Ground Water Movement in a Sand Dyke Subject to Tidal influence

Comportement de l'eau à l'intérieur d'une digue en sable soumise à l'influence de la marée

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Summary

In a continuous sand stratum exposed to the cyclic rise and fall of the tide in a coastal region, gradients are set up accompanied by movement of the water. The flow occurs as a distorted reflection of tidal variations, the nature and extent of the distortion depending on the permeability of the sand.

If an impervious cover is used for slope protection against wave action, the time lag may, during a falling tide, allow the phreatic table inside the cover to stand at a higher level than the free water outside. Excess upward pressure may cause failure of the cover. The technical solution demands that the extent of the upward pressure be known; in other words both the time lag and the attenuation in the tidal wave be determined.

Sommaire

Dans une masse de sable, exposée aux variations des marées, les gradients hydrauliques qui se produisent sont accompagnés par l'écoulement de l'eau interstitielle. Ce mouvement de l'eau se manifeste comme une image distordue des variations du niveau des marées, la nature et l'échelle de la distorsion dépendant de la perméabilité du sable.

Dans le cas des talus, protégés contre l'action des vagues, par une couverture imperméable, la nappe aquifère derrière la couverture se trouve à un niveau plus élevé qu'à l'extérieur pendant la marée descendante. La couverture est alors en danger d'être soulevée par la pression de l'eau exercée à sa base.

La solution technique de ce problème demande qu'on connaisse le décalage dans le temps et l'atténuation de l'onde de la marée à l'intérieur du massif.

L'étude présente une méthode de calcul qu'on pourrait appliquer dans un grand nombre de cas pratiques.

Theory

Darcy's formula gives :

$$q = ky\,\frac{\partial y}{\partial x}$$

$$\frac{\partial q}{\partial x} = k\left[\left(\frac{\partial y}{\partial x}\right)^2 + \left(y\,\frac{\partial^2 y}{\partial x^2}\right)\right] \qquad (1)$$

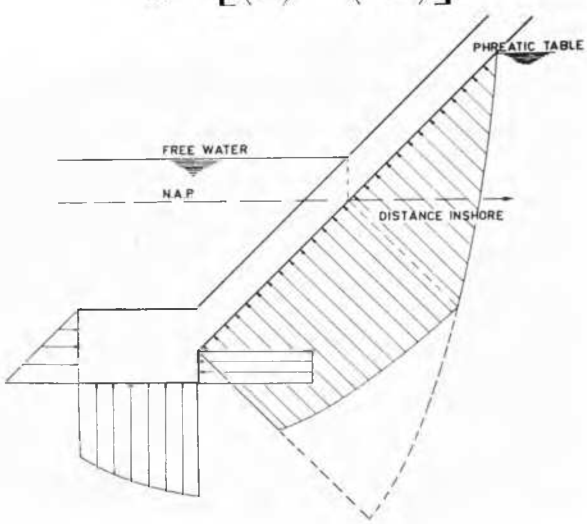

Fig. 1 Forces on bitumen cover at foot of dyke.
Pressions exercées sur la couverture bitumineuse au pied de la digue.

The change in quantity of flow, δq, in the x direction will increase the height of the column at x (see Fig. 2), such that :

$$\delta q = +\,n\left(\frac{\partial y}{\partial t}\right)\delta x,$$

where n is the porosity of the soil.

Then :

$$\frac{\partial q}{\partial x} = +\,n\left(\frac{\partial y}{\partial t}\right) \qquad (2)$$

Combining (1) and (2) gives :

$$\frac{\partial y}{\partial t} = +\,\frac{k}{n}\left[\left(\frac{\partial y}{\partial x}\right)^2 + y\,\frac{\partial^2 y}{\partial x^2}\right]$$

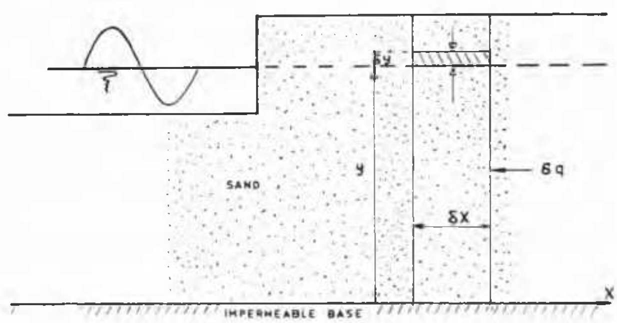

Fig. 2 Change in flow.

We may follow Boussinesq in assuming that :

$$\left(\frac{\partial y}{\partial x}\right)^2 \ll y \frac{\partial^2 y}{\partial x^2},$$

whence

$$\frac{\partial y}{\partial t} = \frac{ky}{n} \frac{\partial^2 y}{\partial x^2}$$

If further we assume that y remains constant, $= h$,

$$\frac{\partial y}{\partial t} = \frac{kh}{n} \frac{\partial^2 y}{\partial x^2} = K \frac{\partial^2 y}{\partial x^2} \qquad (3)$$

Integration of Equation (3).

Since the response must be periodic, we seek a solution of the from :

$$y = u \exp\left[i(\omega t - \theta)\right]$$

where :

$$u = u(x) \text{ only.}$$

This solution will have a period of $\frac{2\pi}{\omega}$. Substituting it into the differential equation (3) gives :

$$\frac{d^2 u}{\partial x^2} = \frac{i\omega}{K} \cdot u \qquad (3)$$

where :

$$K = \frac{kh}{n}$$

The solution of (4) which is finite as $x \to \infty$ is :

$$u = A \exp\left(-x\sqrt{\frac{i\omega}{K}}\right)$$

$$= A \exp\left[-x(1+i)\sqrt{\frac{\omega}{2K}}\right]$$

Thus the solutions of (3) of period $\frac{2\pi}{\omega}$ are :

$$y = A \exp\left(-x\sqrt{\frac{\omega}{2K}}\right) \cos\left[\omega t - \theta - x\sqrt{\frac{\omega}{2K}}\right]$$

and

$$y = A \exp\left(-x\sqrt{\frac{\omega}{2K}}\right) \sin\left[\omega t - \theta - x\sqrt{\frac{\omega}{2K}}\right]$$

The one of these solutions which has the value $A \cos(\omega t - \theta)$ at $x = 0$ is then :

$$y = A \exp\left(-x\sqrt{\frac{\omega}{2K}}\right) \cos\left[\omega t - \theta - x\sqrt{\frac{\omega}{2K}}\right] \qquad \ldots\ldots (5)$$

This derivation, which follows that of Lamb (Hydrodynamics, Section 345) leads to the following conclusions :

I. The amplitude of the tidal oscillations diminishes as

$$\exp\left(-x\sqrt{\frac{\omega}{2K}}\right)$$

and is then attenuated more rapidly for higher frequencies. If the original tidal wave is given by a Fourier series, the higher harmonics disappear more rapidly than the lower harmonics.

II. There is a progressive time lag,

$$x\sqrt{\frac{\omega}{2K}}$$

in the phase of the tidal wave. This lag increases with ω.

III. The tidal wave is propagated in the soil with a celerity

$$\sqrt{2K\omega}$$

It thus appears that if K is known, ω being $\frac{2\pi}{12{\cdot}5}$ hrs^{-1}, the amplitude and celerity of the wave may be determined at any point in distance and time.

Summary of data taken from dyke at Veeregat

1. Boreholes sunk on the site to a depth of 20 m showed continuous sand layers with a few isolated lenses of clay, the latter being too isolated to influence the fluid flow to any appreciable extent.

2. Pore pressure measurements were taken at various points across the dyke, with concentration of measurements at the seaward foot of the dyke. The readings taken at the foot were corrected for temperature and barometric pressure variations.

Analysis of test results, determination of constant K

I. *Propagation of ground water inshore :*

1. *Body of dyke*—Test results give the time taken for the crest of the wave to travel from a free water surface (about station 18) to stations 15 and 16 (Fig. 3).

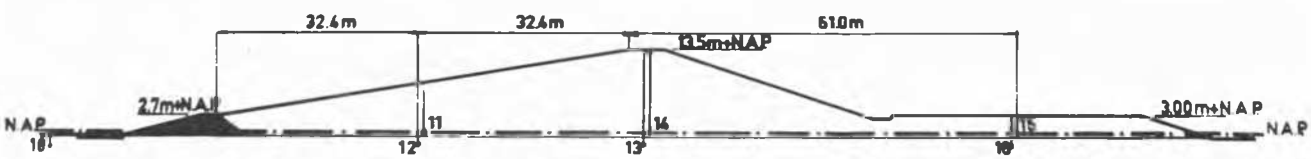

Fig. 3 Cross section at Veeregatdam, with numbers of stations.
Profil de la digue du Veeregat, avec les points de mesure.

The velocity of propagation or celerity, c, of the wave varied from 5·6 m/hour to 17·5 m/hour, this representing an increase in K, given by $K = \frac{c^2}{2\omega}$, from $K = 32$ to $K = 306$. The mean velocity of 9·2 m/hour gives $K = 85$.

2. *Foot of dyke*—The progress of five different waves from free water to station 10 (see Fig. 4) with a celerity variation from 10·8 m/hour to 7·2 m/hour, giving $K = 90$ for an average celerity of 9·5 m/hour. This is the K value used in calculations for the 3 m — tide synthetic phreatic table.

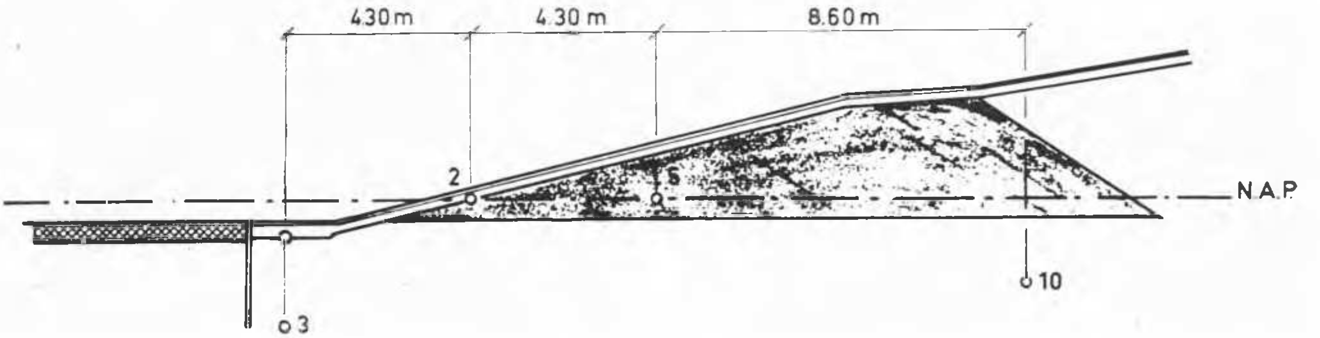

Fig. 4 Detail of foot of dyke.
Détail du pied de la digue.

II. *Attenuation of ground water wave inshore :*

1. *Body of dyke*—Any investigation of the wave attenuation is complicated by the fact that only a half wave initially enters into the system. The negative part of the wave is cut off at 20 cm + N.A.P. at which level lays the sand-bank, this part of the dyke is built on. Thus the amplitudes measured are formed of the envelope of maximum phreatic table levels and some other line representing the level of the table just before the next input. Fig. 5 represents this situation.

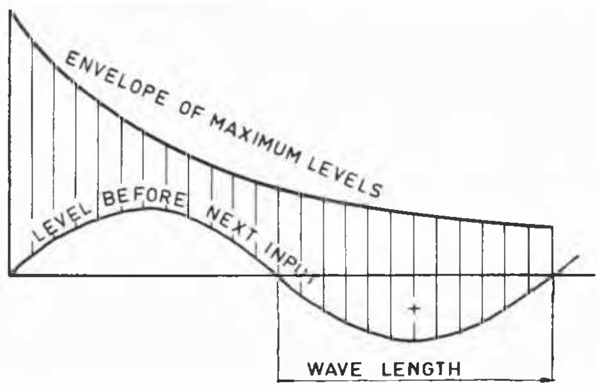

Fig. 5 Attenuation of ground water wave.
Amortissement des vagues de fond.

Although the process of superposition gives a wave form of the type observed, it is very difficult to estimate the shape of the lower line to a sufficient accuracy for calculation. On the other hand the accuracy of the observations does not warrant a calculation based upon an average water level phreatic table, which can be calculated from theory. Although it seems possible that a value of K might be calculated from this information, such a calculation did not seem justified at this stage.

2. *Foot of dyke*—The results obtained at the foot of the dyke were considerably more comprehensive and accurate than those obtained for the body of the dyke; and therefore a trial calculation of K was made. For this purpose Fig. 6 was drawn, whereby the position of the preatic table at high water and N.A.P. + 0·20 could be determined.

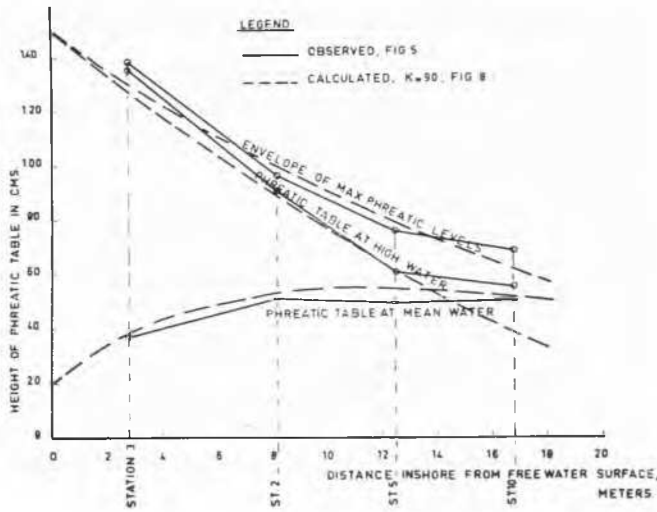

Fig. 6 Comparison of observed results with those calculated theoretically.
Comparaison des résultats des mesures avec ceux obtenus par le calcul.

Using a K-value of 90 the theoretical phreatic tables were calculated and drawn on the same diagram for comparison with a practical case (Fig. 7, where a tidal curve of almost sinusoidal form and of 1·5 meters effective half amplitude obtained).

The method of calculation used is set out in the relevant section below.

The results obtained in Fig. 6 compare favourably with those calculated for $K = 90$, sufficiently for a calculation based on this value to appear justified.

III. *Permeability of sand mass :*

If the value of $K = 90$, derived above is substituted in the expression :

$$K = \frac{k \cdot h}{\mu},$$

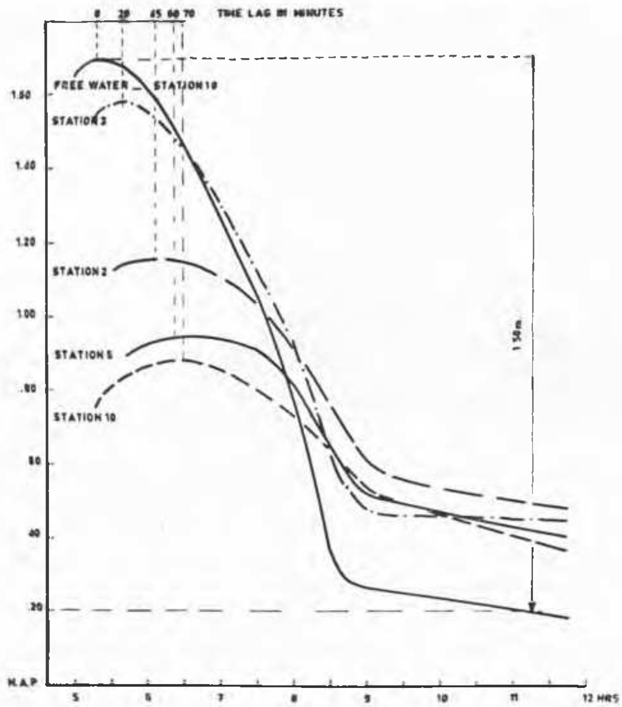

Fig. 7 Observed results at Veeregatdam.
Résultats des mesures dans la digue du Veeregat.

taking h as 48 metres (site observation)
μ as 50 per cent,

then

$$k = \frac{90 \cdot 0{\cdot}5}{48} \text{ metres/hour}$$

$$= \frac{15}{16} \cdot \frac{100}{3{\cdot}600} = \frac{100}{32{\cdot}120} = 0{\cdot}026 \text{ cm/sec.}$$

Results from laboratory tests also gave $K = 0{\cdot}0258$ cm/sec.

IV. *Conclusion :*

The value of $K = 90$ appeared to give a satisfactory agreement with observations and was accordingly adopted.

Effect of capillarity

Fig. 8 shows how the percentage saturation of sand may vary with height above the phreatic table. Consider the effect of "injecting" an additionnal quantity of water of volume V into this system. The height h represents the capillary head before injection. If the water is injected in such a way as to fill the volume V' (Fig. 8) the instantaneous rise in the phreatic table would be h'. However, the forces which created the original distribution would again come into effect, so as to distribute V upwards into the sand mass.

Thus ultimately the water would fill volume V'' when the phreatic table would have risen h'', an amount much smaller than h'. In practice, with considerable variation in the phreatic levels, some intermediate effect may be expected.

From the shape of the curve of Fig. 8 it may be said, qualitatively, that although a small quantity of water rising into the soil may induce a large increase in the level of the phreatic table initially, further quantities will generate commensurably smaller increases (Fig. 9).

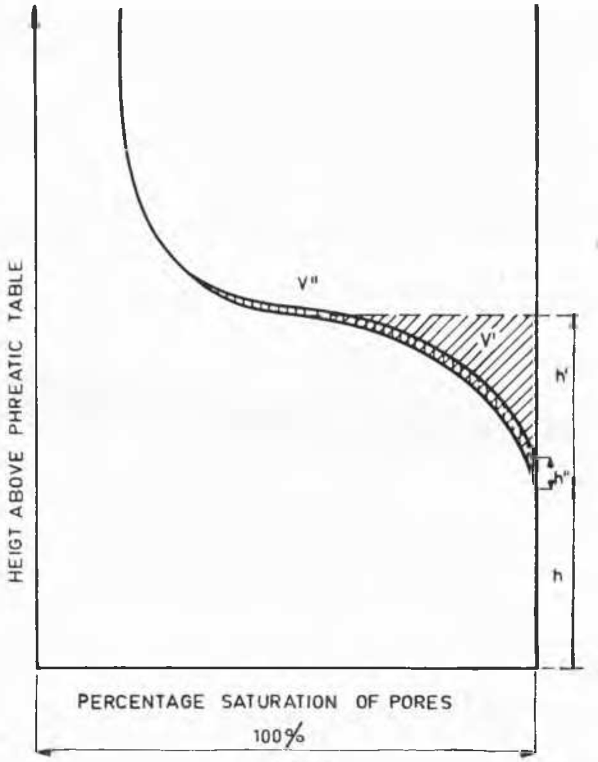

Fig. 8

It thus seems possible to account for the effects of capillarity by adding a constant height, h', to the level calculated without capillary effects. Investigations carried out independently of this report suggest that h' is always less than 10 cms for the site sand, and this value was accordingly taken.

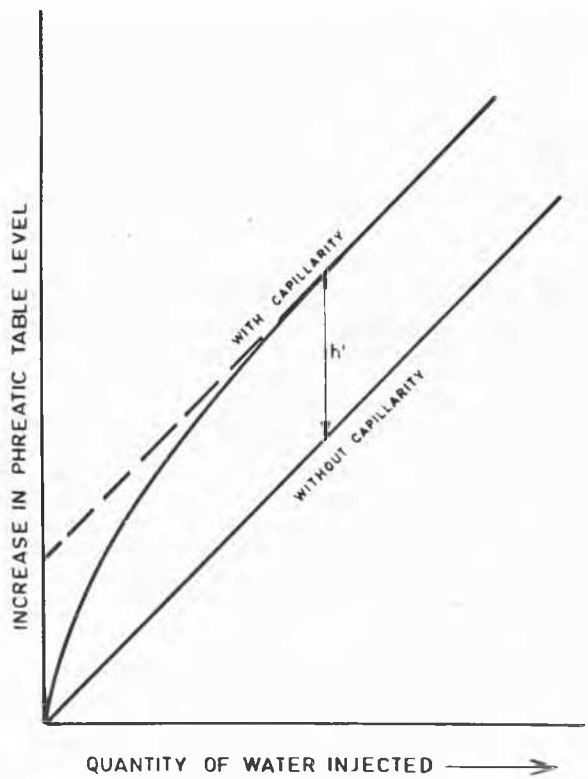

Fig. 9

Calculations

Synthetic phreatic table — 3 m tide :

With $K = 90$.

Amplitude

$$= v\left[\exp\left(-x\sqrt{\frac{\omega}{2k}}\right)\right] = 3{\cdot}0\left[\exp\left(-x\sqrt{\frac{2\pi}{2{\cdot}12,\ 5{\cdot}90}}\right)\right]$$

To this may be added 10 cm for capillary rise (see effect of capillarity).

$$\text{Celerity} = \sqrt{2k\omega}\,, = \sqrt{2{\cdot}90\cdot\frac{2\pi}{12{,}5}}$$

$$\text{Time lag} = x\sqrt{\frac{\omega}{2k}} = x\sqrt{\frac{2\pi}{2{\cdot}12,\ 5{\cdot}90}}$$

These give:

Amplitude $= 3\exp(-0{\cdot}0528\,x)$ meters
Celerity $= 9{\cdot}5$ m/hour
Lag $= 0{\cdot}0528\,x$

These results are plotted in Fig. 10, giving the profiles of the phreatic table across the dyke for a distance of 30 m inshore. (It was from these results that Fig. 6 was determined).

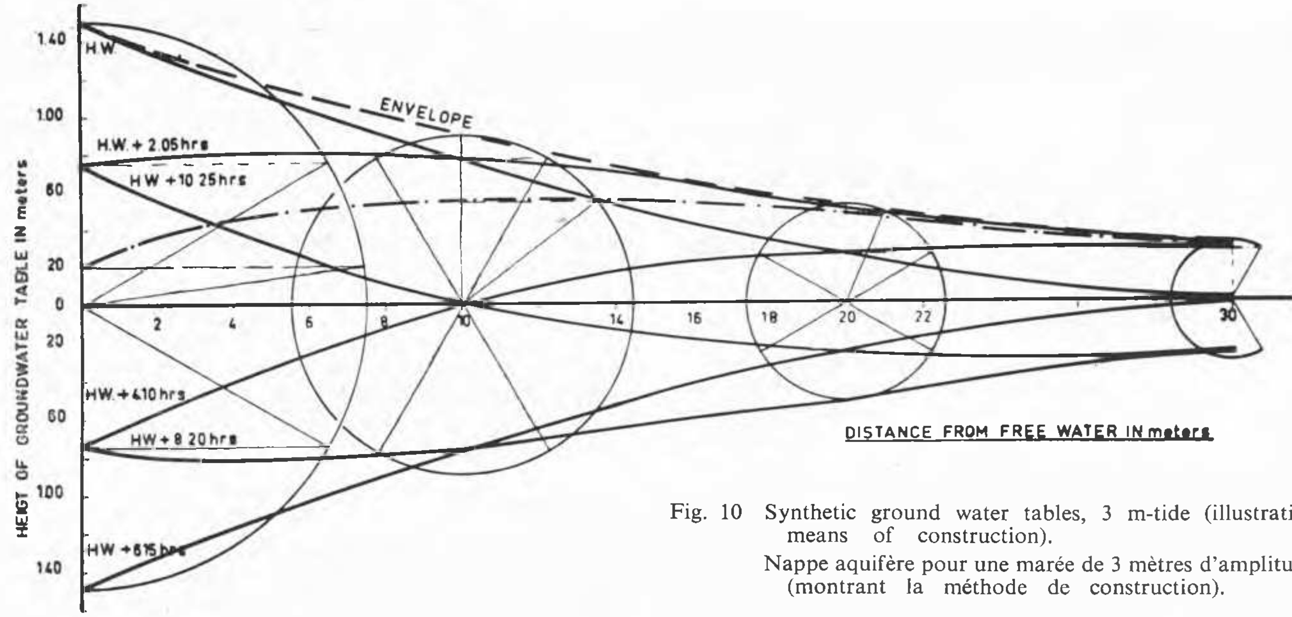

Fig. 10 Synthetic ground water tables, 3 m-tide (illustrating means of construction).
Nappe aquifère pour une marée de 3 mètres d'amplitude (montrant la méthode de construction).

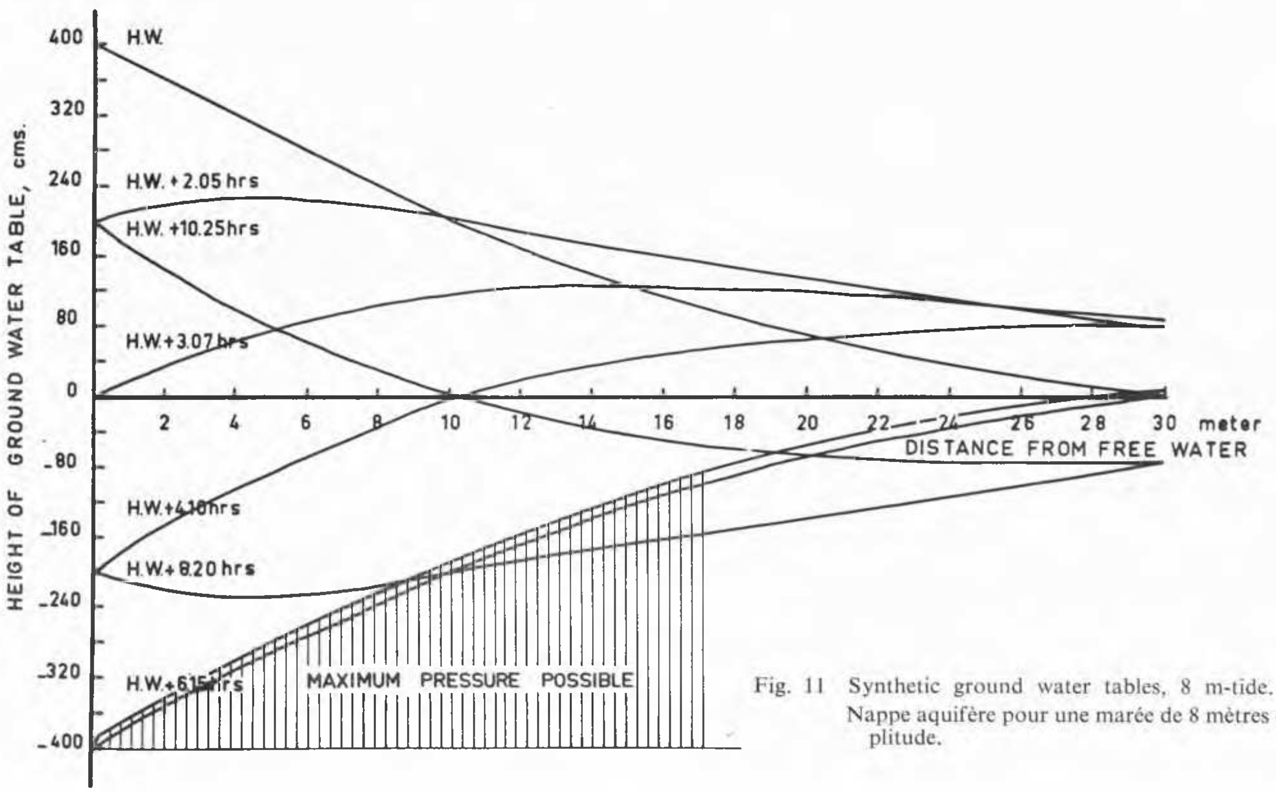

Fig. 11 Synthetic ground water tables, 8 m-tide.
Nappe aquifère pour une marée de 8 mètres d'amplitude.

The phreatic table profile is a measure of the pressure obtaining in the saturated zone beneath the table, and this fact has been utilised in order to construct a pressure at mean water + 0·20. It is suggested that this should be checked in practice with pore pressure meters sealed under a bitumen cover.

Synthetic Phreatic Table, 8 *m tide :*

$$\text{Amplitude} = 8{\cdot}0 \exp\left(-x\sqrt{\frac{2\pi}{2{\cdot}12,\ 5{\cdot}90}}\right) = 8{\cdot}0 \quad \exp$$

(—0·0528 x) meters. Celerity and time lag as before.

This has been plotted in Fig. 11, from which the pressure distribution behind a bitumen cover can be calculated at any stage of the tide effective on the dyke and the foot of the dyke below low water level.

Calculations

The method developed above is open to two main objections, Namely :

1. Differences between the highly idealised mathematical model and the real dyke.

2. The effects of capillarity.

1. In Fig. 12 the mathematical model is compared with the actual dyke. It will be seen that the "geometry" of the two cases differs considerably, so that in *B* a certain part of the tidal wave is entirely cut off. Thus in region "*b*" the process of upward flow cannot occur and the theory is accordingly invalidated. If, however, the region "*b*" is small compared with the length of the tidal wave, the error involved will not be so great, for then the inner flow process will compensate for the flow obstruction under the impervious layer.

2. Above the phreatic table there may exist a saturated capillary zone. In calculations on seepage flow in semi-impervious soils (e.g. clays), overlying impervious beds at shallow depth, this zone may be of considerable importance for seepage

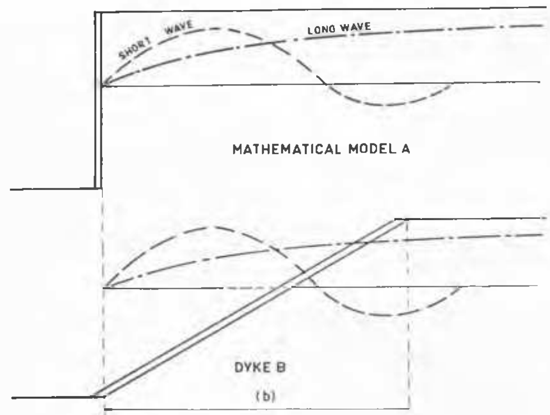

Fig. 12 Comparison of the mathematical model with the actual dyke.
Comparaison du modèle mathématique avec la digue réelle.

calculation. In sand however, especially of the depths such as obtained in and under the Delta dykes, this aspect of capillary action is hardly significant.

Another aspect of capillarity which does enter this study concerns the "magnification" of phreatic level rise due to the low quantity of unsaturated voids available in the soil to absorb upward flow. Allowance can be made for this by the addition of a constant height (in this case 10 cm) to the calculated phreatic level (see Fig. 9). This appears to cover adequately those capillary effects which do occur.

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