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A Laboratory Study of Rates of Consolidation in Clays With Particular Reference to Conditions of Radial Porewater Drainage

Etude de laboratoire de la vitesse de consolidation de l'argile dans le cas particulier d'un drainage radial de la pression interstitielle

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Summary

Consolidation tests are made under the usual conditions of loading and drainage and the results are compared with those obtained in tests in which the same loading conditions are maintained, but where the porewater is induced to flow radially from the sample instead of axially.

A curve fitting procedure for the radial flow case for the determination of coefficients of consolidation is described. This procedure is similar in form to the ' square root of time fitting method'

It is shown that coefficients of consolidation can be determined for the case of radial as well as axial drainage, for application to certain problems of settlement analysis in which the conditions are not those of one-dimensional consolidation in a homogeneous clay.

Sommaire

Des essais de consolidation ont été entrepris dans les conditions ordinaires de charge et de drainage et les résultats ont été comparés avec ceux obtenus dans des essais comportant les mêmes conditions de charge mais où l'eau interstitielle s'écoulait de l'échantillon dans une direction normale et non parallèle à l'axe.

L'auteur décrit la suite des opérations destinées à déterminer les coefficients de consolidation en faisant accorder la courbe expérimentale des vitesses de tassement à la courbe théorique. Cette méthode est semblable à la méthode de la racine carrée du temps.

L'auteur a démontré que les coefficients de consolidation peuvent être déterminés dans le cas du drainage radial ou axial pour être appliqués à certains problèmes d'analyse des tassements pour lesquels les conditions ne sont pas celles de la consolidation à une dimension dans une argile homogène.

Introduction

Since the original presentation of the one-dimensional theory of consolidation by Terzaghi in 1925 [1] * the theory has been extended to cases of two and three dimensional porewater flow. This paper deals with some experimental work on the particular case of the consolidation of cylindrical samples of clays in the œdometer (laboratory consolidation apparatus) under conditions of axial loading and radial porewater drainage.

It was to be expected that the total consolidation would be independent of the direction of porewater drainage, other things being equal, and the results indicated that this was so.

The work outlined here involved a study of the rate of consolidation of a number of clays by comparing the coefficients of consolidation for the axial and radial porewater drainage conditions. These clays varied in structure from apparently homogeneous materials to ones in which laminations were just detectable, the latter being commonly encountered in the Clyde Valley. The laminations were horizontal in the natural soil and this positioning was retained in the test samples.

Classification tests on the range of soils which were studied showed the soils to be inactive inorganic clays of medium to high plasticity.

The tests were carried out in the "standard" type of œdometer, the clay samples being of the order of size of 3 in. diameter and ¾ in. thick. Radial flow conditions were imposed when desired by replacing the standard sample ring with a similar sized one formed of a porous metal, and replacing the upper and lower porous discs with impervious plates of similar dimensions. The metal of the porous ring had a coefficient of permeability of about 4 microns/sec. — many times that of the clays under test. Some further details of the apparatus are given in Appendix I.

Theory

The theory of consolidation is based on several main assumptions, namely, the soil is saturated; both the porewater and the soil grains are incompressible; D'Arcy's law holds; the coefficient of permeability is a constant; the clay is laterally confined; and for every stage of the process of consolidation, an increase in the effective pressure from an initial value p_0 to a final nalue p reduces the void ratio of the clay from an initial value e_0 to a final value e at a constant rate $\partial e/\partial p$ for this range of pressures.

This leads to the differential equation of consolidation for one dimensional consolidation. [2]

$$c_{vc}\,\frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} \quad \text{where} \quad c_{vc} = \frac{k_v}{m_{vc}\,\gamma_\omega} \qquad {}^{1}$$

and is considered to apply to the

Standard oedometer case—The conditions in the standard œdometer test under one dimensional consolidation are that

$$(i)\ \text{at}\ t = 0\ \text{and}\ 0 < z < 2H \qquad u = u_i = \Delta p$$

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* Numbers in brackets refer to the list of references.

[1] Where consolidation symbols bear suffices it is intended that the first suffix (v or r) conveys that the coefficient applies to conditions of vertical or radial drainage and the second suffix (c) conveys conditions of compression.

(*ii*) at $0 \leqslant t \leqslant \infty$ and $z = 0$, $z = 2H$ $\quad u = 0$

(*iii*) at $0 \leqslant t \leqslant \infty$ and $z = H$ $\quad \frac{\partial u}{\partial z} = 0$

(*iv*) at $t = \infty$ and $0 \leqslant z \leqslant 2H$ $\quad u = 0$

The solution of the above differential equation for these conditions leads to the following expression [2] for the average degree of consolidation throughout the clay thickness $2H$ at any time t after the application of stress increment Δp

$$U = 1 - \frac{8}{\pi^2}\sum_{n=0}^{n=\infty}\frac{1}{(2n+1)^2}\ \text{exp.}\left|\frac{-(2n+1)^2\pi^2}{4}T_v\right|$$

where $T_v = \frac{c_{vc}\,t}{H^2}$ termed the "time factor"

The relationship between U and T_v is shown graphically in Fig. 1.

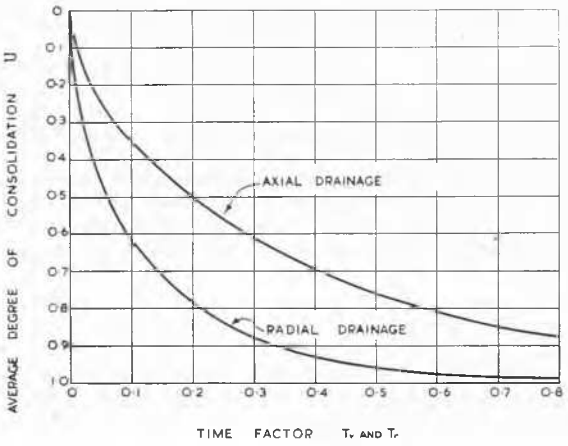

Fig. 1 Theoretical consolidation curves for the axial and radial drainage cases.

Courbes théoriques de consolidation dans le cas d'écoulement axial et radial.

Case of radial porewater flow—For the case where loading and compression are as in the standard test, but where porewater flow is restricted to the radial direction only, it has been shown [2] that the differential equation of consolidation is

$$c_{rc}\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) = \frac{\partial u}{\partial t} \quad \text{where } c_{rc} = \frac{k_r}{m_{vc}\,\gamma_\omega}$$

In the present adaptation of the standard test the conditions are that

(*i*) at $t = 0$ and $0 \leqslant r < R$ $\quad u = u_i = \Delta p$

(*ii*) at $0 \leqslant t \leqslant \infty$ and $r = R$ $\quad u = 0$

(*iii*) at $0 \leqslant t \leqslant \infty$ and $r = 0$ $\quad \frac{\partial u}{\partial r} = 0$

(*iv*) at $t = \infty$ and $0 \leqslant r \leqslant R$ $\quad u = 0$

The solution of this differential equation for the stated conditions leads to the following expression [3] for the average degree of consolidation at any time t

$$U = 1 - 4\sum_{n=1}^{n=\infty}\frac{1}{B_n^{\,2}}\ \text{exp.}\ (-B_n^{\,2}\,T_r)$$

where B_n = n^{th} root of Bessels equation of zero order and $T_r = \frac{c_{rc}t}{R^2}$ called the "time factor".

The relationship between U and T_r for this case is also shown in Fig. 1.

Determination of coefficients of consolidation—Methods of determining the coefficient of consolidation (or swelling) for the standard test conditions, are well known. These involve the fitting of experimental consolidation — time data to the previously described theoretical relationships for this and thereby deducing the appropriate coefficient.

The "logarithm of time fitting method" [2, 4] as used for the one dimensional (axial drainage) case may also be used for the radial drainage case as the theoretical curves, though showing different numerical relationships, have similar characteristics with points of inflection at about $U = 75$ per cent. It is believed that this method has been applied to the radial drainage case. [5, 6]

This curve fitting method using the $\log_{10}$ plot of time requires rather elaborate treatment and the detailed observations (of rate of settlement of the laboratory sample) have to be carried out over a period of at least a day.

The author considered that an adaptation of the "square root of time fitting method" [2, 4] was worth investigating, as this method for the axial case usually gave a readily determinable linear portion on the square root plot and only required detailed observations over a period of say an hour from the time of load application.

The "square root of time fitting method" is based on the observed fact that if U is plotted against against $\sqrt{T_v}$ the plot is linear up to a value of U of 52·6 per cent beyond which the curve tends to an asymptote at $U = 100$ per cent. Further a line drawn from the origin to the point on the curve corresponding to $U = 90$ per cent ($T_v = 0{\cdot}848$) has abscissae 1·15 times the abscissae of the straight part of the $U \sim \sqrt{T_v}$ plot.

The test curve, at any load increment, also shows an initial linear part when the consolidation of the sample is plotted against (time) $^{1/2}$. A line drawn from the intersection of this straight with the axis ($\sqrt{t} = 0$) and having abscissae 1·15 times that of the initial straight on the test curve, is considered to cut the curve at a point corresponding to 90 per cent primary consolidation. This point gives the time taken to reach 90 per cent consolidation, and hence allows the determination of c_{vc} from $c_{vc} = \frac{0{\cdot}848\,H^2}{t_{90}}$.

A similar approach is followed for the case of radial porewater flow. It is shown in Appendix II that if the $U \sim T_r$ relationship is plotted in the form U against $T_r^{\,0{\cdot}465}$ an initial linear part is obtained up to about $U = 50$ per cent. The point on the curve corresponding to $U = 90$ per cent has an abscissa 1·218 times the abscissa of the continuation of the initial straight, and at $U = 90$ per cent $T_r = 0{\cdot}3345$.

Hence the corresponding method of fitting the observed consolidation — time data for any load increment is to plot the consolidation of the sample against (time) $^{0.465}$. This exhibits the expected initial linear part. A line is then drawn as above, but this time having abscissae 1·22 times that of the initial straight on the test curve and is again considered to cut the curve at a point corresponding to 90 per cent primary consolidation. This point gives the time taken to reach 90 per cent consolidation under radial porewater flow conditions and hence allows the determination of c_{rc} from

$$c_{rc} = \frac{0{\cdot}335\, R^2}{t_{90}}$$

In both cases, if determinations of m_{vc} are made from the observed data, the values of k_v and k_r can be calculated at any stage of loading of the samples from

$$k_v = c_{vc}\,\gamma_w\, m_{vc} \text{ and } k_r = c_{rc}\,\gamma_w\, m_{vc}$$

The assumption in the theory of the radial flow case that the porewater does flow entirely radially is one of convenience. In fact a combination of radial flow along the silt layers and axial flow from the clay layers into the silt layers would be expected in the test conditions stated. The time taken to consolidate to any given degree will depend on the relative permeabilities and thicknesses of the clay and silt layers and on the significance of the nominal length of the flow path. (i.e. the radius) [7]. The influence of these features on the results quoted has not yet had adequate study.

Results

The values of coefficient of consolidation, as derived from the foregoing procedures, for any given clay material were reasonably consistent amongst repeat tests. Generally six consolidation tests were made on each clay, three tests with axial drainage and three with radial drainage.

The data from these tests showed that total consolidation was not noticeably affected by the drainage conditions, but that the rates of consolidation differed to a degree dependent on the lack of homogeneity of the clay. Where the clays were apparently homogeneous, little difference showed up in the tests and as the laminated structure became more apparent, this difference became more marked and the general form of the results is as indicated in Fig. 2. The term laminated structure applied to these clays, is intended to suggest one formed of layers of plastic clay about $\frac{1}{8}$ in. thick, separated by dustings of silt sizes and the range of clays studied in these tests was limited to this degree of anisotropy, since satisfactory trimming of the samples was not possible when thicker silt layers were encountered.

LAMINATED PLASTIC CLAY (GOVAN, GLASGOW)				
L L	P L	SIZE FRACTIONS (% BY WEIGHT)		
		CLAY	SILT	FINE SAND
39	23	50	45	5

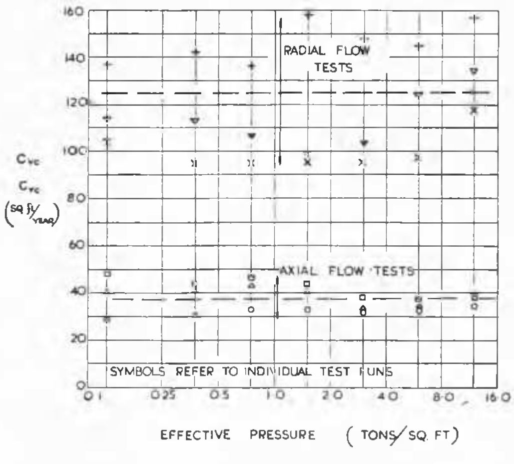

Fig. 2 Results of tests on a clay.
Résultats d'essais sur une argile.

The coefficients of consolidation showed only slight differences throughout the range of pressures — tending to increase a little with increasing pressures but for practical purposes, arguably of constant value. Taking these coefficients to be constant, $c_{rc} = \eta\, c_{vc}$ where $1 < \eta < 4$ in the range of tests carried out.

It follows that if c_{vc} (c_{rc}) is taken as constant throughout the pressure range, the time to reach a specified degree of consolidation is independent of the pressure increment causing the consolidation.

The coefficients of permeability derived from c_{vc} and c_{rc} showed the expected decreases in value when measured at increasing effective pressures though it is implied by the foregoing that $k_r/k_v = \eta$ also.

Two further points of interest emerged from these tests. First, it was noted generally that the proportion of primary consolidation was appreciably greater in the radial flow case, and second, the coefficient of swelling was also greater in this case though less than η times so for laminated clays and was not constant in value, being considerably greater in the higher pressure ranges than in the lower.

A few standard tests were made on samples cut with their axes horizontal so that as positioned in the œdometer cell the laminations lay in vertical planes. This gave a new loading condition, though the drainage was along the laminations as in the radial flow case. The coefficients of consolidation derived from these tests were intermediate in value between those from the two cases previously described though much closer to the radial flow case, as might be expected.

No tests were made under floating ring conditions though results should be similar depending on what relative ring friction effect existed.

Conclusions

For settlement analysis in anisotropic clays a knowledge of c_{vc} and c_{rc} is required. It has been shown [8] that the three dimensional radial and axial flow case can be resolved into its components, so that the average degree of consolidation in three dimensional axially symmetrical flow is

$$U = 1 - (1 - U_v)\;(1 - U_r)$$

in which the variation of U with T_r can be deduced from the equations for U_v and U_r for specified ratios H/R [9].

The parameter for axial flow is obtainable from the standard test and the porous ring method described here provides a simple way of determining the coefficient of consolidation for radial porewater flow. Minor modifications only are required in the standard apparatus and the problem of maintaining the porosity of the rings is no greater than that encountered with the porous discs in the standard apparatus. Likewise, the proposed method of curve fitting is no more complicated than the "square-root" method for the axial case since it is a simple matter to prepare a master chart with abscissae of (time) $^{0{\cdot}465}$ from which any number of copies can be taken.

It appears from the tests that for practical purposes the coefficients of consolidation can be taken as constants and the values for radial porewater flow along the laminations of such clays can readily be distinguished from those for flow across the laminations. For the size of apparatus used in this study, there will be a limit to the scope of the method as the thickness of the clay/silt laminations increases.

The work was carried out in the Soil Mechanics Laboratory of the Royal College of Science and Technology, Glasgow and thanks are due to Professor A.S.T. Thomson and Professor W. Frazer for the facilities provided.

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Appendix I

Details of Apparatus—The loading and cell arrangements of the apparatus do not call for special comment, these being of a general form common to the products of several manufacturers of soil testing equipment.

For both the axial and radial drainage cases, the tests were made under fixed ring conditions.

The usual arrangement of components in the cell was used for the axial drainage case viz. an impervious metal ring containing the sample, with a porous disc above and below the sample, the load being applied to the upper disc, which can enter the ring as the clay consolidates, and the ring and sample resting on the lower disc.

In the radial drainage case, the assembly was essentially the same except that the ring in which the sample was contained was machined from a porous stainless steel, while the loading discs above and below the sample were solid steel. The coefficient of permeability of the porous rings was measured and found to be of the order of 4×10^{-4} cm/sec. — many times that of the clays under test. It was considered that these rings would offer little obstruction to the passage of porewater from the soil.

Some initial difficulty was experienced in that despite careful machining of the rings from the porous metal blanks, the surface pores become blocked and the rings rendered almost impervious. This was overcome by subjecting each ring, after machining to size, to a process of electrochemical corrosion. In brief, the ring was immersed in an electrolyte and made the anode of a circuit with a lead bar as cathode. Direct current to a density of 1 amp./sq.cm. was applied to each ring in turn until inspection showed that the process of corrosion had revealed the true porous structure. In this case, it was found necessary to maintain the current density for about ten minutes. After cleaning the rings, their permeability was measured and found to be as stated above.

The permeability of the rings tended to decrease with repeated use, although it was always much greater than that of the clay, and could be improved again when required by subjecting the rings to further electrochemical corrosion.

Appendix II

Curve fitting method for the radial flow case—The theoretical expression relating average degree of consolidation to time factor for this case has been given with a simple plot of the relationship (Fig. 1) in the text.

The table below gives some corresponding values of U and T_r and shows the linearity of the $U \sim T_r^{0{\cdot}465}$ relationship for values of U up to about 50 per cent.

The linear part approximates to the simple law

$$U = 1{\cdot}83\, T^{0{\cdot}465}.$$

U (1)	T_r (2)	Value of $T^{0{\cdot}465}$		Col. (3)
		From Col. (2) (3)	From $U = 1{\cdot}83\, T^{0{\cdot}465}$ (4)	Col. (4)
0·1008	0·002	0·0556	0·0551	~ 1
0·1687	0·600	0·0925	0·0922	~ 1
0·2153	0·010	0·1175	0·1176	~ 1
0·2986	0·020	0·1621	0·1631	~ 1
0·3598	0·030	0·1958	0·1965	~ 1
0·4096	0·040	0·2238	0·2239	~ 1
0·4521	0·050	0·2484	0·2471	~ 1
0·4894	0·060	0·2702	0·2674	~ 1
0·5228	0·070	0·2903	0·2856	1·016
0·6058	0·100	0·3428	0·3311	1·035
0·7821	0·200	—	—	—
0·8780	0·300	—	—	—
0·9000	0·335	0·6013	0·4917	1·22
0·9316	0·400	—	—	—
0·9616	0·500	—	—	—
0·9932	0·800	—	—	—

Having established the slope of the initial straight on this plot of $U \sim T^{0{\cdot}465}$, the chord from the origin to $U = 90$ per cent ($T_r = 0{\cdot}335$) has abscissae of 1·22 times the abscissae of the straight.

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