Relationship between Apparent Angle of Friction—with Effective Stresses as Parameters—in Drained and in Consolidated-Undrained Triaxial Tests on Saturated Clay. Normally-Consolidated Clay

Relation entre l'angle de frottement apparent et les contraintes effectives en tant que paramètres — dans des essais triaxiaux drainés et non drainés consolidés sur l'argile saturée : argile normalement consolidée

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Summary

When calculating long-term stability of a soil, the shear strength under drained conditions must be known. An adequate triaxial test in the laboratory would then be the drained test. For practical reasons, however, tests on clay must usually be made on consolidated-undrained samples. During the undrained phase, the effective stress then decreases in certain directions in contrast to the behaviour of normally-consolidated clay in nature at drained conditions. Therefore the consolidated-undrained test is not a completely true reproduction of the conditions in the normally-consolidated clay in nature, the stress history being different in the two cases. The author gives a theoretical study of the difference between drained and undrained conditions of normally-consolidated clays.

The shear strength — or shearing stress at the failure surface — in saturated clay may be divided into two parts. One is the true cohesion, which, in theory, is purely a function of the moisture content, and the other is the true friction, corresponding to the true angle of friction. According to the particular stress history, an arbitrary system of effective stresses when acting under drained conditions corresponds to other figures of moisture content and true cohesion than those when acting under undrained conditions. Thus the apparent angle of friction generally is not the same in both cases.

The report shows that in triaxial tests on normally-consolidated clay, the apparent angle of friction is greater in the consolidated-undrained than in the drained test. Finally it is shown how the apparent angle of friction under drained conditions is to be calculated from the data obtained in consolidated-undrained tests ; for this calculation the true angle of friction must be known.

Sommaire

En évaluant la stabilité à long terme il faut connaître la résistance au cisaillement sous des conditions drainées. L'essai triaxial adéquat serait alors l'essai drainé. Pour des raisons pratiques cependant, il faut en général faire, pour ce qui est de l'argile, des essais consolidés non drainés. L'essai non-drainé entraîne une diminution de la contrainte effective dans certaines directions, contrairement à ce qui se passe dans de l'argile normalement consolidée à l'état naturel, dans des conditions non-drainées. L'essai consolidé et non-drainé en laboratoire ne reflète donc pas fidèlement les conditions dans l'argile consolidée normalement à l'état naturel, du fait que le processus de contrainte est différent dans les deux cas. Ce rapport est une étude théorique de la divergence entre les conditions non-drainées et celles drainées de l'argile consolidée normalement.

La résistance au cisaillement — ou l'effort de cisaillement dans la surface de rupture — de l'argile saturée peut se partager en deux parties. L'une des parties est la cohésion vraie qui est la fonction de la seule teneur en eau et l'autre est le frottement vrai correspondant à l'angle vrai de frottement.

Du fait de la différence d'évolution des contraintes la teneur en eau et, partant, la cohésion vraie ont d'autres valeurs lors de l'application d'un système quelconque de contraintes effectives à l'état drainé et à l'état non-drainé. Ceci entraîne que l'angle apparent de frottement n'est généralement pas le même dans les deux cas.

Le rapport démontre que dans les essais triaxiaux sur l'argile consolidée normalement l'angle apparent de frottement est plus grand dans les essais consolidés non drainés que dans ceux drainés. En dernier lieu on montre comment l'angle apparent de frottement dans des conditions drainées doit être évalué en partant des données obtenues dans les essais consolidés non drainés. Pour cette dernière évaluation il faut aussi connaître l'angle vrai de frottement.

Introduction

Osterman has pointed out in (1) that the apparent angle of friction obtained in triaxial tests is often higher in consolidated-undrained tests than in drained tests. He states that the condition can be explained qualitatively by the fact that the true cohesion, which, together with the friction, makes the shearing stress at the failure surface, corresponds to the active effective stresses at the failure in the drained test, while in the consolidated-undrained test it corresponds to the consolidation pressure. The author has calculated the difference between the apparent angles of friction in both types of triaxial tests.

True cohesion

Assume a clay body with a volume and void ratio equal to V_0 and e_0 when the effective stresses are isotropically equal to 0. Consolidation of the clay reduces the volume and the void ratio by ΔV and Δe, and the equation

$$\Delta e = (1 + e_0)\,\frac{\Delta V}{V_0}$$

becomes valid.

The true cohesion c can be expressed

$$c = k \cdot \Delta e$$

or

$$c = k(1 + e_0)\,\frac{\Delta V}{V_0} \qquad (1)$$

where k is a constant, assumed to be chosen so that the rectilinear relationship between c and Δe gives the best approximation to the actual relationship between c and Δe. The approximate expression gives $c = 0$ for $\Delta V = 0$, i.e. for effective stresses isotropically equal to 0; this seems plausible with normally-consolidated clay.

Drained test

With

σ'_1 = axial effective pressure
σ'_3 = radial effective pressure
E_c = "modulus of elasticity" at compression
μ = Poisson's ratio

the expression

$$\varepsilon_1 = \frac{1}{E_c}(\sigma'_1 - 2\mu\sigma'_3)$$

$$\varepsilon_3 = \frac{1}{E_c}[-\mu\sigma'_1 + (1-\mu)\sigma'_3]$$

is obtained for the unit compressions ε_1 and ε_3, and for the unit reduction of volume $\frac{\Delta V}{V_0}$ the expression

$$\frac{\Delta V}{V_0} = \varepsilon_1 + 2\varepsilon_3 = \frac{1}{E_c}(1 - 2\mu)(\sigma'_1 + 2\sigma'_3)$$

By inserting into Eq. (1) the following expression for the true cohesion c prevailing at the stresses σ'_1 and σ'_3 is obtained

$$c = k(1 + e_0)\cdot\frac{1}{E_c}(1 - 2\mu)(\sigma' + 2\sigma'_3).$$

With
φ = the true angle of friction the failure condition (Fig. 1)

$$\frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3 + \frac{2}{\mathrm{tg}\varphi}\cdot k(1 + e_0)\cdot\frac{1}{E_c}(1 - 2\mu)(\sigma' + 2\sigma'_3)} = \sin\varphi$$

is valid which is rewritten

$$\frac{\sigma'_1}{\sigma'_3} = \mathrm{tg}^2\left(45 + \frac{\varphi'_d}{2}\right)$$

$$= \frac{1 + \sin\varphi + 4\cos\varphi\cdot k(1 + e_0)\cdot\frac{1}{E_c}(1 - 2\mu)}{1 - \sin\varphi - 2\cos\varphi\cdot k(1 + e_0)\frac{1}{E_c}(1 - 2\mu)} \qquad (2)$$

Mohr's circles of effective stresses at failure have thus a common tangent running through origin. This tangent slopes at the angle φ'_d towards the σ-axis. Eq. (2) shows directly that the apparent angle of friction $\varphi'_d >$ true angle of friction φ.

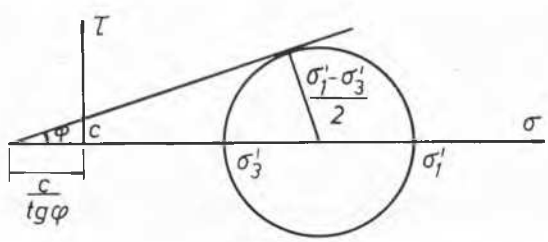

Fig. 1 Effective stresses σ'_1 and σ'_3 at failure. True angle of friction = φ and true cohesion = c.
Contraintes effectives σ'_1 et σ'_3 à la rupture. Angle vrai de frottement = φ et cohésion vraie = c.

Consolidated-undrained test

The sample is first subjected to the isotropically equal consolidation pressure p. While the total pressure often is maintained in a radial direction, the axial pressure is increased till failure occurs. In the undrained phase the following are valid :

σ'_1 = axial effective pressure
σ'_3 = radial effective pressure
$\sigma'_1 - p$ = increase of axial effective pressure from the beginning of the phase
$p - \sigma'_3$ = decrease of radial effective pressure from the beginning of the phase
$p - \sigma'_3$ = neutral pressure

In the undrained phase the unit compression

$$\varepsilon_1 = \frac{\sigma'_1 - p}{E_c} + 2\mu\,\frac{p - \sigma'_3}{E_s}$$

is obtained in axial direction, and in radial direction the unit elongation

$$\varepsilon_3 = (1 - \mu)\,\frac{p - \sigma'_3}{E_s} + \mu\,\frac{\sigma'_1 - p}{E_c}$$

where E_s = modulus of elasticity at swelling.

From the fixed volume condition

$$\varepsilon_1 - 2\varepsilon_3 = 0$$

is obtained

$$\frac{\sigma'_1 - p}{E_c} + 2\mu\,\frac{p - \sigma'_3}{E_s} - 2(1 - \mu)\,\frac{p - \sigma'_3}{E_s} - 2\mu\,\frac{\sigma'_1 - p}{E_c} = 0,$$

whereof with $\lambda = \frac{E_c}{E_s}$,

$$\lambda = \frac{1}{2}\cdot\frac{\sigma'_1 - p}{p - \sigma'_3} \qquad (3)$$

and after rewriting

$$3p = \sigma'_1 + 2\sigma'_3 + 2\,\frac{1 - \lambda}{1 + 2\lambda}(\sigma'_1 - \sigma'_3) \quad \ldots. \quad (4)$$

For the unit reduction of volume $\frac{\Delta V}{V_0}$, calculated from the stressless condition, the expression

$$\frac{\Delta V}{V_0} = \frac{1}{E_c}(1 - 2\mu) \cdot 3p$$

is valid. By inserting into Eq. (1) the following expression for the true cohesion c curing the undrained phase is obtained

$$c = k(1 + e_0) \cdot \frac{1}{E_c}(1 - 2\mu) \cdot 3p$$

which after insertion of Eq. (4) becomes

$$c = k(1 + e_0)\frac{1}{E_c}(1 - 2\mu)\left[\sigma'_1 + 2\sigma'_3 + 2\frac{1-\lambda}{1+2\lambda}(\sigma'_1 - \sigma'_3)\right].$$

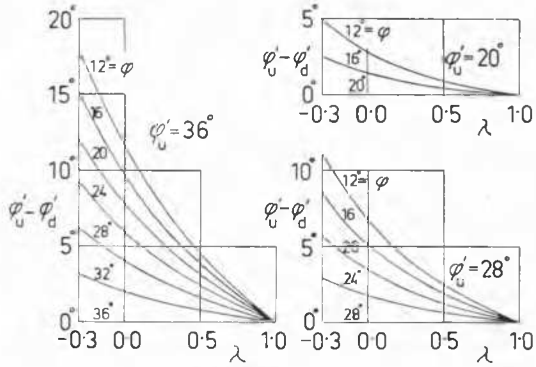

Fig. 2. The correction $\varphi'_u - \varphi'_d$ at various values of φ_u', φ and λ
La correction $\varphi'_u - \varphi'_d$ en fonction de φ'_u, φ et λ.

Thus at failure the condition now valid is

$$\frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3 + \frac{2}{\operatorname{tg}\varphi} k(1 + e_0)\frac{1}{E_c}(1 - 2\mu)\left[\sigma'_1 + 2\sigma'_3 + 2\frac{1-\lambda}{1+2\lambda}(\sigma'_1 - \sigma'_3)\right]} = \sin\varphi$$

which is rewritten

$$\frac{\sigma'_1}{\sigma'_3} = \operatorname{tg}^2\left(45 + \frac{\varphi'_u}{2}\right) = \frac{1 + \sin\varphi + \left(1 - \frac{1-\lambda}{1+2\lambda}\right)4 \cdot \cos\varphi \cdot k(1+e_0) \cdot \frac{1}{E_c}(1 - 2\mu)}{1 - \sin\varphi - \left(1 + 2\frac{1-\lambda}{1+2\lambda}\right)2 \cdot \cos\varphi \cdot k(1+e_0) \cdot \frac{1}{E_s}(1 - 2\mu)} \tag{5}$$

Mohr's circles of effective stress at failure have thus a common tangent through origin, sloping at the angle φ'_u towards the σ axis. The angle φ'_u is the apparent angle of friction in this type of test.

Comparison between the drained and the consolidated-undrained test

With

$$a = 2\cos\varphi \cdot k(1 + e_0) \cdot \frac{1}{E_c}(1 - 2\mu)$$

inserted into Eqs. (2) and (5) are obtained

$$\operatorname{tg}^2\left(45 + \frac{\varphi'_d}{2}\right) = \frac{1 + \sin\varphi + 2a}{1 - \sin\varphi - a} \tag{6}$$

$$\operatorname{tg}^2\left(45 + \frac{\varphi'_u}{2}\right) = \frac{1 + \sin\varphi + 2a - 2\frac{1-\lambda}{1+2\lambda}a}{1 - \sin\varphi - a - 2\frac{1-\lambda}{1+2\lambda}a} \tag{7}$$

From these equations is obtained the expression

$$\frac{1}{\operatorname{tg}^2\left(45 + \frac{\varphi'_d}{2}\right) - 1} - \frac{1}{\operatorname{tg}^2\left(45 + \frac{\varphi'_u}{2}\right) - 1} = \frac{\left(\frac{3}{1+2\lambda} - 1\right)a}{2\sin\varphi + 3a} \tag{8}$$

The value of λ now determine whether φ'_u is greater than, equal to or smaller than φ'_d.

According to Eq. (8) the following relations are valid :

$$\left.\begin{array}{ll} \text{at } -\tfrac{1}{2} < \lambda < 1 & \text{is } \varphi'_u > \varphi'_d \\ \text{at } \lambda = 1 & \text{is } \varphi'_u = \varphi'_d \\ \text{at other values of } \lambda & \text{is } \varphi'_u < \varphi'_d \end{array}\right\} \tag{9}$$

In (2) Skempton gives values that are valid for his pore pressure coefficient A in different clays at the failure phase. In saturated clay the coefficient A expresses the neutral pressure $p - \sigma'_3$ caused by the deviator stress $\sigma'_1 - \sigma'_3$ according to the formula

$$p - \sigma'_3 = A(\sigma'_1 - \sigma'_3)$$

Comparison with Eq. (3) rewritten in the form

$$p - \sigma'_3 = \frac{\sigma'_1 - \sigma'_3}{1 + 2\lambda}$$

shows that transformation from A to λ may be made according to the formula

$$\lambda = \tfrac{1}{2}\left(\frac{1}{A} - 1\right).$$

Skempton's table, complemented with the calculated values of λ is as follows :

Type of clay	*A*	λ
Clays of high sensitivity	$+\frac{3}{4}$ to $+1\frac{1}{2}$	$+\frac{1}{6}$ to $-\frac{1}{6}$
Normally - consolidated clays	$+\frac{1}{2}$ to $+1$	$+\frac{1}{2}$ to 0
Compacted sandy clays	$+\frac{1}{4}$ to $+\frac{3}{4}$	$+\frac{3}{2}$ to $+\frac{1}{6}$
Lightly over-consolidated clays	0 to $+\frac{1}{2}$	$>+\frac{1}{2}$
Compacted clay gravels	$-\frac{1}{4}$ to $+\frac{1}{4}$	$<-\frac{5}{2}$ or $>+\frac{3}{2}$
Heavily over-consolidated clays	$-\frac{1}{2}$ to 0	$<-\frac{3}{2}$

The table and the expressions (9) show now that $\varphi'_u > \varphi'_d$ in normally-consolidated clays and also in highly sensitive clays. The value of φ'_d can now be calculated as follows :

Eq. (7) is rewritten in the form

$$a = \frac{(1 - \sin\varphi)\,\mathrm{tg}^2\left(45 + \frac{\varphi'_u}{2}\right) - 1 - \sin\varphi}{\left(1 + 2\frac{1 - \lambda}{1 + 2\lambda}\right)\mathrm{tg}^2\left(45 + \frac{\varphi'_u}{2}\right) + 2\left(1 - \frac{1 - \lambda}{1 + 2\lambda}\right)} \tag{7a}$$

From the consolidated-undrained test are obtained φ'_u and, by Eq. (3), λ. The true angle of friction φ is determined by special tests. Insertion into (Eq. (7a) gives the value of a and by insertion into Eq. (6) finally, the required value of φ'_d is obtained. The result is shown in Fig. 2.

References

[1] OSTERMAN, J. (1960). Notes on the shearing resistance of soft clays. *Acta Polytechnica Scandinavica*, Ci2, Stockholm.

[2] SKEMPTON, A. W. (1954). The pore-pressure coefficients A and B. *Géotechnique*, 4 : 4 :, pp. 143-147.