Statistical Forecasting of Compression Index

Prévision statistique de l'indice de compressibilité des sols

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Summary

The author presents a study of the correlation of the compression index of two kinds of Brazilian clays with their Atterberg limits and with their natural void ratio, using advanced statistical analysis.

It is proved that within a "Level of Significance" of 0·1 per cent which amounts to a certainty, there is a statistical relationship between the compression index of an undisturbed sample of soil and its liquid limit, as well as between the compression index of a similar sample of soil and its natural void ratio. The author proves statistically that the most adequate mathematical pattern for compression index forecasting would be a "Multiple Regression" between C_c, L_L and e. The compression index of a soil depends not only on its liquid limit but also on its void ratio.

Sommaire

La présente note étudie la corrélation entre l'indice de compressibilité, la limite de liquidité et l'indice des vides pour deux types de sols argileux du Brésil à l'aide de l'analyse statistique.

Il est prouvé qu'avec un taux de probabilité de 99·9 pour cent c'est-à-dire une quasi certitude, il existe une relation statistique entre l'indice de compressibilité d'un échantillon non remanié et sa limite de liquidité, aussi bien qu'entre l'indice de compressibilité et l'indice des vides naturel.

Finalement il est prouvé que la forme mathématique la plus correcte pour fixer à priori l'indice de compressibilité serait une « relation multiple » entre C_c, L_L et e. L'indice de compressibilité d'un sol dépend non seulement de sa limite de liquidité, mais aussi de son indice des vides naturel.

The author gives general conclusions of a statistical investigation on the correlation between compression index, liquid limit and natural void ratio of soils, in which the problem of the compression index forecasting is studied. The study concerns two types of soils from State of São Paulo (Brazil).

A. Clays and silty clays, moderately soft or hard, red or yellow, known as "Motley clays", which constitute the upper horizon of the Tertiary deposits occurring in São Paulo City. The statistical study for this case is based on the results of tests undertaken in the Institute of Technological Research (University of São Paulo) by Milton Vargas and Glauco Bernardo [1]. The Plasticity Chart, Fig. 1, enclosed, facilitates soil identification.

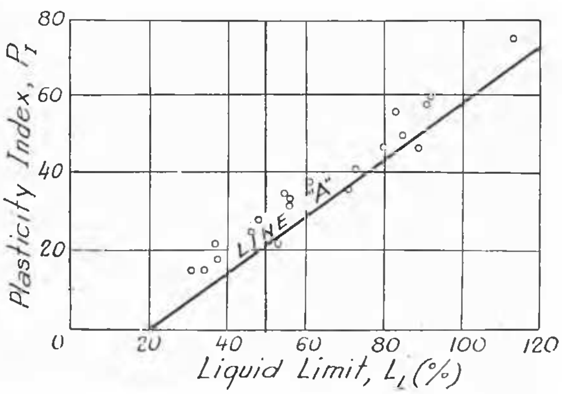

Fig. 1 Plasticity chart for the Motley clay of São Paulo City.
Plasticité de l'argile bigarrée de la ville de São Paulo.

B. Soft silty clay, dark grey, with a low content of organic matter from the Quaternary deposits of marine origin, which occur in the lowlands of Santos. The statistical study for this case is based on the results of tests carried out in the I.P.T. (University of São Paulo) by José Machado [2]. The Plasticity Chart (Fig. 2) also enables the soil to be identified.

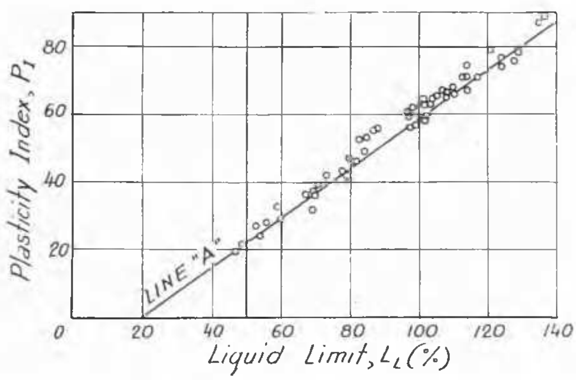

Fig. 2 Plasticity chart for the silty clay of the lowlands of Santos.
Plasticité de l'argile silteuse de la baie de Santos.

In her original study, the author used statistical methods for examining the empirical formulae which would permit a forecast to be made of the compression index of undisturbed samples by correlating this with Atterberg limits and with the natural void ratio of soil.

The determination of the compression index of an undisturbed sample in a Soil Laboratory depends on long and expensive tests, which can be carried out only by experts. This is the reason why several authors examined the possibility of forecasting this compression index with the aid of other soil properties which can be more readily determined. Several formulae have been presented as results of such investigations. Some correlate the compression index with the liquid limit, others with the moisture content of the soil.

The Author has endeavoured to establish statistically, for the two types of soil above mentioned, through "Regression Analysis" the empirical laws which relate the compression index of undisturbed samples of soil with the liquid limit, as well as with the natural void ratio.

The following results were obtained :

A. Motley clays from São Paulo City.

$$C_c = 0{\cdot}0046\,(L_L - 9) \pm 0{\cdot}086 \text{ (see Fig. 3)} \quad \ldots. \quad (A_1)$$
$$C_c = 0{\cdot}256 + 0{\cdot}43\,(e - 0{\cdot}84) \pm 0{\cdot}063 \text{ (see Fig. 4)} \quad .. \quad (A_2)$$

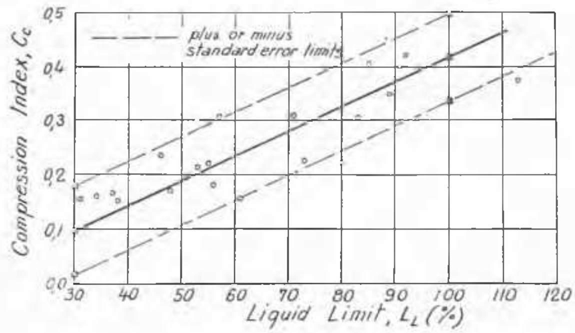

Fig. 3 Regression of the Compression Index C_c upon the liquid Limit L_L, for the Motley clay of São Paulo City.
Relation entre l'indice de compressibilité C_c et la limite de liquidité L_L, pour l'argile bigarrée de la ville de São Paulo.

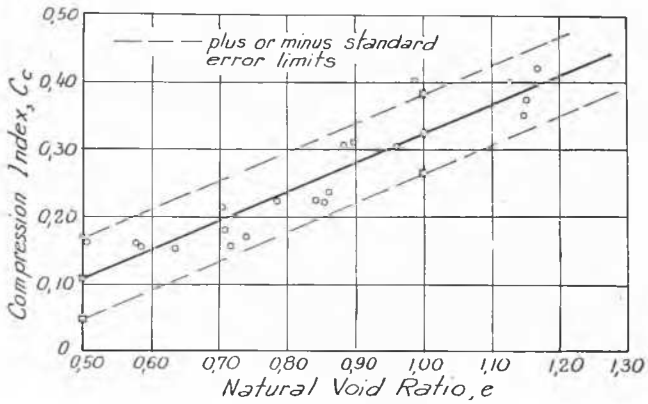

Fig. 4 Regression of the Compression Index C_c upon the natural void ratio e for the Motley clay of São Paulo City.
Relation entre l'indice de compression C_c et l'indice des vides naturel e, pour l'argile bigarrée de la ville de São Paulo.

B. Soft silty clays from the lowlands of Santos.

$$C_c = 0{\cdot}0186\,(L_L - 30) \pm 0{\cdot}41 \text{ (see Fig. 5)} \quad \ldots. \quad (B_1)$$
$$C_c = 1{\cdot}21 + 1{\cdot}055\,(e - 1.87) \pm 0{\cdot}34 \text{ (see Fig. 6)} \quad .. \quad (B_2)$$

The author used statistical " Variance Analysis " to determine, if the above formulae A_1 and A_2 (B_1 and B_2) are "significant" ; that is : if the knowledge of liquid limit, or of natural void ratio, can be used as a basis for forecasting the probable compression index.

The author concluded that almost certainly there is a statistical relationship between the compression index and the liquid limit on one side, and between the compression index and the natural void ratio on the other.

It was therefore concluded that it is possible to forecast the compression index of a soil based on the liquid limit, or on the natural void ratio.

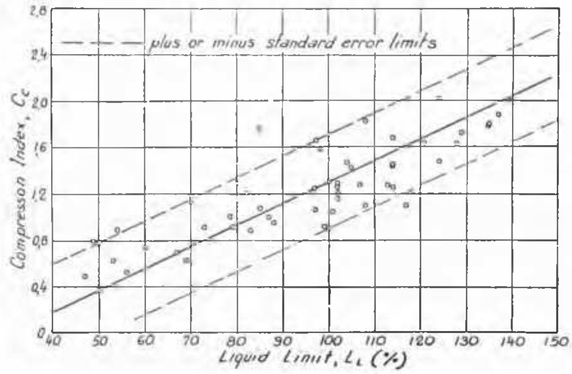

Fig. 5 Regression of the compression index C_c upon the liquid limit L_L for the silty clay of the lowlands of Santos.
Relation entre l'indice de compressibilité C_c et la limite de liquidité L_L pour l'argile silteuse de la baie de Santos.

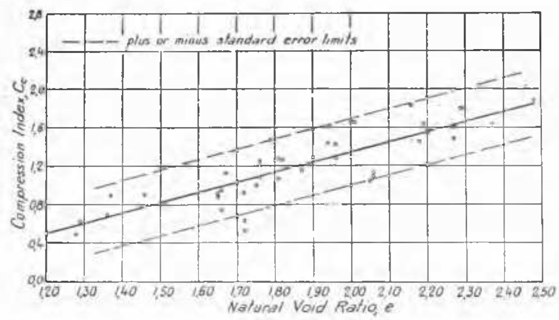

Fig. 6 Regression of the compression index C_c upon the natural void ratio e for the silty clay of lowlands of Santos.
Relation entre l'indice de compressibilité C_c et l'indice des vides naturel e, pour l'argile silteuse de la baie de Santos.

The fact that the compression index can be forecast as a linear function of the liquid limit, as well as a linear function of the natural void ratio, leads to the hypothesis that the most adequate mathematical pattern for compression index forecasting would be a "Multiple Regression" between C_c, L_L and e. This would be possible, since the liquid limit L_L and the natural void ratio e evaluate different soil properties. In fact, while the liquid limit depends on the plasticity of a remoulded sample of soil, the natural void ratio depends on the consistency of soil, and on its natural structure.

By using "Variance Analysis", it was possible to verify the effect of introducing a new term as a function of the natural void ratio in the regression of C_c on L_L. It was proved that the term as a function of the natural void ratio introduced in the regression of C_c upon L_L (A_1 and B_1) was highly significant; that is : with a probability greater than 99 per cent, the compression index of a soil depends on its natural void ratio.

As a final conclusion, it can be stated that the formulae A_1 and B_1 (or A_2 and B_2) in spite of being correct, must be regarded as incomplete.

These established formulae enable the compression index to be computed, the calculations being based on the liquid limit and the natural void ratio :

A. Motley clays from São Paulo :

$$C_c = 0{\cdot}256 + 0{\cdot}00106\,(L_L - 65) + 0{\cdot}32\,(e - 0{\cdot}84) \pm 0{\cdot}063 \quad \ldots. \quad (A)$$

Table 1

Comparaison between several "computed compression index" with their actual values

A. — Clays and silty clays from São Paulo City

Results of Soil tests					Computed Compression Index		
Sample	Atterberg Limits L_L per cent	Atterberg Limits P_L per cent	Compression Index C_c	Natural void ratio e	From L_L (A_1)	From e (A_2)	From L_L and e (A)
2	92	60	0·42	1·17	0·38	0·40	0·39
4	89	47	0·35	1·15	0·37	0·39	0·38
9	73	41	0·23	0·84	0·29	0·26	0·26
14	53	22	0·21	0·71	0·20	0·20	0·20
17	38	18	0·15	0·64	0·13	0·17	0·16

B. — Soft silty clays from lowlands of Santos

Results of Soil tests					Computed compression Index		
Sample	Atterberg Limits L_L per cent	Atterberg Limits P_L per cent	Compression Index C_c	Natural void ratio e	From L_L (B_1)	From e (B_2)	From L_L and e (B)
5	78	36	1·00	2·00	0·88	1·36	1·16
7	102	38	1·30	2·00	1·34	1·36	1·33
10	128	53	1·63	2·37	1·82	1·75	1·71
24	60	31	0·74	1·66	0·56	0·99	0·85
31	114	43	1·46	2·18	1·56	1·54	1·51
36	137	48	1·88	2·48	1·99	1·85	1·84
43	105	40	1·43	1·96	1·40	1·30	1·33
47	85	32	1·08	1·76	1·02	1·09	1·08

B. Soft silty clays from the lowlands of Santos :

$$C_c = 1.21 + 0{\cdot}0072\,(L_L - 95) + 0.53\,(e - 1.87) \pm 0{\cdot}32 \qquad \text{(B)}$$

For a comparison between several computed compression indices obtained when using the formulae A_1, A_2 and A (or B_1, B_2 and B) with their actual values, determined in Soil Laboratory Table I should be consulted.

The results of computation of compression index presented in Table I show that the improvement obtained using the formula A (or B) instead of formulae A_1 and A_2 (or B_1 and B_2) is not very significant in practice. A way of verifying this fact is to observe that the reduction in the "standard error limits" when C_c is computed from (L_L, e) instead from (L_L) only is approximately 10 per cent.

So, although it can be statistically proved that the compression index of a soil depends not only on its liquid limit but also on its natural void ratio, the question that remains is to know whether the increase in work resulting from the use of formulae of type A or B is compensated by the improvement thereby obtained.

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References

[1] Vargas, M. and Bernardo, G. (october 1945). Contribution to the Study of Soil of São Paulo City, *Revista Politécnica.*

[2] Machado, J. Research Report on Settlement of Buildings in Santos (unpublished).