Consolidation and Secondary Time Effect of Homogeneous, Anistropic, Saturated Clay Strata

Consolidation primaire et secondaire des couches d'argile homogènes, anisotropes et saturées

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Summary

The author presents a new theory on the primary and secondary consolidation of homogeneous, anisotropic clay strata ; fundamental assumptions are based on many results of creep, relaxation and oedometer tests and on records of settlement observations published in recent years. He points out that the stress strain relationships for clays as a function of the time approximately can be described by means of linear integral equations. The general differential equations for displacement and water pressure in the anisotropic case are presented, which in the isotropic case are largely simplified. In plane strain, the clay is considered as a porous orthotropic body defined by three independent operators λ', ν', m^8, and two coefficients of permeability k_h and k_v. By means of transformation, similar equations have been obtained as in the isotropic case. Hence solutions in the anisotropic case can be directly derived from the coresponding isotropic solutions ; four methods of testing with triaxial equipments, compression plastometers, and gauge oedometers are suggested. The necessity of rheological measurements is emphasised and the role of these rheological properties in secondary settlement is analysed. An oedometer test with recording of the lateral pressure is discussed in detail ; this test result shows that the ultimate settlement and lateral pressure as a function of the time correspond satisfactorily with theory.

Sommaire

Ce rapport présente une théorie de la consolidation primaire et secondaire des couches d'argile homogènes, anisotropes, saturées ; les hypothèses fondamentales ont été choisies d'après les résultats de très nombreux essais de fluage, de relaxation, d'essais oedométriques et aussi d'observations de tassement communiquées pendant les dernières années. L'auteur remarque, que l'on peut représenter approximativement les relations tension-déformation de l'argile par des équations intégrales linéaires. Les équations générales pour les déplacements et la pression de l'eau interstitielle dans le cas du milieu anisotrope sont présentées ; dans le cas isotrope ces équations se simplifient beaucoup. En déformation plane l'argile est considérée comme un matériau orthotrope poreux, défini par 3 opérateurs λ', ν', m^8 et deux coefficients de perméabilité k_h et k_v. A l'aide de la transformation affine on peut obtenir des équations semblables à celles du cas isotrope ; ainsi on peut déduire directement des solutions du cas anisotrope à partir des cas isotropes correspondants. Quatre types d'essais utilisant des appareils triaxiaux, des plastomètres de compression et des oedomètres à jauges latérales sont proposés pour mesurer les 5 paramètres orthotropes. La nécessité des mesures rhéologiques est accentuée et le rôle des propriétés rhéologiques dans le tassement secondaire est analysé en détail ; cet exemple montre que la correspondance est suffisante entre le tassement final, la pression latérale en fonction du temps et la prévision théorique présentée.

Introduction

In order to meet the increasing demands of the rapid construction on a large scale in China, theoretical and experimental research has been started on the fundamental properties of clays, loess and rocks and their application to engineering. The author presents a part of the research on the properties of anisotropic clay deposits as they frequently occur in his country.

Since 1946 many results of creep and relaxation tests on clay samples have been published ; the deformation in tests under constant stress consists of two parts, vis. an instantaneous deformation and a deformation increasing with the time. Two types of creep-time effects have been reported :

(a) creep at a decelerating rate continued by flow at constant rate ; all these tests have been carried out with the help of the triaxial apparatus (HAEFELI, 1953 ; GOLDSTEIN, 1957), torsion plastometer (GEUZE-TAN, 1953) and compression plastic meters (TAN, 1957-1959) ;

(b) creep increasing with log t ; these tests have been performed with the ring shear apparatus (GEUZE, 1948 ; VYALOV-SKIBITSKY, 1957).

Tests with the help of the relaxation-plastometers (TAN, 1957), whereby the sample is subjected to a constant deformation, and the stress is observed as a function of time show that intensity of this internal stress decreases with the time and for fat clay samples may even be reduced to zero ; this phenomenon supplies further evidence of the tendency of clay to flow continuously under constant deviatoric stress. Further additional data concerning the time-effects have been obtained from vibration tests on clay cylinders, carried out with a special dynamic triaxial apparatus (TAN, 1958) ; these experiments show clearly that damping is sensitive to a change in frequency and this sensitivity is a measure for the visco-elastic properties of clays. In the case of flow at constant rate the viscosities of engineering clays are found to be in the range of $3 \cdot 10^{12}$ to $6 \cdot 10^{14}$ poises (HAEFLI, 1953, TAN, 1954, MASLOV, 1955).

Tests on clay samples and wave velocity measurements in the field have shown that natural clay layers have a certain amount of anisotropy and that this anisotropy increases with the clay content (LAUGHTON, 1957). As the structure of clay is built up of anisometric plates mutually interconnected at many points, it is plausible to expect that these platelets should assume a definite regular orientation, when the clay layer has been subjected to intensive uniaxial compression of sufficiently long duration (contacts type C, TAN, 1957, 1959). Such a state of stress may cause laminated, stratified clay layers with definite orientation of fissures especially under high pressures.

The first attempt to set up the stress strain relationships for porous anisotropic visco elastic materials is due to BILOT (1956). In this paper a new visco-elastic theory on the consolidation and flow of homogeneous, anisotropic saturated clays will be presented, the assumptions in which are based on results of laboratory and field tests published in international literature and on tests carried out in the author's laboratory.

Stress strain relationships

For the majority of cases there is an approximate linear relationship between the stress applied and the deformation observed at definite time intervals; this relationship holds up to a certain yield stress, after which non-linear effects and intensifield flow occurs gradually leading to failure. It has been estimated that this relationship may hold up to from 60-to 80 per cent of the failure stress. Within this linear range the deformation $\varepsilon\ (t)$ from experiments under constant stress σ_0 increases according to the time function $F(t)$ as follows :

$$\varepsilon(t) = \sigma_0 \left[\frac{1}{E} + F(t) \right] \tag{1}$$

and the stress under constant unconfined deformation ε_0 relaxes with the function $R(t)$:

$$\sigma(t) = \varepsilon_0 [E - R(t)] \tag{2}$$

In analogous manner the corresponding test results for simple may be described :

$$\gamma(t) = \tau_0 \left[\frac{1}{G} + \bar{F}(t) \right] \tag{3}$$

$$\tau(t) = \gamma_0 [G - \bar{R}(t)] \tag{4}$$

In the above expression :

E = elastic modulus;

G = shear modulus, $\bar{F}(t)$ and $\bar{R}(t)$ creep and relaxation functions for uniaxial compression and $\bar{F}(t)$ and $\bar{R}(t)$ are the corresponding functions for simple shear. If at the time $t = 0$ the material is initially free of stresses and deformations the above expressions can be generalized :

$$\begin{aligned}
\varepsilon(t) &= \frac{\sigma(t)}{E} + \int_0^t f(t-\theta)\,\sigma(\theta)\,d\theta\,; \\
\sigma(t) &= E\varepsilon(t) - \int_0^t \gamma(t-\theta)\,\varepsilon(\theta)\,d\theta\,; \\
\gamma(t) &= \frac{\tau(t)}{G} + \int_0^t \bar{f}(t-\theta)\,\tau(\theta)\,d\theta\,; \\
\tau(t) &= G\gamma(t) - \int_0^t \bar{\gamma}(t-\theta)\,\gamma(\theta)\,d\theta
\end{aligned} \tag{5}$$

Where :

$f(t) = dF(t)/dt$; $r(t) = dR(t)/dt$; $\bar{f}(t) = d\bar{F}(t)/dt$ and $\bar{r}(t) = d\bar{R}(t)/dt$.

After application of the Laplace transformation :

$$\int_0^t e^{-st} f(t)\,dt = f(s)\,; \qquad f(t) = \frac{1}{2\pi i}\int_{C-i\infty}^{C+i\infty} e^{\omega t} f(\omega)\,d\omega.$$

the above expressions become :

$$\begin{aligned}
\varepsilon(s) &= \left[\frac{1}{E} + f(s) \right] \sigma(s)\,; \\
\sigma(s) &= [\ E - \gamma(s)\ ]\ \varepsilon(s)\,; \\
\gamma(s) &= \left[\frac{1}{G} + \bar{f}(s) \right] \tau(s)\,; \\
\tau(s) &= [\ G - \bar{\gamma}(s)\ ]\ \gamma(s)\,,
\end{aligned} \tag{6}$$

which further can be written in the more rigorous form :

$$\begin{aligned}
\sigma(s) &= \lambda(s)\,\varepsilon(s) \\
\tau(s) &= \psi(s)\,\gamma(s)
\end{aligned} \tag{7}$$

when :

$$\lambda(s) = \frac{1}{E^{-1} + f(s)} = E - \gamma(s)\,;$$

$$\psi(s) = \frac{1}{G^{-1} + \bar{f}(s)} = G - \bar{\gamma}(s)$$

In the following the operators for uniaxial compression $\lambda(s)$ and the shear operator $\psi(s)$ for brevity will be written λ and ψ respectiviely and accordingly all the transformed stresses and deformations will be written without (s). In the isotropic case the six stress-strain relationships in three dimensions can be written :

$$\begin{aligned}
\varepsilon_x &= \lambda^{-1}[\sigma_x - \nu(\sigma_y + \sigma_z)] \quad \ldots\ldots \\
\gamma_{xy} &= \psi^{-1}\tau_{xy}\,; \quad \text{cyclically.}
\end{aligned} \tag{8}$$

in which ν denotes the operators for lateral expansion. So the stress-strain-relaxations are governed by three operatores; from geometrical considerations however the following relationship applies :

$$\lambda = 2\psi(1 + \nu). \tag{9}$$

Further the following expression may be derived :

$$\varepsilon = \varepsilon_x + \varepsilon_y + \varepsilon_z = (\sigma_x + \sigma_y + \sigma_z)/\Theta\,, \tag{10}$$

in which :

$\Theta = \lambda/3(1 - 2\nu)$ is the operator of volume compression; for a material whereby its volume decreases with the time under constant hydrostatic pressure, Θ is an operator such that it decreases with the time approaching a definite minimum value. It becomes clear that it should be $\nu \leqslant 1/2$, for the theoretical case that the deformations under deviatoric stresses for infinite values of the time might become unlimited, thus for $\lambda \to 0$ then ν should approach $\frac{1}{2}$, as ε and thus Θ should be limited. Then the process will approach the state of hydrostatic stress; and the clay assumes ultimately the properties of a thick viscous mass as reported by BERNATZIK (1947).

Now the case of transverse isotropy with the Z-axis as axis of symmetry will be considered; the operational stress-strain relationships can be written in the following matrix form :

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xz} \\ \gamma_{yz} \\ \gamma_{xy} \end{bmatrix} = \begin{bmatrix} \lambda_h^{-1} & -\nu_{hh}\lambda_h^{-1} & -\nu_{vh}\lambda_v^{-1} & & & \\ -\nu_{hh}\lambda_h^{-1} & \lambda_h^{-1} & -\nu_{vh}\lambda_v^{-1} & & & \\ -\nu_{hv}\lambda_h^{-1} & -\nu_{hv}\lambda_h^{-1} & \lambda_v^{-1} & & & \\ & & & \psi_2^{-1} & & \\ & & & & \psi_2^{-1} & \\ & & & & & \psi_1^{-1} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xz} \\ \tau_{yz} \\ \tau_{xy} \end{bmatrix} \tag{11}$$

in which :

λ_h , λ_v = compression operators in horizontal and vertical directions respectively, ν_{hh} = operator of lateral expansion due to horizontal stress in horizontal directions, and ν_{vh} and ν_{hv} are corresponding operators of lateral expansion due to vertical stress in horizontal directions and due to horizontal stress in vertical direction respectively, and ψ_2 and ψ_1 are shear operators acting in vertical and horizontal planes respectively.

Between these seven operators the following two relationships exists :

$$\nu_{vh}\lambda_h = \nu_{hv}\lambda_v ; \quad \lambda_h = 2\psi_1(1 + \nu_{hh}). \tag{12}$$

Further the following restrictions hold :

$$\nu_{hh} + \nu_{hv} \leqslant 1 ; \quad \nu_{vh} \leqslant 1/2$$

The stress-strain relationships can be written also in the following form :

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xz} \\ \tau_{yz} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \varphi_1 & \varphi_2 & \varphi_3 & & & \\ \varphi_2 & \varphi_1 & \varphi_3 & & & \\ \varphi_3 & \varphi_3 & \varphi_4 & & & \\ & & & \psi_2 & & \\ & & & & \psi_2 & \\ & & & & & \psi_1 \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xz} \\ \gamma_{yz} \\ \gamma_{xy} \end{bmatrix}$$

in which :

$\varphi_1 = \lambda_h(1 - \nu_{hv}\nu_{vh})/\Delta$; $\varphi_2 = \lambda_h(\nu_{hh}+\nu_{hv}\nu_{vh})/\Delta$;
$\varphi_3 = \lambda_h\nu_{vh}(1 + \nu_{hh})/\Delta$; $\varphi_4 = \lambda_v(1-\nu_{hh}{}^2)/\Delta$;
$\Delta = (1+\nu_{hh})(1-\nu_{hh}-2\nu_{vh}\nu_{hv})$; $2\psi_1 = \varphi_1 - \varphi_2$

Fundamental equations

Let us now proceed to find the equations for the stresses and deformations, when the soil skeleton is completely saturated with water. Strain components for this skeleton are given in the compatibility equations :

$$\varepsilon_x = -\frac{\partial u}{\partial x} \; ; \; \gamma_{xy} = -\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right), \quad \text{cycl.} \tag{14}$$

where u, v, w represent the displacement field.
The stress-field is subject to the three equibrium equations :

$$\begin{aligned} &\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + \frac{\partial \sigma_w}{\partial x} + X = 0 \\ &\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + \frac{\partial \sigma_w}{\partial y} + Y = 0 \\ &\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + \frac{\partial \sigma_w}{\partial z} + Z_1 = 0 \end{aligned} \tag{15}$$

with σ_w = waterpressure, and X, Y, Z = the body forces per unit mass. In (14) and (15) a shortening should be given the positive sign as pressure is considered positive.

The squeezing of the overburden pore water is governed by two coefficients of permeability k_h and k_v as follows :

$$\begin{aligned} \frac{\partial u_w}{\partial t} &= -k_h\left(\frac{\partial \sigma_w}{\partial x} + \rho_w X^*\right) \\ \frac{\partial v_w}{\partial t} &= -k_h\left(\frac{\partial \sigma_w}{\partial y} + \rho_w Y^*\right) \\ \frac{\partial w_w}{\partial t} &= -k_v\left(\frac{\partial \sigma_w}{\partial z} + \rho_w Z^*\right) \end{aligned} \tag{16}$$

with u_w, v_w, w_w = displacement components and ρ_w unit mass of the water. As the water is incompressible the continuity condition requires :

$$\frac{\partial \varepsilon}{\partial t} = \frac{\partial}{\partial t}\left\{\frac{\partial u_w}{\partial x} + \frac{\partial v_w}{\partial y} + \frac{\partial w_w}{\partial z}\right\}.$$

and then is combination with (16) the following equation in Laplace-transformation may be obtained :

$$-s\varepsilon = k_h\left(\frac{\partial^2 \sigma_w}{\partial x^2} + \frac{\partial^2 \sigma_w}{\partial y^2}\right) + k_v\frac{\partial^2 \sigma_w}{\partial z^2} \tag{17}$$

When constant body forces are assumed. With the help of (13) and (14) the equations (15) can now be written in terms of the displacements :

$$\begin{aligned} &\varphi_1\frac{\partial^2 u}{\partial x^2} + \psi_1\frac{\partial^2 u}{\partial y^2} + \psi_2\frac{\partial^2 u}{\partial z^2} + (\psi_1 + \varphi_2)\frac{\partial^2 v}{\partial x \partial y} \\ &\qquad + (\psi_2 + \varphi_3)\frac{\partial^2 w}{\partial x \partial z} = \frac{\partial \sigma_w}{\partial x} + X \\ &\psi_1\frac{\partial^2 v}{\partial x^2} + \varphi_1\frac{\partial^2 v}{\partial y^2} + \psi_2\frac{\partial^2 v}{\partial z^2} + (\psi_1 + \varphi_2)\frac{\partial^2 u}{\partial x \partial y} \\ &\qquad + (\psi_2 + \varphi_3)\frac{\partial^2 w}{\partial y \partial} = \frac{\partial \sigma_w}{\partial y} + Y \\ &\psi_2\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + \varphi_4\frac{\partial^2 w}{\partial z^2} + (\psi_2 + \varphi_3)\left[\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 v}{\partial y \partial z}\right] \\ &\qquad = \frac{\partial \sigma_w}{\partial z} + Z \end{aligned} \tag{18}$$

For the isotropic case these equations are largely simplified :

$$\begin{aligned} &-\psi\,\nabla^2 u + \left(\Theta + \frac{\psi}{3}\right)\frac{\partial \varepsilon}{\partial x} + \frac{\partial \sigma_w}{\partial x} + X = 0 \\ &-\psi\,\nabla^2 v + \left(\Theta + \frac{\psi}{3}\right)\frac{\partial \varepsilon}{\partial y} + \frac{\partial \sigma_w}{\partial y} + Y = 0 \\ &-\psi\,\nabla^2 w + \left(\Theta + \frac{\psi}{3}\right)\frac{\partial \varepsilon}{\partial z} + \frac{\partial \sigma_w}{\partial z} + Z = 0 \\ &-s\varepsilon = k\nabla^2\sigma_w \end{aligned} \tag{19}$$

The complete process of consolidation and secondary time effects in the isotropic case is thus governed by two operators Θ and ψ. These equations are generalisations of those obtained earlier (Tan, 1953, 1954) where Θ is assumed constant and the process thus is governed by one operator only (Maxwell solid.)

Plane strain

In the case of plane strain the equations (17) and (18) reduce into;

$$\varphi_1 \frac{\partial^2 u}{\partial x^2} + \psi_2 \frac{\partial^2 u}{\partial z^2} + (\psi_2 + \varphi_3) \frac{\partial^2 w}{\partial x \partial z} = \frac{\partial \sigma_w}{\partial x} + X;$$

$$\psi_2 \frac{\partial^2 w}{\partial x^2} + \varphi_4 \frac{\partial^2 w}{\partial z^2} + (\psi_2 + \varphi_3) \frac{\partial^2 u}{\partial x \partial z} = \frac{\partial \sigma_w}{\partial_e} + Z; \quad \ldots . \ (20)$$

$$- s\varepsilon = k_h \frac{\partial^2 \sigma_w}{\partial x^2} + k_v \frac{\partial^2 \sigma_w}{\partial z^2} = 0$$

It will be shown further on that these equations can be simplified by performing affine transformation. The stress-strain relationships in plane strain :

$$\begin{aligned} \varepsilon_x &= \frac{1}{\lambda_h^*} \sigma_x - \frac{\nu_{vh}^*}{\lambda_v^*} \sigma_z \ ; \\ \varepsilon_z &= - \frac{\nu_{hv}^*}{\lambda_h^*} \sigma_x + \frac{1}{\lambda_v^*} \sigma_z \ ; \\ \gamma_{xz} &= \psi_2^{-1} \tau_{xz} \end{aligned} \qquad (21)$$

are governed by the shear operator ψ_2 and 3 new operators for plane strain λ_h^*, λ_v^*, ν_{hv}^*, which are related to the operators in (11) in the followinng way :

$$\lambda_h^* = \frac{\lambda_h}{1 - \nu_{hh}^2} \ ; \ \lambda_v^* = \frac{\lambda_v}{1 - \nu_{vh}\nu_{hv}} \ ; \ \nu_{hv}^* = \frac{\nu_{hv}}{1 - \nu_{hh}} \qquad (22)$$

whereas : $\nu_{vh}^* \lambda_h^* = \nu_{hv}^* \lambda_v^*$

Next the clay will be considered as a porous orthotropic visco-elastic body described by four operators λ', ν', ψ', m^8 of which only three are independent, since the following relationship will be assumed :

$$\lambda' = 2\psi'(1 + \nu')$$

This assumption is reasonable as in many natural clay deposits the degree of anisotropy has been measured no to exceed the ratio $\lambda_h/\lambda_v \leqq 3$. Let the ratio λ_h^*/λ_v^* be denoted by m^8, then the following relationships will be introduced :

$$\begin{aligned} &\lambda_h^* = m^4 \lambda' \ ; \ \lambda_v^* = \lambda'/m^4 \ ; \\ &\nu_{vh}^* = \nu'/m^4 ; \ \nu_{hv}^* = m^4 \nu' \end{aligned} \qquad (23)$$

Hereafter affine transformation will be performed to the equations (20) which is defined by :

$$x = mx' ; \quad z = z'/m ; \quad u = u'/m ; \quad w = mw' \quad \ldots . \ (24)$$

This transformation involves the following operations :

$$\begin{aligned} \varepsilon_x &= \varepsilon_x'/m^2 \ ; & \varepsilon_z &= m^2 \varepsilon_z' \ ; \\ \sigma_x &= m^2 \sigma_x' \ ; & \sigma_z &= \sigma'_z/m^2 \ ; \\ k_h &= k'_h/m^2 \ ; & k_v &= m^2 k'_v \ ; \\ X &= mX' \ ; & Z &= Z'/m \ ; \end{aligned}$$

$$\begin{aligned} &\gamma_{xz} = \gamma'_{xz} \ ; \ \tau_{xz} = \tau'_{xz} \ ; \ \psi_2 = \psi' \ ; \\ &\bar{X} ds = m \bar{X}' ds \ ; \ \bar{Z} ds = \bar{Z}' ds/m \ ; \\ &\frac{\partial \sigma_w}{\partial x} = m \frac{\partial \sigma_w'}{\partial x'} \ ; \ \frac{\partial \sigma_w}{\partial z} = \frac{1}{m} \frac{\partial \sigma_w'}{\partial z'} \end{aligned} \quad \ldots . \ (25)$$

It may be noticed that this transformation from the coordinate axes $x - z$ to the new axes $x' - z'$ does not affect the shear, the shearing stress and the shearing operator. (13), (21) and (23) it may be derived further that :

$$\varphi_1 = \frac{m^4 \lambda'}{1 - \nu'^2} \ ; \ \varphi_2 = 0 ; \quad \varphi_3 = \frac{\lambda' \nu'}{1 - \nu'^2} ;$$

$$\varphi^4 = \frac{\lambda'}{m^4 (1 - \nu'^2)} \ ; \ \psi' = \frac{\lambda'}{2(1 + \nu')} . \qquad (26)$$

Substitution of (24) and (25) and (26) into (20) gives directly;

$$- \psi' V^2 u' + \frac{1 + \nu'}{1 - \nu'} \psi' \frac{\partial \varepsilon'}{\partial x'} + \frac{\partial \sigma_w'}{\partial x'} + X' = 0 \ ;$$

$$- \psi' V^2 w' + \frac{1 + \nu'}{1 - \nu'} \psi' \frac{\partial \varepsilon'}{\partial z'} + \frac{\partial \sigma_w'}{dz'} + Z' = 0 \ ; \qquad (27)$$

$$- s\varepsilon' = k_h' \frac{\partial^2 \sigma_w}{\partial x'^2} + k'_v \frac{\partial^2 \sigma_w}{\partial_e'^2} .$$

when : $\varepsilon' = \varepsilon'_x + \varepsilon'_z$

From the first two equations (27) it may be derived :

$$V^2 \varepsilon' = - \frac{1 - \nu'}{2\psi'} V^2 \sigma'_w \qquad (28)$$

For the solution of the problem the affine transformation should be performed also on the boundary conditions. Then the solution referred to the new transformed coordinate axes $x' - z'$ can be found. For the final solution in the original $x - z$ system, however, back transformation with the help of (24) and (25) should be carried out, and hereafter the deformation, stress-distribution and water pressure as a function of the time can be obtained by inversion of their Laplace transformations. It occurs that the first two equations of (27) referred to the transformed $x' - z'$ system is similar to the corresponding equations in the isotropic case referred to the original $x - z$ system. This becomes clear when in (14) the isotropic plane strain parameter $\nu' = \nu_c/(1 - \nu_e)$ will be substituted. Hence solutions of the isotropic case in the original $x - z$ system are simultaniously also solutions of the transverse isotropic case in the $x' - z'$ system, provided the influence of the water-pressure σ_w can be neglected as it is the case with the calculation of instantaneous settlement, the maximum settlement due to consolidation and the settlement due to secondary time effects.

If only roughly approximate solutions are wanted, then it may be tried :

$$- s\varepsilon' = k'_h \frac{\partial^2 \sigma_w'}{\partial x'^2} + k'_v \frac{\partial^2 \sigma_w}{\partial_z'^2} \sim \sqrt{k'_h k'_v} \, V^2 \sigma_w' \quad \ldots . \ (29)$$

and the complete process including the primary, secondary consolidation and the water pressure as a function of the time can be derived directly by back transformation from the corresponding isotropic case.

Experimental determination of the rheological operators

It may be recalled that the process of consolidation and secondary time effects of homogeneous, orthotropic clay-layers in plane strain is governed by three independent operators : λ', ν', m^8 and two coefficients of permeability k_h and k_v. The measurement of these five quantities will not be a simple procedure, as clay is a two-phase material, viz. a porous soil-skeleton saturated with water. For the evaluation of these three operators from experimental data, the following relationships may be summarized :

$$m^8 = \frac{\lambda_h^*}{\lambda_v^*} = \frac{\lambda_h}{\lambda_v}\,\frac{1 - \nu_{hv}\nu_{vh}}{1 - \nu_{hh}^2} = \frac{\varphi_1}{\varphi_4} \;;$$

$$\nu' = m^4\nu_{vh}^* = \nu_{hv}^*/m^4 \;;\; \nu_{hv}^* = \frac{\nu_{hv}}{1 - \nu_{hh}} = \frac{\varphi_3}{\varphi_4} \;;$$

$$\lambda' = \sqrt{\lambda_h^*\lambda_v^*} = m^4\lambda_v^* = \lambda_h^*/m^4 \;;$$

$$\lambda_v^* = \frac{\lambda_v}{1 - \nu_{vh}\nu_{hv}} \;;\; \lambda_h^* = \frac{\lambda_h}{1 - \nu^2_{hh}}.$$

In these expressions we have to deal with froe operators λ_h, λ_v, ν_{hh}, ν_{vh}, and ν_{hv}, of which ohly four are independent because of the relationship (12). Thus it is necessary to perform at least four independent types of tests as for instance :

I. Uniaxial compression tests on clay cylinders loaded along their symmetry-axis of anisotropy Z,

II. Similar tests in a direction perpendicular to this Z axis ;

III. Confined consolidation tests in Z direction;

IV. Similar tests in a direction perpendicular to the Z axis.

For the experiments type I and II use can be made of triaxial equipments, and of compression plastometers, which have been designed especially for creep-and flow tests. The following loading system will be recommended : the sample is subjected to a constant vertical stress for so long a period that its continuous flow effects can be recorded adequately; then the vertical stress is increased by the same amount during the following time interval. This loading system is continued until failure; at every interval the deformation is recorded as a function of time. In this manner it is possible to study the instantaneous deformations and the creep-flow effects and to measure the failure strength on the long term. These tests data enable us to determine E and the flow function $F(t)$ and thus to calculate λ_h and λ_v in the linear range of the stress-strain-relationships. These creep-test data are required for the computation of instantaneous settlement and the secondary time-effects due to deviatoric stresses.

The tests type III and IV will enable us to determine ν_{hh}, ν_{vh} and ν_{hv}, by measuring φ_1, φ_4 and φ_3/φ_4. It may be pointed out that φ_1, φ_3 and φ_4 are operators; since they vary with the duration of loading and moreover the time-effect of the waterpressure is always related to these time effects, the measurement of these operators becomes very intricate. These quantities, however, can readily be determined for small and very large values of the time by measuring the settlement and the lateral pressure. For this purpose the gauge œdometer has been developed, which enables us to record the lateral pressure as a function of time (TAN, 1957, 1958). For the analysis of these test data the following formulae have been derived :

(a) Tests type III :

for $t \to 0$:

$$w_0 = \frac{2q}{\varphi'_4}\sqrt{\frac{c_v t}{\pi}} \;;\; \bar{\sigma}_e = q - 2q\left(1 - \frac{\varphi_3'}{\varphi_4'}\right)\sqrt{\frac{C_v t}{\pi h^2}} \quad \ldots\ldots (30a)$$

for $t \to \infty$: $\quad w_\infty = \dfrac{hq}{\varphi_4''} \quad ; \quad \bar{\sigma}_e = \dfrac{\varphi_3''}{\varphi_4''}\,q, \quad \ldots\ldots (30b)$

where $C_v = k_v\varphi_4'$;

(b) Tests type IV :

for $t \to 0$: $\quad u_0 = \dfrac{2q}{\varphi_1'}\sqrt{\dfrac{C_h t}{\pi}} \quad \ldots\ldots (31a)$

for $t \to \infty$: $\quad u_\infty = \dfrac{hq}{\varphi_1''} \quad (31b)$

where $C_h = k_h\varphi_1'$.

In above formulae : u_0, u_∞, w_0, w_∞ settlements in x and z directions : q = loading intensity; h = thickness of sample; C_v and C_h coefficients of consolidation in vertical and horizontal directions; σ_e = mean lateral pressure; φ_1', φ_3', φ_4' are constants denoting the approximations for φ_1, φ_3, φ_4 for small values of time, whereas these operators in the ultimate state are approximated by the constants φ_1'', φ_3'' and φ_4'' respectively. The quantities φ_1', C_h, φ_4' and C_v can be obtained by applying Taylor's curve fitting method after plotting u_0 and w_0 against $\sqrt{t'}$; simultaneously k_h and k_v can be calculted. Further φ_3'/φ_4' can be computed from the slope of the $\sigma_e - \sqrt{t'}$ curve. The quantities φ_1'' and φ_4'' can be obtained from long duration tests. For the isotropic case the following reductions hold :

$$\varphi_1' = \varphi_4' = \varphi' = \frac{3(1 - \nu_e)}{(1 + \nu_e)}\,\Theta \;;\; \varphi_1'' = \varphi_4'' = \varphi'' = \Theta \;;$$

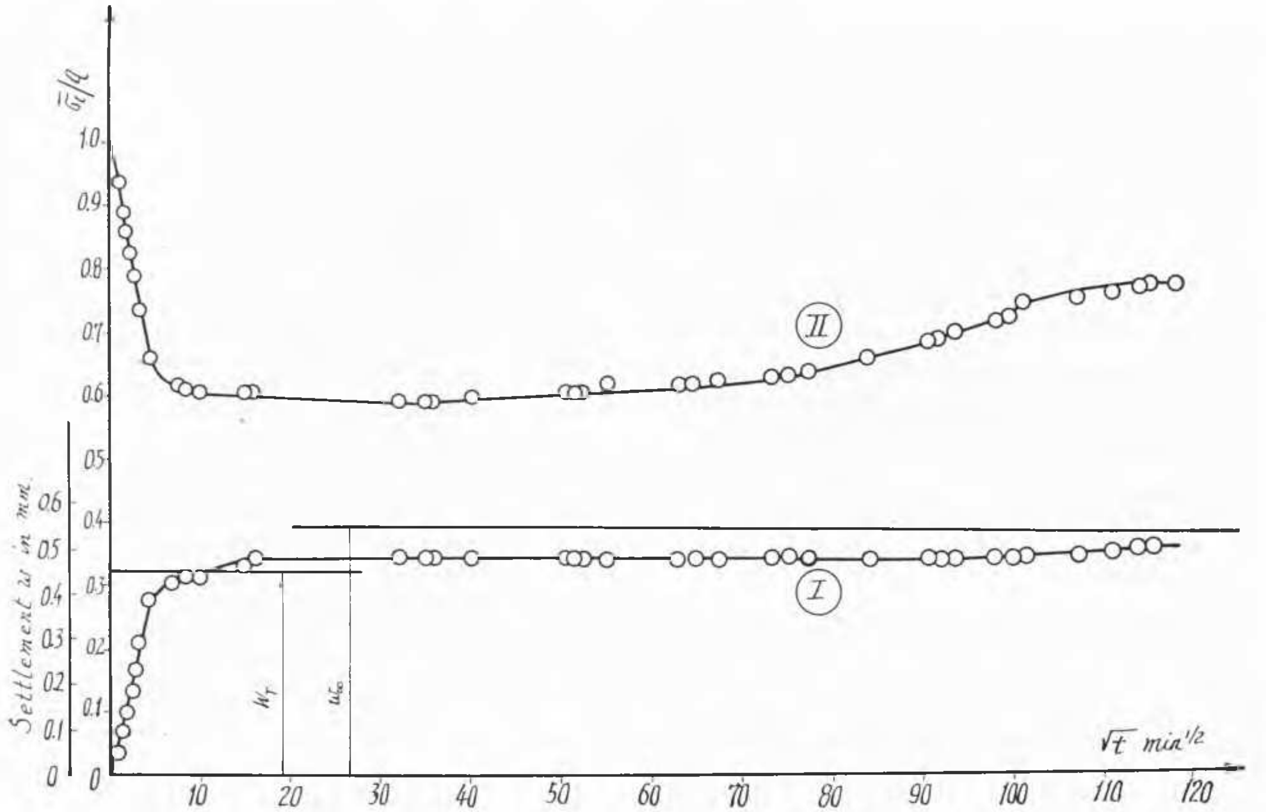
$$C_v = C_h = \frac{3(1 - \nu_e)}{1 + \nu_e} \;;\; \varphi_3'/\varphi_4' = \frac{\nu_e}{1 - \nu_e} \quad (32)$$

and it may be recalled; Θ = compression modulus, ν_e = elastic part of operator of lateral expansion ν. This isotropic case renders it possible to compute φ'' from φ', when ν_e has been determined from the $\bar{\sigma}_e - \sqrt{t}$ curve.

For practical purposes the displacements on the long term are the most important and then it well be sufficient to take only the values of the operators for large values of the time. In practice samples are taken usually from vertical boreholes with a diameter of less than 15 cm. Then the creep tests type II can not be performed easily and we are restricted to the tests type I, III and IV. As sufficient data no longer can be provided for the computation of the 5 operators, it will be proposed to introduce the approximation $\nu_{vh}\nu_{hv} \sim \nu_e^2$ and to estimate φ_1', φ_1'', φ_4' and φ_4'' with the help of (30, 32) and (31, 32). This computation will be illustrated now by means of the following experimental example. The figure shows the $w_0 - \sqrt{t}$ curve (1) and the $\bar{\sigma}_e/q - \sqrt{t'}$ curve (II) for a dense, stiff clay specimen with a watercontent of 21·5 per cent and a height of 2 cm loaded by q = 2·1 kg/cm². Firstly the ultimate settlement after Terzaghi W_T can be calculated according to

Taylor's method giving $W_t = 0{\cdot}45$ mm, and then the following quantities can be determined : $\varphi'_4 = 93$ kg/cm^2; $C_v = 1{\cdot}8 \times 10^{-3}$ cm/sec^{-1}; $h_v = \dfrac{k}{\rho_w} = 9{\cdot}1 \times 10^{-6}$ cm/sec^{-1}.
It occurs that the settlement exceeds this theoretical value and obviously approaches another asymptote, which can be estimated with the help of (30) and (32). Now determine φ_3'/φ_4' from the slope of the $\bar\sigma_e/q$-$\sqrt{t'}$ curve. If we further assume $\varphi_3'/\varphi_4' \sim \nu_e/(1 - \nu_e)$ then we find $\nu_e \sim 0{\cdot}41$ and the settlement after the author's isotropic theory can be calculated $w_\infty = 3(1 - \nu_e)/(1 + \nu_e) \cdot W_T \sim 0{\cdot}55$ mm and thus $\varphi_4'' \sim 76$ kg/cm^2.

The lateral stress decreases initially with time until a definite minimum is reached, but then gradually rises to a maximum. This test-result is common for dense clays, but this increase has been not yet observed for loose clays. The decrease is due to the waterpressure, which relaxes with time. Simultaneously shear and volume creep occur, thereby gradually an increasing part of the stress is transferred to the rigid ring of the gauge œdometer ; so the interrelation of the two complex processes, consolidation and flow, has been measured. This test result is in agreement with the theory presented here, which predicts that the operator of lateral expansion ν should increase with the time leading to a gradual rising of the lateral pressure to its ultimate maximum. In above example the operator ν increases from 0·41 to 0·44, when computed with the help of (30b) and (32).



Settlement as a function of $\sqrt{t}$ (Curve I)
Ratio lateral/vertical pressure $\bar\sigma_e/q$ as function of $\sqrt{t}$ (Curve II)

Tassement en fonction de $\sqrt{t}$ (Courbe I)
Rapport pression latérale/verticale $\bar\sigma_e/q$ en fonction de $\sqrt{t}$ (Courbe II)

Conclusion

In the discussions during the Fourth International Conference in London 1957 it was concluded that the majority of settlements observed in practice is larger than predicted by the current methods of computation. It has been pointed out that this fact may be due to the following factors : (a) effect of deviatoric stresses causing an instantaneous deflection followed by secondary time effects due to lateral flow ;

(b) secondary time-effects due to volume creep as measured in œdometer tests. Therefore it is necessary to determine the rheological properties and the measuring technique suggested above is fairly simple. In practice two main types of secondary time effects have been recorded (Terzaghi, 1953) : the settlement proceeds at constant rate or increases linearly with log t. As it has been analised above this secular settlement is mainly governed by the shear operator ψ. In the first case of constant rate the assumption of a porous Maxwell body may be satisfactory and the shear operator may be wriiten : $\psi = \eta G s/(G + \eta s)$; in the second case a good approximation may be : $\psi = \eta G/(G - \eta \log cs)$;

For the determination of ultimate settlement in the one dimensional case, which is larger than predicted from Terzaghi's theory, simultaneous measurements of lateral pressure and settlement are required, but the calculation itself is a simple procedure. In order to estimate secondary settlement in more dimensional cases long term creep and flow tests are required ; the computation of settlement due to deviatoric stresses is similar to that in the elastic theory except that for the shear modulus G the shear operator ψ should be substituted. For plane strain in the transversely isotropic case the stress and deformation fields can readily be derived from the corresponding isotropic case by back affine transformation.

The author states that in China multi-layered clay deposits hundreds of metres deep frequently occur, consisting of regularly alternating plane parallel layers several metres deep. By ensuring the continuity of displacements of the consecutive layers it is possible to compute the equivalent anisotropic parameters of the entire clay mass and to bring this problem back to the case of transverse isotropy.

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