than in other construction materials. In saturated soils they are of particular interest and importance in connection with the occurrence of pore water pressures. If we want to study the stress-strain relations hips in natural soils we should, therefore, distinguish between two kinds of strain : volume change, and deformation. Each of these may occur without the other, for instance : volume change without deformation caused by an overall pressure, and deformation at constant volume during an undrained test in saturated soil.

By assuming the coefficients E and ν in the theory of elasticity, the two kinds of strain mentioned above are connected with each other. This procedure may be useful for other construction materials, where volume changes are relatively unimportant, but not for natural soils. We should, therefore, substitut E and ν by two other coefficients. One of them should be the volume change taking place without deformation (bulk modulus K), and the other the resistance against deformation at constant volume.

We consider volume changes of a soil in elastic state, replacing the natural soil by an ideal material which behaves in a different way for first loading and unloading. For virgin loading it is assumed that the theory of elasticity and the law of superposition may be applied approximately. The relationship between an increasing allround pressure and the porosity is given by the bulk modulus K. K is actually not a constant but might be considered as such for a limited range of pressures.

A general state of stress is given by three principal stresses, the major σ_1, the intermediate σ_2 and the minor principal stress σ_3. In agreement with the theory of elasticity we assume that the decrease of volume due to compression depends on the sum of the three principal stresses $\sigma_1 + \sigma_2 + \sigma_3$. This means that a stress state consisting of three different principal stresses σ_1, σ_2 and σ_3 produces the same decrease of volume as an equivalent overall pressure of the magnitude

$$\sigma' = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3).$$

In Fig. 1, the major principal stress σ_1 is plotted against the minor one, σ_3. We consider a specimen under the overall

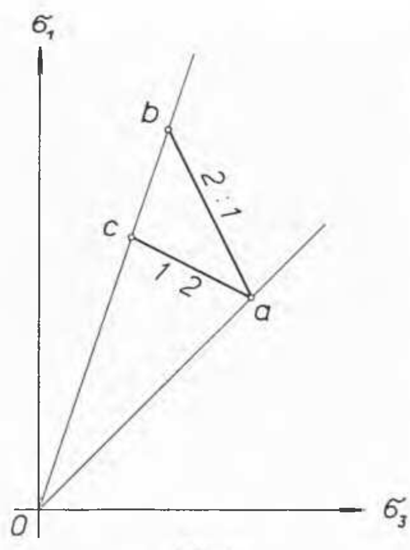


Fig. 1

pressure $\sigma_1 = \sigma_2 = \sigma_3$ corresponding to point a in Fig. 1. If we increase σ_1 by $\Delta\sigma_1$ compression will occur. But if we want to keep the volume constant we have to reduce the other principal stresses. Diminishing both, σ_2 and σ_3, by the same amount we obtain $\Delta\sigma_2 = \Delta\sigma_3 = -\frac{\Delta\sigma_1}{2}$ represented by the straight line *a-b* in Fig. 1. On the other hand, if we increase σ_1 and σ_2 equally ($\Delta\sigma_1 = \Delta\sigma_2$) we have to reduce σ_3 by $\Delta\sigma_3 = -2\,\Delta\sigma_1$ in order to keep the volume of the specimen constant (straight line *a-c* in Fig. 1). For any intermediate principal stress whatever, there is no compression possible on the left hand of the straight line *a-c* and no expansion on the right hand of *a-b*.

According to the behaviour of natural soils we assume that our ideal material behaves in a different way for unloading and for virgin loading. For this reason, the straight line *a-b* cannot be valid for reducing σ_2 and σ_3 by $\Delta\sigma_2 = \Delta\sigma_3$ after initial compression under the overall pressure $\sigma_1 = \sigma_2 = \sigma_3$. But if we remember that no compression at all is possible on the left hand of *a-c* (which means expansion only) we may conclude that the straight line *a-c* is the lower boundary for expansion, i.e. for two axial unloading.

We now consider two specimens under the same overall pressure corresponding to point a in Fig. 2. We load one of

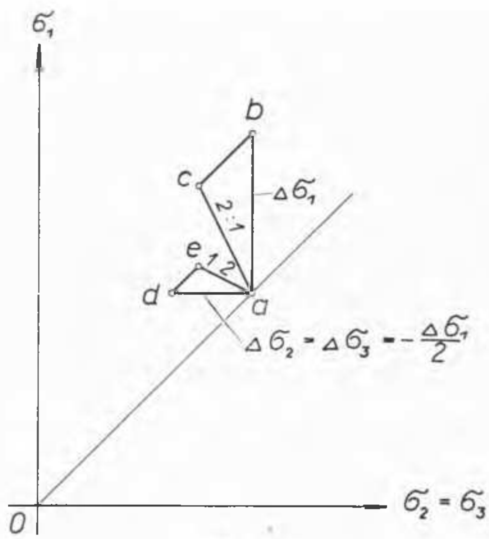


Fig. 2

them uniaxially by increasing σ_1, and unload the other one biaxially by reducing $\sigma_2 = \sigma_3$ in such a way that $\Delta\sigma_2 = \Delta\sigma_3 = -\frac{\Delta\sigma_1}{2}$ with $\Sigma\,\Delta\sigma = 0$. In the first specimen a compression occurs that is proportional to the distance *b-c*. The other specimen expands proportionally to the distance *d-e*. As *d-e* is one half of *b-c*, the modulus for threeaxial unloading is twice bulk modulus *K* for threeaxial loading. Applying the law of the constant sum of principal stresses for constant volumes, there is no difficulty of correlating the cases of uni —, bi — and threeaxial loading and unloading if we proceed from an initial virgin allround pressure.

If we increase σ_1 in a specimen under an initial virgin allround pressure corresponding to point a in Fig. 3 we have to reduce $\sigma_2 = \sigma_3$ according to the straight line *a-b* in order to keep the volume constant. It is reasonable to assume that, starting from point *b*, the line *b-a* corresponds to biaxial loading and *b-c* to uniaxial unloading at constant volume. In similar manner, it can be assumed that the line *c-b* belongs to uniaxial reloading and *c-d* to biaxial unloading. Since the volume of the specimen is the same at points *a* and *c* it must be possible to produce in the laboratory overconsolidated soil by loading and unloading at constant volume, and to study its mechanical properties.

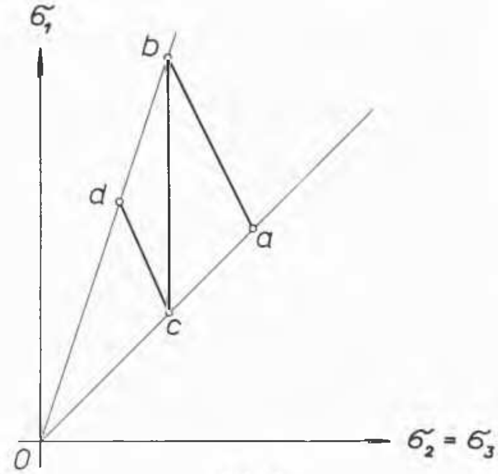


Fig. 3

Due to the variety of soils available, we cannot expect that our considerations are even approximately valid for every soil. Neither anisotropy caused by stratification due to orientation of the grains, nor the vibration effect in sand has been considered. But by considering the difference in the behaviour of natural soil from that of the ideal medium, we can obtain some information about these other influences.

An extensive research programme has been started to clear up these problems. As test materials a fine sand and a relatively fine silty clay were used. The programme was based mainly on shear tests carried out with the ring shear apparatus. During the test, the volume of the specimen was kept constant by removing or adding vertical load at the moment the dial gauge indicated the beginning of any volume change. After the first shearing, the shear load and such a part of the vertical load was removed so that the volume remained constant and another shear test was performed at constant volume, in the same or reversed direction of shearing. This procedure was repeated several times. After that, the specimen was reloaded up to the initial vertical load, thus causing an overconsolidation of the specimen. Shear tests could thereby be performed with heavily overconsolidated soils, using one specimen only, and eliminating scattering caused by lack of homogeneity of various soil samples.

First, constant volume shear tests with sand were performed. As there is no really essential difference in the behaviour of dry and saturated sand, the shear tests were carried out under water since this made it easier to keep the volume constant. In Fig. 4a the test result obtained with a loose, saturated sand is shown. Most of the vertical load had to be removed during the test in order to avoid compression.

The shear strength of a dense sand is believed to consist of two parts : internal friction and interlocking effect. The internal friction is proportional to the vertical load only and is given by the angle φ of internal friction which equals the shear angle of the sand in loose state. In addition, the angle φ also limits the elastic state in the significance used above.

In Fig. 4b, the relationship between vertical load and shear load at constant volume is indicated for a dense saturated sand. After consolidation under the vertical load of 5 kg per sq cm corresponding to point a, the stress state is known approximately only by estimating the coefficient at rest. For small shear loads, the constant volume line is curved up to point b, which means that mainly the direction of the principal stresses is changed but not their magnitude. Between points b and c, the constant volume curve is almost a straight line. At the elastic limit (point c), the shear strength is not yet reached.

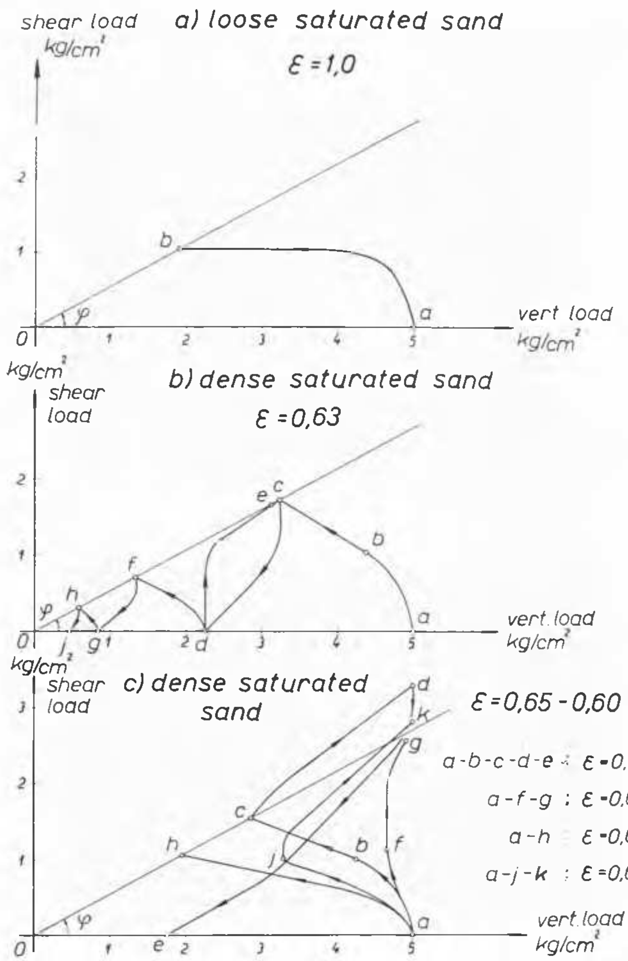


Fig. 4

If we want to remain within the elastic limit 0-c, the shear load must be removed and also partly the vertical load according to curve c-d. The volume at points a and d is the same. If we carry out another shear test, starting at point d, we can retain the direction of shearing and obtain the curve d-e. But if the direction of shear is reversed the relationship between shear load and vertical load at constant volume is given by curve d-f. The difference between the curves d-e and d-f is caused by increased anisotropy the direction of which has changed with regard to point a.

We can repeat this procedure and we obtain the curves f-g and h-j for unloading and g-h for shearing at reversed direction. The points c, f and h are located on a straight line passing through the origin. At point j, the initial vertical load is almost completely removed although the colume equals that at point a.

In Fig. 4c, the result of another test carried out with dense saturated sand is represented. After consolidation corresponding to point a, the curve a-b-c resulted from a constant volume shear test. At the elastic limit (point c) the strain-stress relationship is changed quite abruptly. For increasing shear loads, we have also to increase the vertical load corresponding to curve c-d. No shearing can occur as long as the volume is kept constant. After removing the shear load, the vertical load corresponds to point e at constant volume. If the specimen is reloaded under compression of the sample, a constant volume shear test gives the curve a-f-g maintaining the shearing direction and the curve a-h for reversed direction. But after repeated shearing up to the elastic limit only and reloading to 5 kg per sq cm, the curve a-j-k was obtained, which indicates a change of the strain-stress relationship already at point j. The influence of the overconsolidation and isotropy on the stress-strain relationships and the final peak load of shearing is still under consideration.

Constant volume shear tests on clay are even more interesting than those of sand, because at constant volume no pore water pressure can occur. This means that a quick test can be performed instead of a slow test without the development of water pressure. This is true only to a certain degree. It has been observed that especially during the first shearing after consolidation of a disturbed saturated clay specimen a time influence still exists. However, by removing part of the vertical load, we are able to carry out a shear test avoiding pore water pressures within a reasonable time. As it was possible to produce sand specimens with the same porosity under very different vertical loads, the same method was used for clay in order to separate cohesion and internal friction, as friction depends on the vertical load only and cohesion on porosity.

In Fig. 5 test results performed with one clay specimen placed under semifluid condition into the ring shear box and

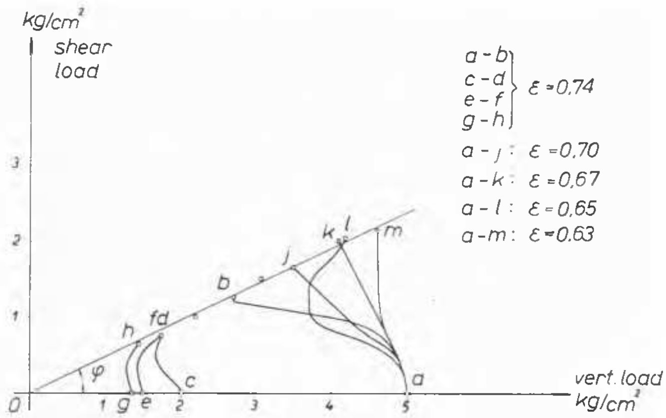


Fig. 5

consolidated under the vertical load of 5 kg per sq cm are shown. The curve a-b represents the relationship between shear and vertical load while keeping the volume of the specimen constant during the test. After unloading and changing the direction of shear, the curves c-d, e-f and g-h were observed. As clay has no dilative properties, the beginning of failure occurred actually at the points b, d, f and h. After having reconsolidated the specimen under 5 kg per sq cm, the peak values of shear correspond to points j and l for unchanged direction of shear respectively to k and m for reversed direction.

As can easily be seen from Fig. 5, the peak values of shearing are located on a straight line rising under 25·5°. This happens at any rate if the shear tests are performed in the way mentioned above independent of porosity, temperature, or other potential influences. According to this result, the shear strength of disturbed saturated clay depends on the normal load only. If this is considered to be the criterion of a friction medium, then a saturated disturbed clay is such a material. A cohesion depending on the porosity could not be observed even for heavely overconsolidated clay.

The location of the peak value on the straight line may vary according to porosity, direction of shearing, temperature or other potential influences. For this reason, the shear strength determined from a conventional consolidated quick shear test may scatter over a wide range.

If we accept these conclusions, the shear strength s of a disturbed saturated clay is given by the formula

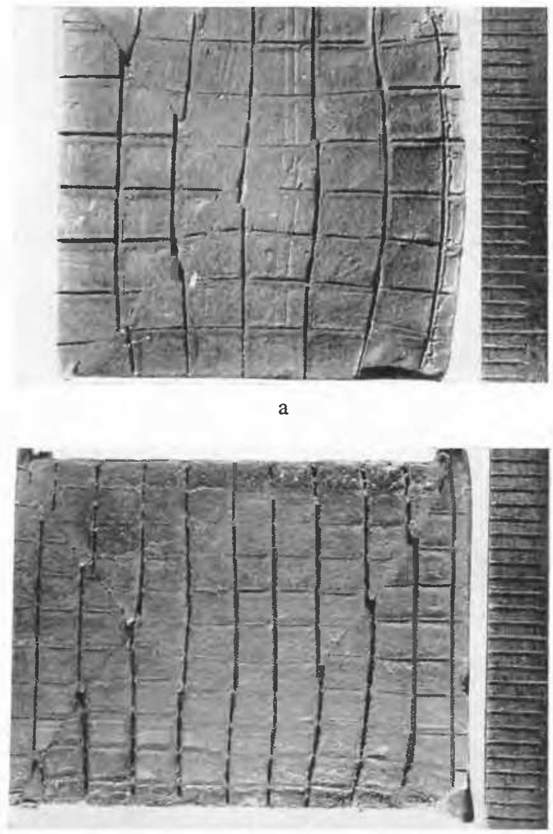
$$s = \sigma_e \text{ tg } \varphi = \eta \sigma \text{ tg } \varphi,$$

φ being the angle of internal friction, σ_e the effective pressure normal to the shear plane, σ the total pressure, and η the ratio of effective and total pressure. Only φ is a constant for a certain clay but η is not. It depends on the history, temperature and direction of shearing etc. For overconsolidated clay, η may approach or even surpass unity. From the first shearing under the virgin consolidation load of 5 kg per sq cm, the shear strength of 1,25 kg per sq cm resulted at a vertical load of 2,7 kg per sq cm corresponding to $\eta = 0{\cdot}54$. The unconfined compressive strength of such a clay is about one half of the consolidation pressure, or 2·50 kg per sq cm. According to the $\varphi = 0$ method, the shear strength of the clay would be 1,25 kg per sq cm. Hence, one half of the unconfined compression equals approximately the shear strength determined from a slow constant volume shear test. But this statement is true only for virgin loading and first shearing. For overconsolidated clay, the shear strength was found to be much less than one half of the unconfined compressive strength.

The objection could be made that the inclination of the rupture plane after an unconfined compression test is contradictory to such a high angle of internal friction. For this reason, numerous such tests were performed with the clay under consideration. In accordance with former test results, the angle of internal friction, deduced in conventional manner from the inclination of the failure plane, scattered between 0 and 40°. This is due mainly to the fact that the sample at failure is as a whole in the plastic and the wedge, produced at failure, evades unilaterally and asymmetrically, altering the inclination of the failure planes. In order to avoid an asymmetric rupture and to enforce failure in one direction only, cubes of saturated consolidated clay were subjected to ubconfined compression tests and guided laterally by means of two parallel glass plates. In Fig. 6a the failure pattern is represented without greasing the top and bottom surfaces of the specimen. As can be seen, the final rupture planes connect approximately opposite edges of the cube. As the inclination of the failure planes in deformated condition is about 45°, the angle of internal friction determined in the conventional manner would equal zero. The top and bottom surfaces of the specimen shown in Fig. 6b were greased. The rupture planes, separating an almost unjolted wedge, are much steeper and correspond to an angle of internal friction of about 25°, which is well in agreement with that obtained from constant volume shear tests. This proves that the inclination of the rupture planes depends to a high degrees on the boundary conditions at the top and bottom surfaces of the specimen.

The water content of the tested clay equaled 54·5 per cent at the liquid limit and 24 per cent at the plastic limit. From a routine slow shear test using a common shear box resulted a shear angle of 23° and from a consolidated quick shear test an angle of 16° as an average.

A more detailed description of the test practice, further test results with its discussion will be given in the third paper of "Mitteilungen des Institutes für Grundbau und Bodenmechachanik an der Technischen Hochschule Wien".



a

b

Fig. 6 Unconfined compression test with clay samples:
(a) without greasing the top and bottom surfaces;
(b) with greasing the top and bottom surfaces;

Essai de compression sans contrainte latérale sur des échantillons d'argile:
(a) sans graisser les surfaces supérieure et inférieure.
(b) après graissage des surfaces supérieure et inférieure.

References

[1] H. Borowicka. The mechanical properties of soils.