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The Investigation of Mechanical and Hydrological Properties of Soils Based on Surface Tension Forces Consideration

L'étude des propriétés physiques et hydrologiques des sols basée sur la considération des forces superficielles

by Prof. S. V. NERPIN, Dr of Technic. Sci., Head of the Chair of the Institute of Water Transport
and
Prof. B. V. DERJAGUIN, Corresponding Member of the USSR Acad. Sci., Dr of Chem. Sci., Head of the Laboratory of Surface Phenomena

Summary

The authors consider that the study of soil mechanics is inadequate, and that it is necessary to make allowances for the mechanical properties of soil and the mechanism involved.

An account is given of surface tension effects in contact zones between the soil particles and in water film interfaces. The authors try to relate chemical and morphological peculiarities of separate soil particles to the behaviour of the soil whilst under load, during the period of unloading, under shear load and when water is percolating throught it.

The use of scale model research on disc-shaped particles is considered.

Results of percolation studies on soils having different moisture contents are given as well as an explanation of percolation anomalies observed during water filtration through clay strata.

Sommaire

Ce rapport signale l'insuffisance de la méthode descriptive adoptée dans certaines recherches de mécanique des sols et la nécessité d'étudier l'origine de leurs propriétés mécaniques et les mécanismes correspondants.

Les auteurs montrent que l'analyse de l'action des forces superficielles dans les zônes de contact des particules du sol entre elles et entre les particules du sol et les filtres d'eau interposés permet de relier les particularités chimiques et morphologiques des particules au comportement du sol lors de la charge, de la décharge, du glissement et de la filtration de l'eau à travers ce sol.

Ils étudient le processus de consolidation d'un modèle de sol constitué par des particules en forme de plaques et de disques.

Ils donnent les résultats des recherches sur la filtration pour différentes teneurs en eau, de même que l'explication des anomalies de filtration à travers les sols argileux.

Introduction

Modern experience in the building construction indicates that the present study of soil mechanic is not longer adequate and that a more detailed investigation is necessary of the mechanical properties of soils and of the mechanism of the processes which take place in them.

The mechanical properties of soils depend to a considerable extent on the nature of the particles, their shape and size, but both the degree and the nature of these differ substantially in various soils.

This paper deals with highly dispersed systems, the properties of which depend mainly on surface phenomena.

Interaction between solid particles immersed in a liquid

To give a correct estimate of the significance of different factors in determining the various properties of soils it is necessary to examine the interactions between separate particles which statistically account for the mechanical properties of the total system.

The role of various kinds of forces acting in the zones of contact between particles varies according to the distance between their surfaces. When the surfaces come into direct contact there will arise elastic forces of resistance to tension, compression and shear. At larger distances these forces disappear, but the effect of the closeness of the interphase surfaces may make itself felt up to distances of the order of 10^{-5} to 10^{-4} cm.

Closeness of the boundary surfaces changes the chemical potential μ in the thin liquid layers separating the particles, and it becomes unequal to the chemical potential in the liquid bulk :

$$\mu_0 = \frac{F + PV}{N},$$

where F is the free energy, P, the pressure, V, the volume and N, the number of gram-molecules in that volume.

Apparently the condition of thermodynamic equilibrium μ = const can be restored by appling a certain load of intensity P (Fig. 1) to the particles. In this case the following

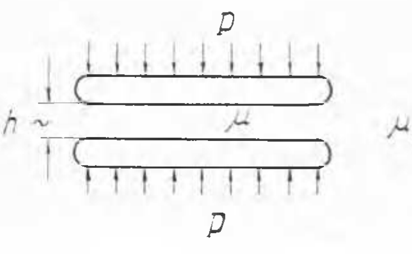

Fig. 1 The disrupting pressure in a system of parallel disk-like particles.

Les pressions interfaciales dans un système de particules lamellaires.

condition of equality between the chemical potentials in the layer and in the bulk μ_0 can be written :

$$\frac{\mu + PV}{N} = \mu_0, \tag{1}$$

whence :

$$P = \frac{\mu_0 - \mu}{V_m}$$

where : V_m is the molecular volume.

When $\mu_0 > \mu$, the external load must act in the direction indicated in the figure to secure equilibrium, and therefore in this case the system of particles is capable of causing a swelling pressure based on the disjoining action of thin layers first discovered by one of us [1] and investigated in several studies. [2].

It was subsequently shown that the disjoining pressure is an interphasal pressure drop at the boundary of a thin layer to the Laplacian pressure drop on an interphasal surface, but depending on the layer thickness instead of the surface curvature [3].

In the general case the disrupting pressure is determined by forces of a variable nature.

The difference between the chemical potential in the layer separating the particles and its value in the bulk is due, first of all, to the existence of distant action molecular forces which come into play when the two phases are separated by a layer of small thickness. A strict theory of molecular interaction between macroscopic bodies was developed in recent years by E. M. LIFSHITZ [4]. The authors demonstrated [5] that the chemical potential in a layer separating two bodies can be expressed approximately by the relationship :

$$\mu_1 = \mu_0 + \frac{A}{h^3} \tag{2}$$

where A — is a constant depending on the surface energies of the approaching bodies. For two identical bodies the constant A is proportional to the surface energy value σ at the interphasal boundary. For approaching bodies of various nature it is proportional to the value $\Delta\sigma = \sigma_{1-3} - (\sigma_{1-2} + \sigma_{2-3})$ which may be positive or negative.

The second factor causing the inequality of chemical potentials is the existence of equilibrium ionic atmospheres at the particle surfaces.

As was shown at an earlier date [6], the disjoining pressure effects arising due to this factor may be of different signs. But if the charges on the approaching surfaces are equal the effect is always that of repulsion. In this case the following expression may be written for the chemical potential of the solvent molecules in the layer :

$$\mu = \mu_0 - kT(C - C_0)V_m\,, \qquad \ldots\ldots \tag{3}$$

where : μ is the chemical potential of the solvent molecules in the bulk of the liquid ; μ_0 , the ionic concentration in the bulk ; C_0, the ionic concentration in the plane of symmetry of the layer (at this point the condition $E = 0$ is fulfilled, E being the electric field) ; T, the absolute temperature ; k, the Boltzman constant.

In the above expression $C = \frac{n_p}{n}$, where n_p and n are respectively the ionic and molecular concentrations of the solvent in the solution, in molecules per cubic centimeter.

In the general case the factor $\mu(h)$ can be found in the parametric form ; however, for several cases the chemical potential in the layer can be expressed directly through the layer thickness [2]. For example, with relatively large values of the potential ψ at the boundary surface ($\psi > 100$ mv) and a small h — value with respect to the thickness of the ionic atmosphere [7] :

$$\mu = \mu_0 - B/h^2$$

where

$$B = \beta D \left(\frac{kT}{ze}\right)^2 V_m$$

here, in addition to the previous notations, z = electrovalence of counter ion ; D = dielectric permeability ; e = electron charge ; β = numerical coefficient depending on the valency ratio of the two ions.

The third factor determining inequality of the chemical potentials in the bulk and within a thin layer, is the specific physical state of polar liquids near boundary surfaces.

It has been demonstrated experimentally [8] that the solvate layer, possessing a specific structure, has a sharp boundary with the rest of the liquid, and may be regarded as a specific boundary phase. As phase equilibrium occurs only at a definite thickness of the boundary layers, the chemical potentials around such a layer at other thicknesses will be unequal to that in the bulk. Thus, when the particles are brought together to distances less than twice the thickness of the boundary layers there will arise a disrupting effect analogous to the action of an adsorption monolayer of surface-active substances The value of the disrupting pressure will then be expressed by the relationship :

$$P_c = \frac{\mu_0 - \mu_c}{V_m} \tag{4}$$

where μ_0 and μ_c are the chemical potentials in the bulk and in the boundary solvate layer, respectively.

Unfortunately, the present-day theory of the liquid state does not yet allow quantitative determination of the value μ_c.

The nature of the forces of interaction between the particles is shown in Fig. 2 ; section *a-b* characterizes elastic forces. Fig. 3. shows the curves of interaction between particles.

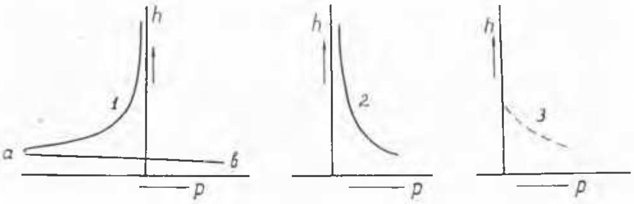

Fig. 2 The particle interaction :
1) Molecular forces ;
2) The forces of electrostatic interaction of ions ;
3) Forces due to the interaction of solvate layers.

Interactions mutuelles des particules :
1) Forces moléculaires ;
2) Forces provenant des interactions électrostatiques des ions ;
3) Forces provenant des interactions des couches de solvation.

Curve I corresponds to the case where after disappearance of the forces of elastic repulsion the polecular and ionic electrostatic forces give rise at all distances h to an attraction effect between the particles. Such particles are in an aggregated state and only a change of the dispersion medium (including such a change due to the introduction of exchange ions into the solution) can alter their state.

Curve II refers to the case where repulsion forces acting at large distances pass, with decreasing distance, into attraction forces, up to the appearance of elastic repulsion forces. This case may be considered to include a system consisting of kaolinite particles, which possesses limited swelling power.

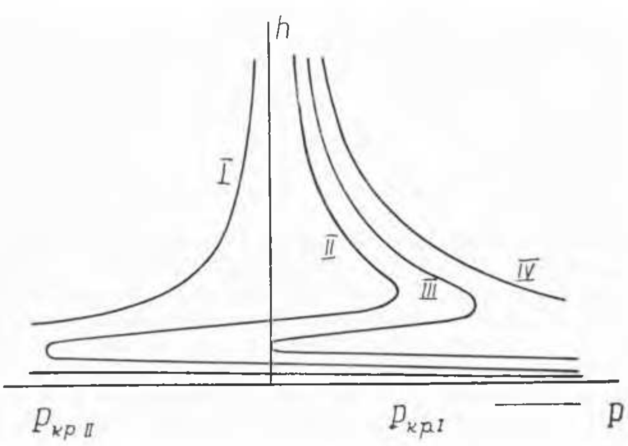

Fig. 3 The Curves of resultant interaction of particles. Interactions mutuelles des particules.

Curves III and IV characterize systems in which various kinds of repulsion forces prevail at all distances. Curve IV refers to the case where the repulsion effect caused not only by iono-electrostatic interaction forces, but in addition by forces of molecular interaction in the dispersion medium.

All the curves between III and IV characterize minerals of the montmorillonite type, capable of unlimited swelling, up to breakdown into elementary packets.

The statistical distribution of particles in soils according to mineralogical composition may lead to simultaneous existence of all the forms of interaction indicated above; but the properties of any given soil must depend on their interrelation.

Along with the action of the surface forces discussed above, cementing of the particles may occur due to the plastic properties of the layers separating them. For the case of flat particles the adhesion force per unit area is expressed by [9]:

$$P\tau = \frac{C}{h} \qquad (5)$$

where C is a constant depending on the form of the contact area and the ultimate shear strength τ_0 of the liquid in the layer separating the particles.

The capillary forces acting between particles have been studied by many authors. For the case of two spherical particles of equal size, the force acting at a single contact grows with decreasing moisture content, tending towards the value :

$$f = 2\pi\sigma R \qquad (6)$$

where σ is the surface energy at the water-air interface and R is the radius of the spherical particles.

Considering the projections of all the interaction forces on a normal to the cross section, and referring them to 1 cm^3 of its area, we can write the equilibrium condition as :

$$q + q_m + q_p + q_c \pm q\tau = 0\,, \qquad \ldots . \quad (7)$$

where q external load; q_m = molecular elasticity forces; q_p = long range action forces (including those of molecular origin); q_c = capillary forces; $q\tau$ = plastic resistance to approachment (+) and departure (—) of the particles.

The condition of shear strength of the soil can be written as :

$$\tau \leqslant (q + q_{m,c} + q_p + q_c + \varepsilon q\tau)\,\mathrm{tg}\,\varphi \qquad (8)$$

Here, besides the above notations, τ is the shearing stress; $q_{m,c}$ is the molecular tensile strength of the contacts; φ is the angle of internal friction in the soil; ε is a co-efficient smaller than unity, taking into account the possibility of shearing to full development of plastic adhesion forces.

Compression properties of clay soils

If on reduction of load, the disrupting pressure tends to return the system to its initial state * the molecular bonds at the points of direct contact between the particles (curves I and II in Fig. 3), as well as the plastic adhesion forces, will hinder swelling of the soil when the load is reduced.

Considering the interactions between individual particles which make up statistically the properties of the bulk system, it is interesting to note the existence of two barriers, a right-hand and a left-hand one, on the curves of the II type. As long as the external force acting on the contact does not exceed the value of the first right-hand barrier, reversible increase of the distance between the particles is possible after the load has been removed or reduced **.

If the force acting on the contact reaches the value of the right-hand barrier $P_{c,\mathrm{I}}$ the particles will approach each other spontaneously to a value of Δh corresponding to the distance along the co-ordinate h from the right-hand barrier to the lower part of the curve, characterizing the elastic repulsion forces. If after this, the load is not merely reduced but completely removed, the distance change will no longer be determined by the part of the curve above the right hand barrier, but by its lower elastic part. Only after a tensile force equal to the value of the left-hand lower barrier $P_{c,\mathrm{II}}$ is applied, will the distance between the particles increase to infinity.

Thus, if on compression of the system the contact stresses exceed the first barrier, the particles fall, as it were, into a "molecular trap" which gives rise to molecular cohesion, making the system resistant to rupture and manifesting itself when the latter swells.

Though the mechanism of particle interaction during the compaction and swelling of soils indicates the nature of the observed hysteresis, the difference between the compression and decompression curves must be due not only to this.

Flocculation processes during the period of soil formation, and subsequent processes inevitably result in pore-size heterogeneity, and therefore in heterogeneity of the whole system.

Inconsistancy of system rigidity results, in its turn, in deviations of the normal stress values from the average value and to the appearance of shear stresses in the system along its microplatforms even in the case of compression of the soil under a constant pressure ***. With each new step of the load, both the condition of instantaneous soil strength on its microplatforms determined by equation (8) may be disturbed, and the threshold of creep may be exceeded.

In the first case relatively rapid microshifts may arise. In the second case they may occur over a considerable period of time. Shear deformations should cease at the moment when as a result of a new, more uniform pore-size distribution and strengthening of the soil due to the particles coming closer to each other, the condition of strength will not only be observed along the microplatforms, but the creep threshold will not be exceeded.

The complexity of the above mechanism of compaction and swelling of a soil and the exceedingly complex nature of the statistics of the particles and pores makes it impossible, at least at present, to build up a theory of compaction on the basis of quantitative accounting of particle interaction in combination with the statistical characteristics of the system.

It is much easier to carry out calculations for a simplified model system, and then, regarding this solution as a method of approximation, to apply the relationships obtained to practical systems.

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* Which corresponds to a tendency of the free energy of the system to decrease to its initial level.

** In this reasoning we make no allowance for adhesion of the particles due to the plastic properties of the liquid, which is determined by the dependence (5) given above.

*** This fact was also indicated by Prof. G. M. Lomisé in his works.

It was shown [10] that for a system consisting of disc-shaped plates the following expression can be obtained :

$$q — q_p = \frac{2}{3}\frac{\tau_0}{\eta} r - \frac{3}{2}\frac{\eta}{h^3} r^2 \cdot \frac{dh}{dt}. \qquad \ldots. \quad (9)$$

Here, besides the previous notations, r is the plate radius and η is the viscosity.

Expression (9) is interesting because its right side contains both terms depending on the rate of compaction and terms which do not depend on it, *i.e.*, in the general form this expression may be written as :

$$q = f(h) + \varphi\left(h, \frac{dh}{dt}\right), \qquad \ldots. \quad (10)$$

but with $\frac{dh}{dt} = 0$, $q = f(h)$. Therefore, the value $f(h)$ is the compression factor $q(h)$ considered above.

With small variations of h the compression equation may be written as follows :

$$q - q_0 = \frac{dg}{dh}(h - h_0). \qquad (11)$$

Here q_0 and h_0 are the initial load and thickness of the layer.

Denoting $\frac{dq}{dh}$ by a, we write :

$$q(h) = - ah + ah_0 + q_0 \qquad \ldots. \quad (12)$$

Substituting (12) into (9), we get :

$$\frac{C}{h^3}\frac{dh}{dt} = ah + A \qquad (13)$$

where $C = \frac{3}{2}\eta r^2$ and $A = — ah_0 — q_0 + q$.

Writing (13) as $\frac{Cdh}{h^3(ah + A)} = dt$ and integrating from h_0 to h and from 0 to t, we find

$$t = - \frac{C}{A^3}\left[\left(\frac{A}{h} - \frac{A}{h_0}\right)\left(\frac{1}{2}\frac{A}{h} + \frac{1}{2}\frac{A}{h_0} - a\right) + a^2h\left(\frac{A + ah}{A + ah_0}\right)\frac{h_0}{h}\right] \qquad \ldots. \quad (14)$$

The resulting expression is the equation of compaction as a function of time, not connected with the filtration resistance of the system to compression, but taking into account only of its compressive characteristics and the properties of the films separating the particles.

Filtration properties of soils

The flow of a liquid through completely saturated soils can be expressed by formula [11] :

$$Q = \frac{\alpha}{\eta s^2}\partial^3 \operatorname{grad}\theta + \varepsilon\beta\partial \operatorname{grad}\psi \qquad \ldots. \quad (15)$$

where η = viscosity of liquid ; S = "kinetic" specific surface area ; ∂ = active porosity ; θ = moisture potential ; α = numerical co-efficient ; ε = pore tortuosity factor ; ψ = any of the three values : temperature, solute concentration, electric field potential ; β = corresponding factor : thermoosmotic, capillary-osmotic or electro-osmotic.

Introducing the notations :

$$\frac{\alpha\partial^3}{\eta S^2} = k \,;\ \operatorname{grad}\theta = J \,;\ - \frac{\varepsilon\beta\eta S^2}{\alpha\partial^2}\operatorname{grad}\psi = J_0 \ ,$$

we can write for the flow of a liquid a formula of the following type :

$$q = k\,(J — J_0) \qquad \ldots. \quad (16)$$

An empirical expression for filtration of a liquid through clay soil.

It follows from this expression that the anomalies observed in filtration through clay soil may be attributed to effects of boundary slip, the theory of which was given in earlier papers [12]. In this case the condition $q = 0$ is a state of quasi-equilibrium, under which there exist two mutually compensating flows : one moving under the influence of the moisture potential gradient, and the other, a counterflow, moving under the action of slip of the liquid in relation to the surfaces of the particles.

Several papers assumed [13] that a second reason for the anomalies of filtration of a liquid through narrow-pore filters may be its plastic properties characterized by an ultimate shear strength τ_0 of the order of 3×10^{-2} dyne $\times$ cm^2, the appearance of which may be due, for example, to contamination of the water with colloidal particles [14]. In this case, the theory of filtration should not be based on the Newtonian law of friction in liquids, but on the Shvedov-Bingham law :

$$\tau = \tau_0 + \eta\frac{dV}{dy}.$$

For moisture flow through unsaturated soils, when its magnitude is determined by the conductivity of the films on the particle surfaces, and the solute concentration is constant along the flow, we may write the following expression :

$$q = \frac{\alpha Sh^3}{\eta}\left(\frac{\partial\theta}{\partial w}\frac{\partial w}{\partial x} + \frac{\partial\theta}{\partial\sigma}\frac{\partial\sigma}{\partial T}\frac{\partial T}{\partial x}\right) - \frac{\beta Sh^2}{\eta}\frac{\partial\sigma}{\partial T}\frac{\partial T}{\partial x} + \gamma Svh\frac{\partial T}{\partial x}. \qquad \ldots. \quad (17)$$

Here, in addition to previous notations, h = film thickness ; W = percentage moisture content ; σ = surface tension ; T = temperature ; α, β, γ = numerical coefficients ; v = thermoosmotic factor.

The authors have dealt only with several problems concerning the nature of the mechanical properties of soils, but investigation of surface forces makes it possible to examine many other problems as well, for example, the function or the shape and size of the particles, the influence of salting of desalting of soils on their properties, and others.

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