Research on the Texture of Granular Masses

Étude sur la texture des substances granulées

by Torsten KALLSTENIUS, Techn. Lic., Head of Mechanical Department
and
Werner BERGAU, Dr. Ing., Head of Measuring Section, Swedish Geotechnical Institute

Summary

The authors consider that the nature of granular masses is not properly taken into account when calculating their mechanical behaviour; as part of a wider research programme, they have investigated the texture of masses of glass balls.

This texture was studied layer by layer, applying a tetrahedral lattice based on the spacing of the balls, from which the shapes of the tetrahedra were evaluated. The balls were allowed to fall freely under the action of gravity and the base plane was horizontal. Both theory and practical tests prove that there is a tendency for the balls to arrange themselves in "chains", and the number of balls per horizontal unit area is found to be nearly constant in all layers. If this number is deliberately changed in one layer, it will be repeated again after a few more layers.

It was also discovered that the mass is anisotropic to the extent that the height of each tetrahedron is less than required for isotropy. Close to a plane boundary surface the balls arrange themselves in a definite manner for six or ten layers thereby producing a "wall effect". This must be taken into account in laboratory tests.

Sommaire

Dans la détermination habituelle du comportement mécanique des substances granulées, on n'accorde pas, suivant l'opinion des auteurs, une attention suffisante à la nature de ces substances. C'est pourquoi, dans le cadre d'un programme de recherches plus vastes, ils ont étudié la texture de masses de billes de verre.

Cette texture a été étudiée couche par couche et rapportée à une structure tétraedrique basée sur les centres des billes. La forme de ces tétraèdres a été étudiée. On a fait tomber les billes une par une, sous la seule action de la pesanteur. Le fond du récipient était horizontal.

La théorie et les essais montrent, tous deux, que les billes ont tendance à se disposer en « chaînes ». Le nombre des billes par unité de surface horizontale fut trouvé presque constant dans toutes les couches. Si ce nombre est modifié à dessein, dans une couche, on retrouve la valeur normale au bout de quelques couches.

On a trouvé aussi que la masse est anisotrope et que la hauteur de chaque tétraèdre est moindre que celle qui correspondrait à l'isotropie.

Près de la surface limite, les billes s'arrangent en couches plus nettes sur une épaisseur de 6 à 10 couches, ce qui produit un « effet de mur » dont on doit tenir compte dans les essais de laboratoire.

Introduction

In connection with soil pressure cell calibrations (S.G.I. Proc. N°. 12, 1956), the authors felt that there was a need to explain the mechanical behaviour of granular masses with more attention being paid to their true nature than is usually the case. Mr. Justus Osterman, Director of the Swedish Geotechnical Institute, and the authors have collaborated in an extensive, but yet incomplete, study of these matters (cf. J. Osterman, Some aspects on the properties of granular masses, Svenska Nationalföreningen för Mekanik, Reologisektionen, Meddelande 1, 1959).

It has been common practice, when trying to base calculations on the texture of a granular mass, to assume regular patterns of spheres. It is not possible here to mention all authors, but Bullets "Traité d'Architecture-Pratique" (1691) seems to have been one of the first known. Even a brief consideration indicates, however, that regularity is quite improbable under natural conditions. Nevertheless, masses built up from grains of similar size and approximately spherical shape may be found in nature. Experiments with lead shot by W.O. Smith, P.D. Foote and P.F. Busang (Physical Review, Vol. 34, 1929) may be mentioned here. The authors started with a study of the texture of a mass built up of approximately equally large spheres, under conditions similar to slow sedimentation.

They considered a plane horizontal surface with spherical grains falling freely on to that surface. As far as the grains can hit the surface directly, without touching other grains, distances between grains can be assumed to depend entirely on chance. On the other hand, if they touch grains they will be guided sideways and assume positions where interaction between grains plays an important role.

When so many grains have reached the base plane that no other grain can reach this, the base layer is complete. The grain pattern in this layer can be described if triangles are formed between the centres of the grains. Triangles can be formed more or less at will, but there must be a restrictive regulation to the effect that the triangles shall be formed between adjacent grains in the way which best avoids angles greather than 90°. The shortest diagonal between four adjacent grains was therefore chosen. This is based on the consideration that the grains forming the next layer will normally rest on three base-grains each, but cannot find a stable base if the base-grains form a triangle containing an angle greater than 90°.

If the grains have a radius "r" we can call the sides of the base triangles $a{\cdot}r$, $b{\cdot}r$ and $c{\cdot}r$ where "a" always indicates the longest side and "c" the shortest side.

If a triangle forms the base for a fourth grain, the position of this grain is geometrically determinad as its centre will be situated at the distance 2 r from each of the base grain centres. The distance $h{\cdot}r$ from the base triangle can therefore be calculated.

Fig. 1 shows an elementary tetrahedron as determined

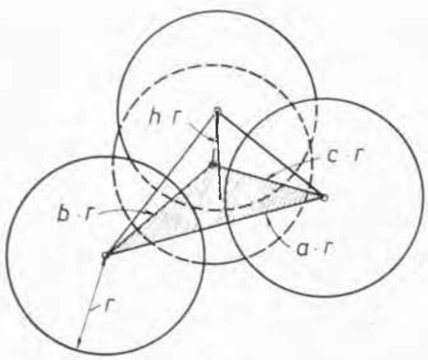

Fig. 1 Elementary Tetrahedron.
Tétraèdre élémentaire.

by coefficients a, b, c and h. The tetrahedra formed by the base layer and the layer immediately above this are called base-tetrahedra. The statistical variation of the dimensions of these base-tetrahedra has been studied by a graphical method. In the granular mass, similar but not identical tetrahedra must exist and the authors have studied them empirically.

Graphical Method

It is not practical to use probability calculations for the texture of the base layer of grains, as the aforementioned interaction effects are complicated. Therefore a combination of graphical solution and calculations has been chosen in order to reach a theoretical understanding.

The interaction of grains may be of two different kinds as illustrated in Fig. 2. Simple interaction occurs when one grain hits another grain and is rolled off in a plane through both grain centres, until both grains remain on the base plane in mutual contact. The distance between the grains is then $2r$, which will determine leg "c" of a base triangle. Double interaction occurs when one grain first hits another grain but is then guided by a third grain to a final position in contact with both grains. Here a triangle is formed where $c = b = 2$. The two types of interaction may combine in chains.

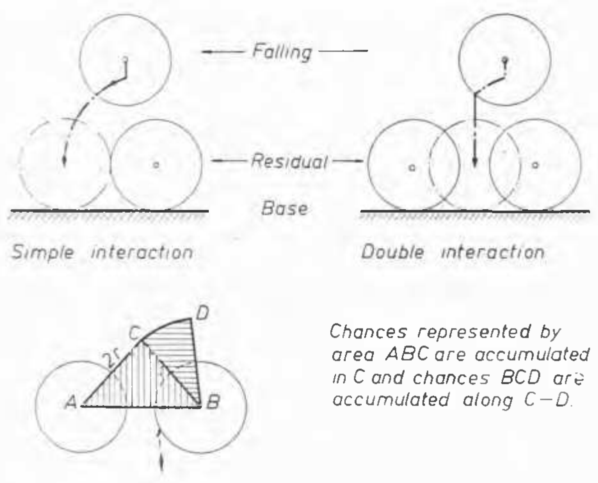

Fig. 2 Interaction between Grains.
Action mutuelle entre les grains.

If we assume the probability to be equal for a grain to reach any part of the base surface, the interaction excepted, we can study the frequency of possible base triangles by dividing the base surface into unit areas and solving the problem graphically. We have chosen a network with a spacing of 0·1 r. Thus our unit area (which represents one chance in our system) is 0·01 r^2.

For each possible unit value of a we studied all possible combinations of b and c but had to consider the following limitations :

(1) $b \leqq a$ due to definition.

(2) $c \leqq b$ due to definition.

(3) $2 \leqq a, b, c$ as no grains can come closer than contact.

(4) Due to definition a fourth grain must not be able to pass the base triangle.

(5) Cases where an angle of the base triangle exceeds 90° cannot be used. Here one must reckon with the chance that a fourth grain will be rolled off to the opposite side of line "a", permitting the formation of two new triangles a_1, b_1, c_1 and a_2, b_2, c_2 (here called redivision).

(6) Due to symmetry only half of the triangles need be checked.

The above limitations are shown graphically in Fig. 3. Fig. 4 shows the basic graphical procedure to determine the number of chances for different combinations of "b" and "c" for the arbitrate a-values 3·6 and 2·6. As stated above triangles with an angle exceeding 90° are not included here. Even if for high a-values the number of chances giving such triangles is comparatively high they are not so frequent in the complete network of the base layer. The reason lies in the aforementioned procedure to select the diagonal between four adjacent grains which produces the minimum possible number of triangles with angles exceeding 90°.

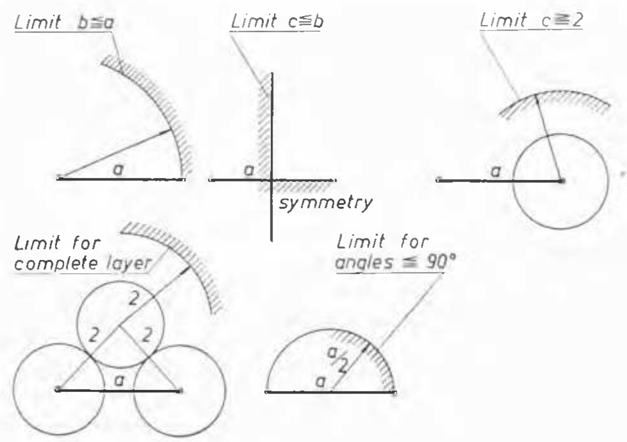

Fig. 3 Limiting Values. (Graphical Evaluation.)
Valeurs limites. (Évaluation graphique.)

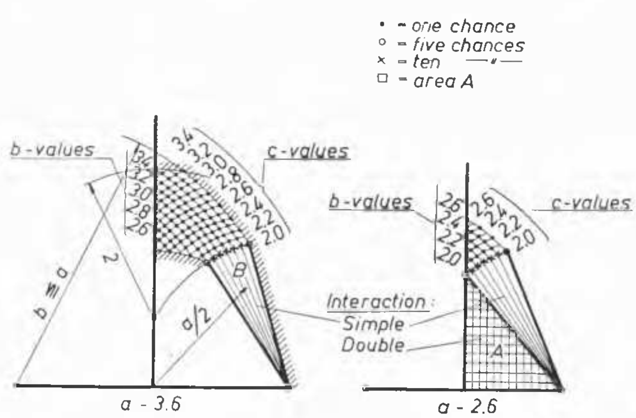

Fig. 4 Determination of Direct and Accumulated Chances.
Détermination de probabilités simples et cumulées.

Fig. 5a illustrates how two triangles ABC and ABD on the common base "a" are redivided by diagonal CD to two other triangles ACD and CBD. As far as D is situated within the area ABE_1, the new triangles contain no angle exceeding 90°. These chances have then already been counted for smaller a-values and nothing new is introduced concerning the base pattern. If D has hit surfaces AD_2D_3 or $BD_6D_5D_4$ we obtain one triangle with all angles smaller than 90° (which has already been counted), and one triangle with an angle exceeding 90° for each unit area. The latter triangles cannot form bases for tetrahedra but will exist in the base pattern between the tetrahedra. The probability for angles exceeding 90° is governed by the ratio between the areas giving such triangles and the total possible area below AB. An important requirement is that no diagonal should exceed "a".

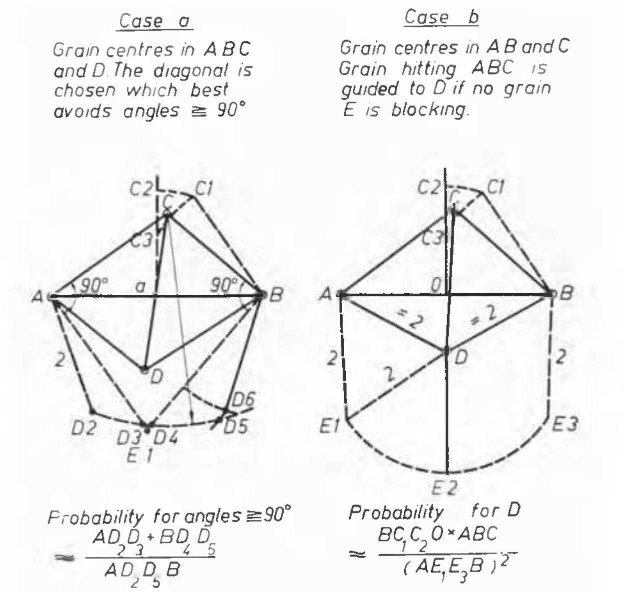

Fig. 5 Redivision of Base Triangles.
Division des triangles de base.

A special effect is demonstrated in Fig. 5b. If a grain hits surface ABC and no grain is present in $AE_1E_2E_3B$, it will be guided by double interaction to a position D. However, it is necessary that grain C reaches its position before a grain hits $AE_1E_2E_3B$ and that a grain then hits surface ABC before a grain hits $AE_1E_2E_3B$, which means two consecutive events and a corresponding small probability. The process means more probability for the formation of triangles ACD and CBD.

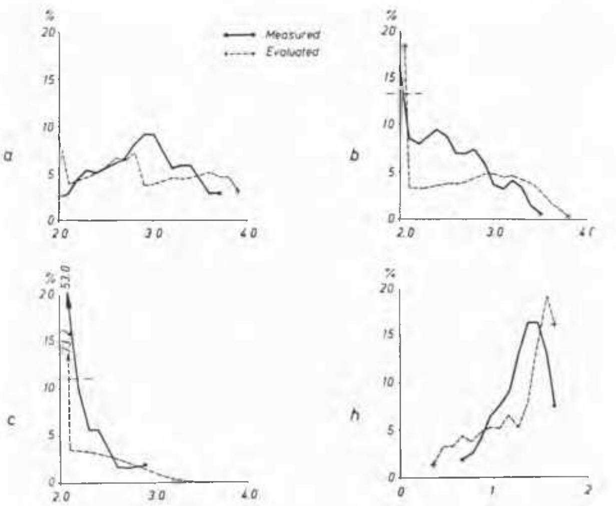

Fig. 6 Frequency Distribution
a b c and *h*
Distribution de fréquence
a b c et *h*

Certain special cases with small probability have not been described here although they were considered in our studies.

Frequencies for different combinations of a, b, c and h and their average values have thus been calculated from the individual triangles. In Fig. 6 are given the frequency distributions for the base layer and the base tetrahedra together with measured values from layers 5 and 7 in test B which will be described below.

In Table 1 we have given the frequencies of different constellations of grains in the base layer.

Table 1

a-value	Frequency % Constellations of grains in triangle			*Reflexion*	*Angles 90°*
	All three free	*One free, two in contact*	*One in contact with two*		
3·9	1·8	1·3	—	—	0·9
3·8	2·7	1·9	—	—	1·1
3·7	2·8	1·8	—	—	1·2
3·6	3·0	2·1	—	—	1·0
3·5	2·8	2·1	—	—	0·9
3·4	2·5	2·1	—	—	0·8
3·3	2·3	2·1	—	—	0·8
3·2	2·0	2·5	—	—	0·6
3·1	1·7	2·5	—	—	0·4
3·0	1·5	2·5	—	—	0·2
2·9	1·2	2·5	—	0·1	0·1
2·8	0·9	2·6	3·8	0·5	—
2·7	0·7	2·3	3·4	0·3	—
2·6	0·5	2·4	3·7	0·8	—
2·5	0·3	1·9	3·5	0·7	—
2·4	0·2	1·3	3·4	0·6	—
2·3	0·1	1·1	3·4	0·6	—
2·2	—	0·7	3·6	0·7	—
2·1	—	0·2	3·6	0·7	—
2·0	—	0·1	8·7	1·8	—
Sum	27·0%	35·9%	37·1%	6·8%	8·0%

There is a great probability of grains coming in contact with each other. The constellation "one in contact with two" means a tendency to form chains. Fig. 7 shows a base layer formed under conditions similar to the assumed and one can there directly see the tendencies indicated in Table 1. As triangles with an angle exceeding 90° cannot form bases for tetrahedra and only about every second triangle of the base layer can be included in a base-tetrahedron, the frequencies for different values of a, b and c will differ in the interspaces from the values given in Fig. 6.

The total number of chances included in this study was about 3100.

Tests

Tests were performed to study the building up of a granular mass, layer by layer.

A glass plate with a layer of glue formed the horizontal base plane. Glass balls with an average diameter of 5·9 mm were dropped carefully on the surface.

Two series of tests, A and B, were performed. Every second

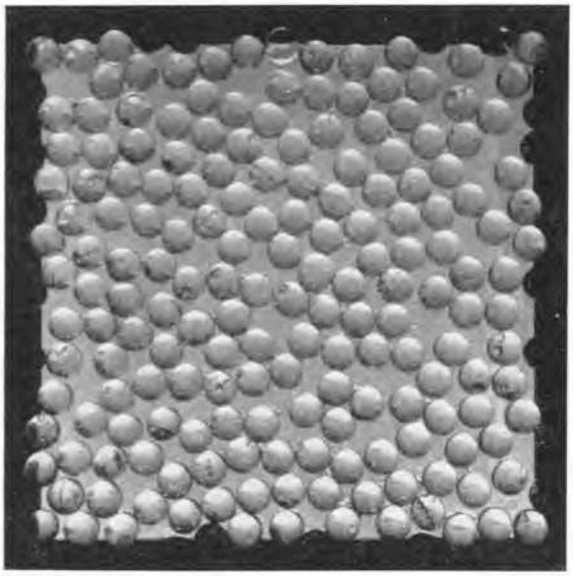

Fig. 7 Base Layer, Test A.
Couche de base, essai A.

layer was uncoloured and every second layer blackened. For each layer the number of balls within a standard area 82 × 82 mm were counted, and the distance of the ball tops to the base plate were measured.

In test *A* the base layer — formed by chance — is shown in Fig. 7. After six layers, balls were poured carefully over the layers to a height of 135 mm above the base plate and two consecutive layers ($n + 1$ and $n + 2$) were counted.

In test *B* the base layer was purposely arranged in the densest possible regular order and seven consecutive layers were counted and measured. The results of the counting are shown in Table 2.

Table 2

Layer	*Number of balls within standard area*	
	Test A	*Test B*
1	180	232
2	184	169
3	176	172
4	168	168
5	183	182
6	184	181
7	—	186
$n + 1$	176	
$n + 2$	185	

We note the nearly equal number of balls for different layers of Test *A* and how the extremely high density of layer 1 in test *B* has been counteracted by an extra low density in layer 2. After a few layers the numbers counted in test *B* are the same as in test *A*. The influence of chance is constant for the given conditions of formation.

For layers 5 and 7 in test *B* the triangles formed by the ball (grain) centres were measured. The statistical distribution of the measured values *a*, *b* and *c* and the corresponding calculated *h*-values are given in Fig. 6. A comparison with the results of the graphical evaluation of the base layer shows an evident similarity even if the concentration towards small values due to interaction is not as obvious in the test. This should be so, as in layers above the base every grain will be guided by the base grains to a position differing slightly from that selected if the base were a plane surface.

Fig. 8 shows the distribution of grain centre positions above the base surface. The influence of the limiting plane surface has a far-reaching effect and indicates a "wall effect" on the texture of the mass. At a greater distance from the surface the layers begin to mix.

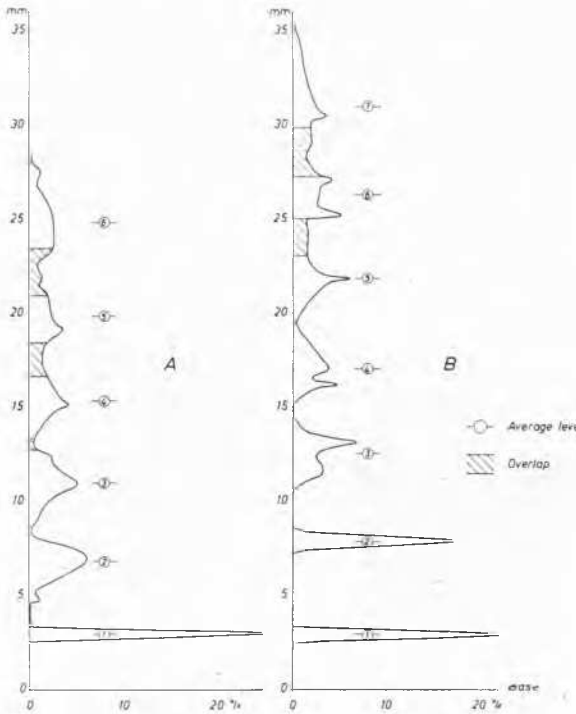

Fig. 8 Distribution of ball centres.
Distributions des centres des billes.

Discussion

The statistical distribution of *a*, *b* and *c*-values and the corresponding *h*-values of the tetrahedra in the mass can be expressed in approximate mathematical shape but at present the authors prefer to describe the texture by average values. Considering the frequencies of different values *a*, *b* and *c* they arrived at the mean values given in Table 3.

Table 3

Shape of average tetrahedron	a_a	b_a	c_a	h_a
Base tetrahedra, graphical evaluation	2·93	2·59	2·18	1·30
Layers 5 — 7, Test B	2·86	2·53	2·12	1·35

The h_a-values in the table were calculates to suit a_a, b_a and c_a. For a graphical evaluation, the authors have calculated the void ratio "*e*" of the average tetrahedron and found it to be $e = 0{\cdot}51$. This was compared with a calculation for all individual tetrahedra, which gave $e = 0{\cdot}50$. It thus seems possible to use the average-tetrahedra for void ratio calculations.

In the course of their calculations, the authors discovered special influences affecting the texture in the mass. As far as the *h*-values are concerned, each triangle of grains can be in contact with one grain on every side, thus forming a pentahedron. When *h* is smaller than unity, pentahedra are no longer possible which means that the formation of tetrahedra is restricted. h-values smaller than unity must therefore be considered with only half their basic frequency. Therefore the average h-value when calculated for the average values a_a, b_a and c_a is about 0·04 greater than the average h which might be calculated from Fig. 6.

The rather flat shape of the tetrahedra and the fact that the base triangle can only heel little to cause the top grain to roll off (cf. the angle of natural slope as indicated by Fig. 9) leads us to the conclusion that the texture of the mass is un-isotropic (it is not possible to imagine our average tetrahedron standing with its base in a vertical direction). An indication of un-isotropy is the observation in Test A that the number of grains in contact with an area 82 × 82 mm was 179 for a horizontal area but 205 for a vertical area. In this case, however, we were not free from wall effects.

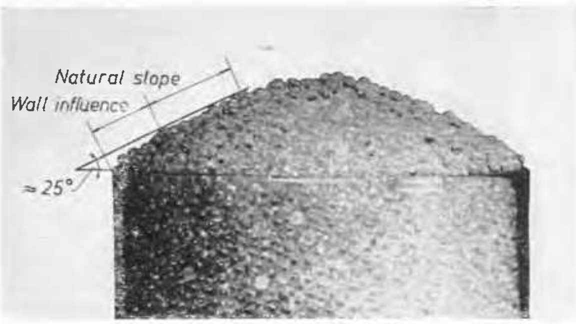

Fig. 9 Angle of natural Slope for Glass Balls.
Angle de talus naturels pour des billes de verre.

This un-isotropic texture may be an explanation of the consolidation-test curve shape. If the mass of tetrahedra is subjected to stress conditions introducing a stress ellipsoid with altered orientation, the structure may stand an alteration only to a limited extent. Above that a breakdown and re-orientation of the structure follows. A direct impression of that is obtained when deforming a mass of billiard balls held together by a net. The authors consider that the orientation of the texture should be allowed for by suitable coefficients when calculations are undertaken.

The authors have studied the influence of wall effects on void ratio determinations. In Test A the average void ratio (including wall disturbances) was found by experiment to be $e = 0{\cdot}61$ in a container measuring 187 × 144 × 130 mm. They determined the void ratios nearest to the limiting surfaces from the measured dimensions of each layer and the counted number of grains. For horizontal surfaces they could check 5 1/2 layers, but for vertical ones only 1 1/2 layers. For the half-layer between the limiting surfaces and the centres of the peripheral grains the void-ratios were very high ($e_h = 1{\cdot}20$; $e_v = 0{\cdot}82$). On the other hand the void ratio in the space between the first and second layers of grain centres was $e = 0{\cdot}41$ for both horizontal and vertical surfaces.

The corrected void ratio for the central mass in Test A was estimated to be $e = 0{\cdot}62$, which is very near the uncorrected value. Figures indicate that the wall-influence may be considerable in relatively smaller vessels (cf. K. Schubert, Einfluss der Versuchszylinderabmessungen auf die lockerste Lagerung rolliger Böden, Zschr. f. Bauwesen, Cottbus, H.2 1957-1958).

The high void ratio in the outermost half-layer is known to affect permeability tests, and the very dense and ordered next layers form a skin which affects compression tests and strength tests (cf. S.G.I. Proc. No. 12 p. 44, Fig. 28).

The obtained void ratio $e = 0.62$ may be compared with the void ratio calculated from the average tetrahedron in layers 5-7, Test B, which was $e = 0{\cdot}60$.

The tetrahedra seem to be guided by the plane limiting surfaces in such a way that the inter-spaces between the tetrahedra (which need not have tetrahedral shape) are smaller on the average than the tetrahedra. In the mass unaffected by wall effects tetrahedra and interspaces are more equal in volume.

Wall effects are noteworthy in several different respects. The influence on texture as indicated by Fig. 8 seems to have the most far-reaching effect. The influence on void ratio consists of, firstly, one very special limit effect in the half-layer close to a plane surface and secondly, less important changes in the void ratio of the consecutive layers.

Additional Tests

The authors have here dealt with glass balls and one set of building-up formations where every grain was permitted to find an individual stable position. It is difficult to obtain smaller void ratio by simple packing. Penetration tests in similar materials show a great increase in penetration resistance when the same void ratio is approached ($e \approx 0{\cdot}60$). Table 4 shows the void ratios for glass balls, two types of quartz sand and one type of gravel for different conditions of formation (cf. also Kolbuszewski, Proc. 2nd Int. Conf. on Soil Mech. and Foundation Engng., Vol. 1, pp. 158-165).

Table 4

Material	*Tests in ∅ 75 mm glass cylinder*				*Tests in rubber sack*	
	Slowly poured	*Quick-ly poured*	*Tem-pered*	*Rolled*	*Atmo-spheric pressure*	*Va-cuum*
∅ 2 mm glass balls	0·59	0·65	0·59	0·70	0·53	0·50
∅ 4 mm glass balls	0·61	0·66	0·61	0·71	0·61	0·51
∅ 6 mm glass balls	0·61	0·66	0·59	0·75	0·60	0·50
Normal Sand A	0·61	0·83	0·59	0·83	0·54	0·51
Normal Sand B	0·68	0·93	0·66	0·93	0·61	0·59
Gravel ∅ 2-4mm	0·63	0·90	0·60	0·92	0·60	0·51

Fig. 10 shows how the texture of a base layer is changed when stresses are introduced in the plane. One can observe a crystalline pattern with amorphous zones between the "crystals". Fig. 11 shows how a similar pattern is formed when putting a granular mass under stress. This external "skin" must influence laboratory tests very much (cf. L.C. Graton and H.J. Fraser. J. of Geology Vol. 43, Nov.-Dec. 1935).

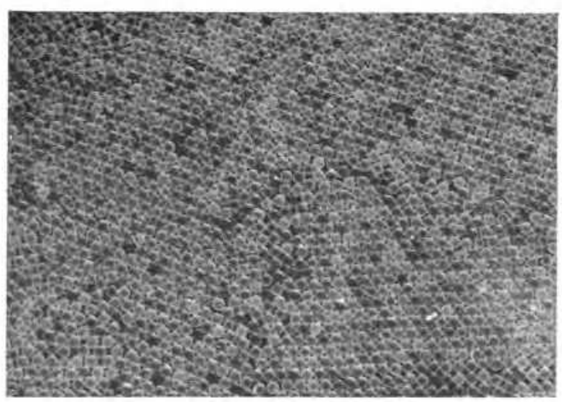

Fig. 10 Base Layer Subjected to Stresses in the Plane.
Couche de base soumise à des tensions planes.

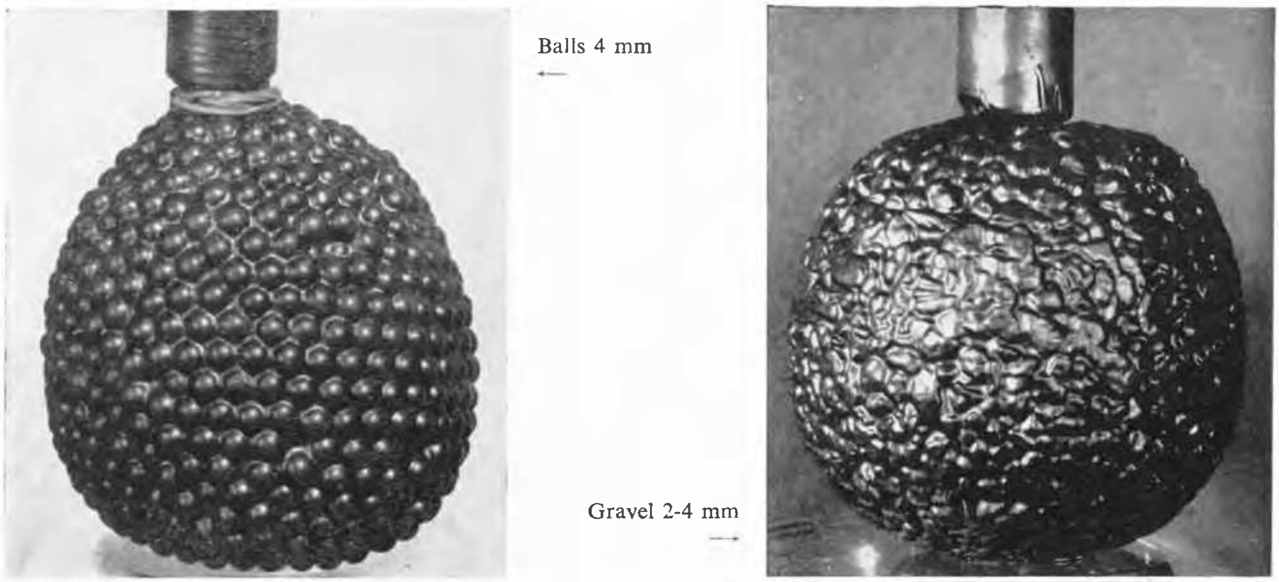

Fig. 11 Skin Effect (Granular Masses under Vacuum).
Effet de peau de substances granulées mises sous vide.

Recapitulating the investigations, it can be said that the idea of examining grain masses both by means of statistical analyses and by experiments seems to the Authors to be a useful attempt to an understanding of the behaviour of granular materials, and may help to avoid experimental errors, for instance due to boundary conditions.