Problems of the Rheology of Soils

Sur la rhéologie des sols

by S. S. Vialov, Professor, Dr. of Technical Sciences, Frozen Ground Research Institute, U.S.S.R. Academy of Sciences, Moscow
and
A. M. Skibitsky, Engineer, Research Division, Hydroproject, Moscow

Summary

Clay soils and all frozen soils possess pronounced rheological properties. The authors give results of research devoted to problems of the rheology of soils, representing a further development of their work published in the Proceedings of the Fourth Intern. Conference [1].

The initial rheological relationships, are considered in the first section the second section being concerned with the rheological phenomena applicable to a compound stress condition. Principles for calculating creep and continuous strength of soils are given. In the third section, the peculiarities of creep in dense clays are considered. These are explained on a rheological model, and a law of variation of viscosity is established.

Sections 1 and 2 were written by S. S. Vialov, and section 3 by A. M. Skibitsky.

Sommaire

Les sols argileux, aussi bien que tous les sols gelés, ont des propriétés rhéologiques définies. Le présent rapport expose les résultats de recherches expérimentales et théoriques consacrées aux questions de rhéologie des sols ; ils représentent le développement des recherches des mêmes auteurs qui ont été publiées dans les travaux du IV^{e} Congrès [1].

Dans la première partie de ce rapport on donne les relations rhéologiques de base ; dans la seconde partie sont exposées les règles du processus rhéologique dans le cas de contraintes complexes ; cette partie comprend également les principes du calcul du fluage des sols et leur résistance dans le temps. Les particularités du fluage des argiles denses sont examinées dans la troisième partie : elles sont représentées par un modèle rhéologique ; on a aussi établi une loi de la variation de la viscosité des argiles denses.

Les deux premières parties sont dues à S. S. Vialov. L'auteur de la troisième est A. M. Skibitsky.

1. Rheological processes in soils.

Many soils — clays, silts, and all frozen soils — have rheological properties, i.e., the ability to develop creep deformations and reduce strength under continued loading. The character of the rheological phenomena depends upon soil properties, but they generally obey common laws that are reflected by typical experimental curves, Fig. 1.

The process of creep characterized by curves (a) differs depending on the magnitude of the load : if $\tau < \tau_\infty$ (where τ_∞ is the ultimate continuous strength), the deformations are damped and failure does not occur, but if $\tau > \tau_\infty$, then non-damping deformations occur. The latter include the following stages : OA — shock loading, AB — variable, BC — steady (plastic-viscous flow at a constant rate), and CD — progressive (at an increasing rate). It has already been shown [1] that the principal stage in weakly sensitive diluted soils is that of steady flow, which does not necessarily pass into the progressive stage. In the case of plastic and frozen soils, we find both the variable and the steady flow stages develop, with the latter inevitably passing into the progressive stage, culminating either in brittle failure or in total plastic deformation of the sample (without disrupting continuity). In the case of dense clays [1], the principal stage is the variable stage, which, for $\tau > \tau_\infty$, leads to failure (and, for $\tau < \tau_\infty$, to stabilizing of deformations).

In the general case, the relationship between stress τ and strain γ is non-linear and is reflected by a family of mutually similar curves, the parameter of which is the time during which the load acts (Fig. 1b). Experiments [2] show that these curves have points of inflection at $\tau = \tau_s$ (variable yield point), in other words, they consist of two portions of different curvature, the first portion $O\tau_s$ being nearly linear for several soils.

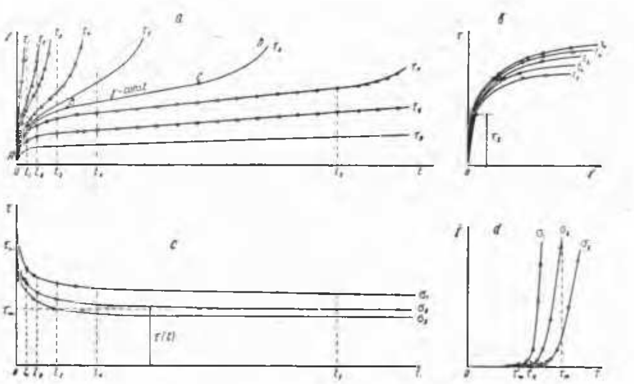

Fig. 1 Rheological curves (shear test data of frozen sandy soil).
(a) Creep curves at $\tau_1 > \tau_2 > \tau_3 > \ldots$;
(b) stress-strain relation at various instants of time $t_1 < t_2 < t_3 \ldots$;
(c) curves of continuous shear strength at various normal stresses $\sigma_1 > \sigma_2 > \sigma_3$;
(d) shear stress in relation to the rate of plastic-viscous flow at various values of σ.

Courbes rhéologiques (Essais de cisaillement sur limon sableux gelé).
(a) Courbes de fluage pour $\tau_1 > \tau_2 > \tau_3 \ldots$;
(b) relation entre la tension (τ) et la déformation aux temps $t_1 < t_2 < t_3 \ldots$;
(c) courbes de résistance au cisaillement pour différentes contraintes normales ;
(d) relation entre l'effort de cisaillement (τ) et la vitesse d'écoulement visco plastique (γ) pour différentes valeurs de σ.

The process of diminishing strength is reflected by curve (*C*) — the magnitude of the failure stress relative to time is shown during which progressive flow (failure) sets in. The initial ordinate of the curve determines the conditional-instantaneous strength τ_0, and the ordinate of any point determines the continuous strength $\tau(t)$ at time t, while the asymptote of the curve determines the ultimate continuous strength τ_∞. The ratio τ_∞/τ_0 fluctuates approximately between 0·8 and 0·5 for dense and medium-dense clays and 0·08 for frozen soils. The principal reduction in strength occurs in the initial period of time. For the majority of soils, τ_∞ is a practically real value, the greater the more sensitive the soil. For ice, $\tau_\infty = 0$, for diluted soils, τ_∞ is measured in hundreds of kg/cm², for plastic soils, tenths of kg/cm², for dense clays and plastic frozen soils, up to 1-2 kg/cm², and for solidly frozen soils, up to 5-10 and even 20 kg/cm².

The relationship between the stress and the constant rate of steady flow $\gamma = \frac{d\gamma}{dt}$ = a constant, is reflected by the rheological curve (Fig. 1). Inasmuch as such flow arises when $\tau > \tau_\infty$, and when $\tau < \tau_\infty$, only damping deformations are possible, the curve $\dot{\gamma}$-τ begins from point τ_∞ i.e., τ_∞ may be identified with the Bingham limit shear stress. Experiments |2| show that in the general case the $\dot{\gamma}$-τ relation is non-linear. Within a certain range of stresses $\tau_\infty < \tau < \tau_k$, the flow (according to Rebinder) occurs without noticeable failure of the structure and is approximately linear (Bingham). When $\tau > \tau_k$, intensive failure begins, the rate of flow increasing and the coefficient of viscosity η varying between wide limits, being a function of τ. In the case of large $\tau > \tau_m$, there occurs a limiting failure of the structure accompanied by shear failure in sensitive soils, or flow with max η = a constant in the case of wet soils.

2. Methods for calculating creep and continuous strength

Engineering calculations of soils that possess rheological properties must be carried out with respect both to continuous strength and to creep. The strength calculation consists in determining the ultimate (breaking) load at which the stresses at a given instant do not exceed the continuous strength of the soil at this instant of time. Deformation calculations involve determining the load at which the deformations of creep (or their rates) at a given instant of time do not exceed the permissible limiting (specified) value. Two cases are possible :

(1) only stabilizing deformations are permissible, and

(2) non-damping deformations (of restricted magnitude) are permissible. In the first case, the stresses should be less than the ultimate-continuous (τ_∞), and in the second they can be greater than the latter.

The calculations may be performed on the basis of general methods of plasticity theory, but supplemented by a condition that determines the variation of the stress-deformation state of the soil with time. The stressed state of a mass of soil is complicated, and the foregoing condition should relate the intensity of stresses :

$$T = \sqrt{\frac{2}{3}(\tau_1^2 + \tau_2^2 + \tau_3^2)}$$

the intensity of deformations :

$$\Gamma = \sqrt{\frac{2}{3}(\gamma_1^2 + \gamma_2^2 + \gamma_3^2)}$$

and the intensity of rates :

$$\dot{\Gamma} = d\Gamma/dt.$$

The corresponding graphs will be analogous to Fig. 1. The values of T, Γ, and $\dot{\Gamma}$ are determined from a triaxial test under conditions of creep.

The rheological equation relating the intensity of deformations $\Gamma(t)$ at any instant of time with the time-varying intensity of stress may be written in integral form (Volterra-Boltzman-Rabotnov) and is of a form similar to equation (4) given in reference [1]. When T = a constant, this equation takes on the form :

$$\Gamma(t) = \frac{T}{2G} + \varphi(T)\int_0^t k(t)\,dt = \Gamma_0 + T^{1/\alpha}\int_0^t k(t)\,dt \quad \ldots. \quad (1)$$

Where $\Gamma_0 = T/2G_0$ is the conditional-instantaneous deformation, and the function $\varphi(T)$ characterizes the relationship between T and Γ and is determined from a graph of the type given in Fig. Ib; in the general case, it may be taken as $\varphi(T) = T^{1/\alpha}$ (see equation (2)). The function $K(t) = \dot{\Gamma}/\varphi(t)$ is the creep function determining the rate of deformation with time in the case of a single stress. It is determined experimentally from the creep curves (Fig. 1a) reconstructed in $\dot{\Gamma} - t$ coordinates. For damping creep, $k(t)$ is taken by different authors (Florin, Vialov, Skibitsky, Goldstein) in the form either of a hyperbolic function

$$K(t) = \frac{a}{t^n + b}$$

or an exponential function $K(t) = at^{m-1}e^{-(nt)^m}$ or in combination

$$K(t) = \frac{ae^{-nt}}{t^{1-\mu}}$$

($n = m = 1$, $b = \mu = 0$, $b = e/c$ are special cases). Investigations into type (1) equation and its application to the solution of problems of soil mechanics are given in several works by Soviet scientists (Florin, Arutunyan, Vialov, Goldstein, etc.). Equation (1) takes into account the variation in compaction of soils during deformation. Using Arutunyan's theory, we should regard $k(t)$ and G as varying with time ν according to a given law corresponding to the law of compaction. This problem together with that of creep in the soil as well as percolation have been considered by Florin and Meshan.

The function $\int_0^t k(t)\,dt$ may also be given in parametric form, thereby simplifying calculation. To do this, as is shown in |2|, the authors proceed from a graph of the type in Fig. 1b, constructed in $T - \Gamma$ coordinates. According to experiments |2|, the curves $T - \Gamma$ may be described by the law

$$\Gamma(t) = A(t)T^{1/\alpha} \quad \text{or} \quad T = \beta(t)\Gamma^\alpha, \qquad (2)$$

where $\beta(t)$ is the coefficient of deformation corresponding to each of the curves. It is necessary to introduce into the calculations those values of $\beta(t)$ which correspond to the given time t. The calculation itself does not differ in any way from that dealing with the theory of plasticity with strengthening.

If steady flow continues in the soil for a considerable time and if the soil during this time is assumed to be stressed, the strain calculation may be performed on the basis of the rate of this flow (which must not be greater than specified) assuming |1|.

$$\dot{\Gamma} = \frac{d\Gamma}{dt} = k(T - T_\infty)\beta,$$

where the parameters k and β are determined from a graph of the type in Fig. 1c. For $\beta \to 0$ (3) passes into the Bingham equation, which also at times applied (Maslov, Geuse) to soils.

Experiments [2, 3] show that the rate of flow due to shear depends on the normal stress σ_n (Fig. 1c).

For cohesive soils an allowance must be made both for cohesion c and for "friction" φ; this was first done by Maslov [3,4]. He proposed an equation that combines the conditions of Coulomb and Bingham, with due allowance for variable viscosity.

$$\dot{\gamma} = \frac{\tau - (c + \sigma_n \operatorname{tg} \varphi)}{\eta_\infty - (\eta_\infty - \eta_0) e^{\mu t}} \tag{4}$$

The effect of "friction" may likewise be accounted for by proceeding from the generalized condition of a limiting state (Stroganov, Grechishchev), by putting in (3),

$$T_\infty = \bar{c} + \bar{\sigma} \ \operatorname{tg} \bar{\varphi}, \tag{5}$$

where $\bar{\sigma}$ is the mean stress, and $\bar{c}$ and φ are parameters of a straight line constructed on the $\tau - \bar{\sigma}$ coordinates.

Strength computations with account of creep may be performed in the usual manner on the condition of the limit stress state [5], but with the time factor taken into consideration.

In the creep process two critical states are operative [1]: the origination of steady plastic-viscous shearing flow and the transition of this flow to the progressive stage which leads to failure (Fig. Ia). It is this latter stage that should be regarded as the condition of limiting stress. It sets in when the acting tangential stresses prove to be equal to the shear strength at the given instant of time. Account of the variation in time of the shear strength may be taken, as suggested in [2], in parametric form; it is then possible to assume the usual condition of limiting stress, thus :

$$\tau(t) = c(t) + \sigma_n \operatorname{tg} \varphi(t) \tag{6}$$

the only difference being that the parameters C and φ vary with time, from the maximum, "instantaneous" values to the minimum ultimate-continuous values (it should be noted that C varies more than φ).

The envelopes of Mohr's circles (that determine C and φ) are constructed as follows. For each value σ_n we build a family of creep curves (Fig. 1a), and from them, the curves of continuous shear strength (Fig. 1c); the law of time-variation of $\tau(t)$ is given in reference [1], equation (7).

Rearranging the latter curves in the $\tau - \sigma_n$ coordinates, we obtain the family of envelopes of Mohr's circles, the parameter of which is the time t of action of the shearing stress (Fig. 2). In the general case, the equation of these curves will be non-linear $\tau = F(\sigma_n, t)$. In the special case when the envelopes are rectilinear, we obtain equation (6). We may, similar to (5), proceed from the generalized condition of the state of limiting stress, and construct the family of envelopes in $T - \bar{\sigma}$ coordinates.

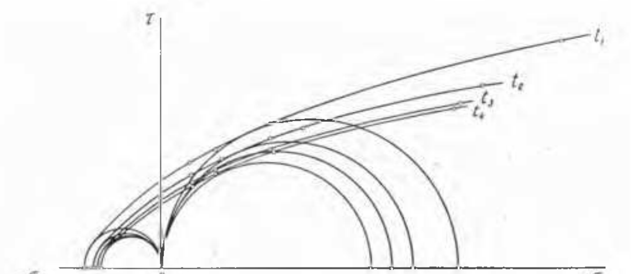

Fig. 2 Envelopes of Mohr's circles for various instants of time; $t_1 > t_2 > t_3 > \ldots$.
Enveloppes de Mohr à des instants différents.

In the computations, we use those values of $C(t)$ and $\varphi(t)$ that correspond to the given instant of time. In the case of a continuous applied load, the computation is conducted on the basis of the ultimate continuous values C_∞ and φ_∞, i.e., the ultimate continuous strength (this precludes the possibility of non-damping creep appearing). The same applies to those soils (for instance, dense clays) for which the strength reduction to τ_∞ takes place very rapidly and which should not be calculated for intermediate values $\tau(t)$.

3. Peculiarities in the creep of dense clays

As has already been shown [1], shear deformation of dense clays with time may be described by the following equation :

$$\gamma = \gamma_0 + b \lg(1 + t). \tag{7}$$

which combines the Buisman equation (1936) with the Pokrovsky equation (1934).

This equation corresponds to equation (1) when :

$$k(t)\, \varphi(\tau) = \dot{\gamma} = \frac{b}{1+t} \cdot \frac{1}{ln 10} \tag{8}$$

New data were obtained as a result of investigations in 1958 and 1959. Numerous experiments were carried out with continuous shear tests for various τ, σ, and $\Delta\tau$, consolidation tests at various $\Delta\sigma$, and for relaxation in undrained triaxial and shear tests.

It was found that for τ = a constant and for one and the same σ, the rates of creep for the same time t are approximately proportional to τ. The coefficient of rate of creep "b" as a function of τ is shown in the graph in Fig. 3. From Fig. 3 it will be likewise seen how significantly "b" depends upon σ.

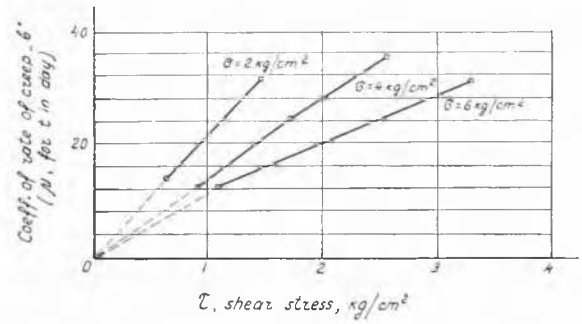

Fig. 3 Creep rate versus shearing stress τ for various σ (dense clay).
Relation entre la vitesse de fluage et l'effort de cisaillement τ pour des différentes valeurs de σ (argile dense).

Thus, for different τ = a constant and for one and the same σ we may assume :

$$\gamma = \gamma_0 + \tau \cdot b_0 \lg(1 + t) \quad \ldots . \tag{7a}$$

On the other hand, investigations into the process of secondary compression in consolidation tests showed proportionality of the rates of compression not to the total value of σ, but to the magnitude of the step of σ, i.e., $\Delta\sigma$. Whence, for compression, we have :

$$\gamma_i = \gamma_{0i} + \Delta\sigma_i\, b_0 \lg(1 + t) \tag{7b}$$

The method of step loading was also used in continuous shear tests, with each step τ maintained during the time

Δt = 5-15 days, at Δt = a constant. The experiments were :

(a) parallel tests for various $\Delta\tau$ = const (0.5 - 0.25 - 0·125 kg/cm^2) and

(b) tests in which $\Delta\tau = A$ and $\Delta\tau = 2A$ were alternated.

The experiments showed that for dense clays there exists a proportional relationship between the coefficient of the creep rate "b" and the magnitude of a step $\Delta\tau$ (for the same τ).

Also recorded was a distinctly pronounced proportional relationship between the coefficient "b" and the total value of τ for $\Delta\tau$ = const. Generally, by diminishing the value of $\Delta\tau$ as τ increases (with $\Delta\tau \cdot \tau$ held constant), it was possible to maintain a practically constant "b" within a broad range of values of τ.

At present, similar tests are being conducted on a triaxial apparatus.

We can assume that the strengthening of the dense clays in the process of creep (which strengthening leads to creep ceasing at $\tau < \tau_\infty$) is associated mainly with diminishing viscosity as γ increases. Such strengthening is also suggested by continuous shear tests, in which the regular step τ was preceded by a stepwise unloading of the specimen to zero, with subsequent increase of τ to the former value by the same steps. In the case of repeated loading, the value "b" proved to be considerably less for the same τ (Fig. 4).

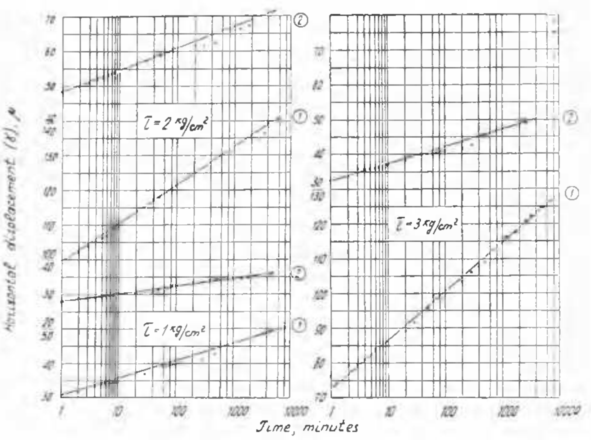

Fig. 4 The effect of the history of loading dense clay on the rate of creep :
1) first loading; 2) second loading.

Influence des conditions de chargement de l'argile dense sur la vitesse de fluage :
1) premier chargement; 2) second chargement.

Thus, as a first approximation, the creep process of dense clays may be described on the basis of a rheological model that provides for redistribution of stress between the elastic and viscous elements in the cours of creep, i.e., a model similar to the Kelvin body model. By describing the part of τ in elastic elements of the soil by a certain function $f(\gamma)$ and including, conditionally, in $f(\gamma)$ the frictional resistance (which in clays may be considered mobilized also in line with deformation) we shall have

$$\dot{\gamma} = \frac{\tau - f(\gamma)}{\eta}. \qquad (9)$$

In the Kelvin body model, it will be recalled, $\eta = a$ constant, which is not confirmed for colloidal systems by Rebinder's studies. To determine the law of $\eta(t)$, we equate (9) and (8) and, solving the resulting equation for η, when $b = \tau \cdot b_0$, we have

$$\eta = \frac{\tau - f(\gamma)}{\tau} \cdot \frac{(1+t) ln 10}{b_0}. \qquad (10)$$

The results of computations on the basis of (10), carried out on the assumption that $f(\gamma) = \alpha\gamma$, show that η increases (up until creep cease) over 1000 times as compared with η by the end of the first minute, varying approximately from 1×10^{10} to 1×10^{14} pauses.

In the absence of strengthening with increasing γ, i.e., when $f(\gamma) = 0$, we have

$$\eta_0 = \frac{(1+t) ln 10}{b_0}. \qquad \text{.... (10a)}$$

This expression gives the variation of viscosity.

In the case of stepwise loading, the time t is recorded from the time of application of each successive load, in other words, the variation of η with time is repeated for each step.

The influence of strengthening on the magnitude of η is accounted for by the factor

$$\frac{\tau - f(\gamma)}{\tau}.$$

The physical meaning of this factor is that soil creep is mainly a contact process, regulated both by the viscosity of an individual contact and by the number of these contacts. During creep (with unloading of the viscous component, which, as may be supposed, has an initial shear strength) this number will inevitably diminish with consequent diminution of the factor

$$\frac{\tau - f(\gamma)}{\tau}.$$

However despite this reduction, η will increase in approximate proportion to $t^{3/4}$, and a sharp decay of η will begin only when a state of static equilibrium is approached.

The Kelvin body model which is used to explain creep corresponds to non-relaxation bodies. When the viscous element in this model is replaced by an elasto-plastic viscous element (its counterpart is the Bingham-Schwedoff body) it is possible to describe processes of relaxation. For a complete

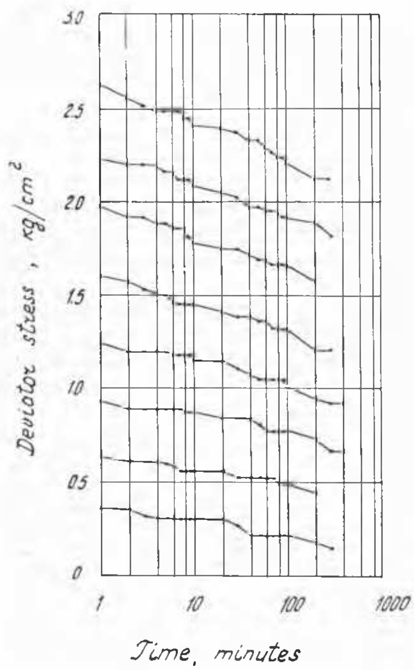

Fig. 5 Relaxation curves of deviator stresses in the triaxial test (under σ_3 = 3 kg/cm^2).

Courbes de relaxation du dérivateur dans un essai triaxial (σ = 3 kg/cm^2).

description of the rheological properties of soil, we must take into account the elasticity of the mineral particles. To do this, the Kelvin model should be supplemented by an elastic element connected in series with it, capable of relaxing when linked with the viscous element of the model.

Experiments show that the process of relaxation of dense clays (based on observational data for the first 100 minutes) is described by an equation similar to (7).

The nature of the curves of relaxation is illustrated in Fig. 5, which gives the observational results of the relaxation of deviator stresses in the triaxial test.

References

[1] S. S. Vyalov and A. M. Skibitsky (1957). Rheological Processes in Frozen Soil and Dense Clays. *Proc. Fourth Intern. Conferen. on Soil Mechanics*, Vol. 1.

[2] S. S. Vyalov (1959). The Rheological Properties and Bearing Power of Frozen Soils, *U.S.S.R. Academy of Sci.*, Moscow.

[3] N. N. Maslov (1958). On the problem of the conditions of manifestation of the creep of clays. *Science Reports of Higher Schools*, Stroitelstvo (Construction), No. 1, Moscow.

[4] N. N. Maslov (1958). Some questions of the stability and deformation of retaining walls. *Conference of Problems of Earth Pressure.* Brussels.

[5] N. A. Tsytovich (1951). Soil Mechanics, Moscow.