The Vibrational Behaviour of Soil in Relation to its Properties

Le comportement vibratoire d'un sol en fonction de ses propriétés

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Summary

Measurements were carried out on various types of subsoil to determine their vibrational behaviour. During this work amplitudes in the range of frequency of 10-80 Hz were measured at the point of excitation and at different distances from it, and the velocity of propagation of the seismic waves thus generated was also determined.

The measuring apparatus used consists of an oscillator with variable weight (1·8 to 3·5 t.) and variable surface for transmitting harmonic oscillations to the subsoil, and of a car in which the electric measuring instruments and the power generator are housed. The oscillator is conveyed by a special carrier provided with hydraulic equipment so that it can be rapidly loaded and unloaded. Measurements were taken on sandy, sandy-gravelly, silty and silty-clayed subsoil.

The resonant frequency and the measured amplitudes enable the type of soil, its coefficient of swelling and its angle of internal friction to be determined.

In the case of loose soils, their density is a good guide to their oscillatory behaviour. Good density ($D > 0{,}7$) is not only desirable for increased load bearing capacity and sensitivity of settlement, but also to ensure that the vibrational behaviour will not change with time.

Sommaire

On a procédé à des mesures sur différentes espèces de sol, pour se faire une idée de leur comportement oscillatoire. On a mesuré l'amplitude dans la bande de fréquence de 10 à 80 Hz au point d'excitation et à différentes distances de celui-ci, ainsi que la vitesse de propagation des ondes excitées sismiquement.

L'appareillage comporte un excitateur d'oscillation d'un poids variable (1,8 à 3,5 t.) ainsi qu'une plaque de base, dont la forme et l'excentricité sont variables, pour la transmission d'oscillations harmoniques au sol, et un camion laboratoire, équipé d'appareils de mesure électriques et d'une génératrice électrique fournissant le courant nécessaire. L'oscillateur est transporté sur un véhicule spécial, pouvant être chargé et déchargé rapidement moyennant une installation hydraulique.

Les mesures ont été effectuées sur des sols sableux, sable-graveleux, limoneux et limono-argileux.

La fréquence de résonance et l'amplitude permettent de tirer des conclusions selon le type de sol, sur son coefficient de gonflement et son angle de frottement interne.

Lorsqu'il s'agit d'espèces de sols non consolidés la compacité représente un critère satisfaisant en ce qui concerne le comportement oscillatoire. Une grande compacité ($D > 0{,}7$), est nécessaire, non seulement pour réaliser une capacité portante plus élevée et diminuer la sensibilité au tassement, mais pour que le comportement oscillatoire reste invariable en fonction du temps. S'il faut remplacer de mauvaise couches de sous-sol il est recommandable de remblayer avec un sol non cohérent à degré élevé de non uniformité.

1. Problem.

An extensive static calculation as well as one based on data from vibration measurements are indispensable for the design and the construction of foundations under dynamic stress. The dimensioning of the structure should enable the subsoil to bear the necessary load without excessive settlement. The values of the natural frequencies should be remote from those of the exciting frequencies of the vibrator, thus avoiding any resonant frequency with a wide margin. This sequence of calculations is possible only when considering the properties of subsoil, and the effect of forced vibrations on its behaviour, including the data obtained from the corresponding measurements in the calculation.

2. Review of previous vibration methods.

A simple vibration system consists of a vibrator, the foundation, and the subsoil. For the general case, the subsoil stressed has to be regarded as a mass of variable consolidation modulus and density. The subsoil does not represent an isolated system.

Some simplifying assumptions have to be made with a varying degree of simplification. These assumptions are necessary in order to give sound practical information on vibrational behaviour.

Previous work in this field can be classified into the following groups :

2.1. The foundation, including any machinery which it supports, can be considered as a single-mass system whereby the elastic reactive force is proportional to the deformation while the resultant inertia forces of the subsoil at the point of application are proportional to the acceleration. Finally, the forces necessary for overcoming the internal friction and the seismical radiation are proportional to the rate of deformation. Assuming that the system has only one degree of freedom for motion, the well-known differential equation for the problem results from the equilibrium conditions as follows :

$$\frac{G}{g} \cdot \frac{d^2u}{dt^2} + k \cdot \frac{du}{dt} + C_f \cdot u = p(t) \qquad \dots. \quad (1)$$

Let :

ω_0 be the natural frequency and

ω_1 the exciting angular frequency.

Then we obtain the following solution :

$$u = a_1 \frac{1}{\sqrt{\left[1 - \left(\frac{\omega_1}{\omega_0}\right)^2\right]^2 + \left(\frac{k}{m \cdot \omega_0}\right)^2 \left(\frac{1}{\omega_0}\right)^2}} \cdot$$

$$\cdot \sin \left(\omega_1 \cdot t - \arctan \frac{\omega_1 \cdot k}{m(\omega^2 \cdot - \omega_1^2)}\right) \qquad (2)$$

This system has been dealt with by HERTWIG, FRÜH, LORENZ [1], RAUSCH [2], and by other authors.

2.2 LORENZ [3] has shown that the foundations can be regarded as an oscillating mass point mounted on an imperfect elastic spring. By the application of this assumption, a spring function is obtained that depends upon the amplitude of the oscillations. A good agreement is attained between the results of the measurements and the calculations.

Then the spring function becomes :

$$C(x) = \frac{d\sigma_x}{dx} = c_1 + \frac{c_2 \cdot c_3}{(c_3 + x)^2} \qquad (3)$$

where

c_1, c_2, c_3 are free constants.

2.3. The forces are transmitted to the elastic isotropic semi-infinite medium by friction. This theory formulated by REISSNER [4] has been further developed by ARNOLD, BYCROFT, and WARBURTON [5] as well as by SECHTER [6].

3. Vibration measurements.

For better evaluating the behaviour of the subsoil the Institut für Grundbau und Baugrundmechanik, Technische Hochschule Dresden, carried out dynamic and static investigations on soils and foundations for about 5 years. For every test point, amplitude measurements were made at the vibrator and at various distances from it within the frequency range of 10 to 80 c/s. Simultaneously the progation velocity of the seismic waves was measured.

3.1. *Vibrator.* — The vibrator used for the measurements consists of a fixed casing containing two movable discs which carry 2 sliding revolving masses. These masses can compensate the horizontal forces by their mutual rotation. The residual force is a common vertical component having a harmonic characteristic. No forces occur when the two movable revolving masses are just in opposition. When moving the revolving masses together the largest out-of-balance force (centrifugal force) will result. The weight of the vibrator can be changed from 1·8 tons to 3·5 tons. There are three different areas of baseplate, namely : 0·25,0·5, or 1·0 square metres respectively. Alternative shapes are a square, a rectangle or a circle.

Fig. 1 Vibrator in operating position and the transport vehicle.
Vibrateur en position avec son transporteur.

Fig. 2 Vibration measuring unit consisting of a vibrator, the measuring station car, the switching unit, and the vibration frequency meter.

Unité de mesure vibratoires comprenant : un vibrateur, la voiture laboratoire, l'enregistreur de vibrations, la fréquence mètre.

Rotational speed can be controlled by a newly developed unit of special design (see Fig. 2), allowing a convenient adjustment of speed with sufficient stability to load variations as well as accurate maintenance of a constant speed for a long time.

The vibrator is mounted on an special vehicle provided with hydraulic equipment for quick hauling and placing of the vibrator unit (see Fig. 1).

The measuring station car used as the tractor accomodates the diesel-electric generator supplying direct current for driving the vibrator motor. The alternating current required for the oscillograph is generated by means of a transformer.

3.2. *Vibration meter.* — The vibrations are recorded by inductive vibration meters operating on the inertia principle, developed by Professor MARTIN of Jena. The natural frequency of the vibration meters is 3 to 5 c/s which represents a range of values sufficiently remote from the frequencies being measured. The device mentioned enables the both vertical and horizontal vibrations to be recorded.

The voltage induced in the vibration meter, due to a magnet with annular clearance, attains a level that does not require the use of a tube amplifier. This fact eliminates any possible source of error.

The waves received by the pick-up are recorded by an oscillograph with eight loops giving a photographie record of the wave forms.

A time mark of 500 c/s is provided for interpretation. Another time mark of 2 c/s which must be calibrated frequently is available from a rectifier. The transition zero of the vibrating body is recorded on one loop by a sliding contact and the propagation velocities of the waves can be measured by means of this zero value.

For the analysis of data, the measuring devices including the shielded lead cables have been calibrated, thus allowing an absolute indication of amplitude values. The evaluation can be amplified considerably in the same way by establishing a constant for the measuring arrangement. The calibration is performed on an oscillatory table using the same circuit with all the meter multipliers used for the subsequent measurements (Fig. 3). Fig. 4 shows some characteristic calibration curves employed for the determination of the measuring arrangement constant.

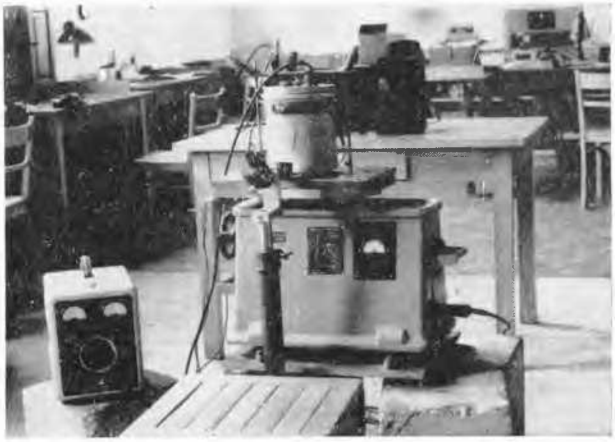

Fig. 3 Calibration device.
Installation d'étalonnage.

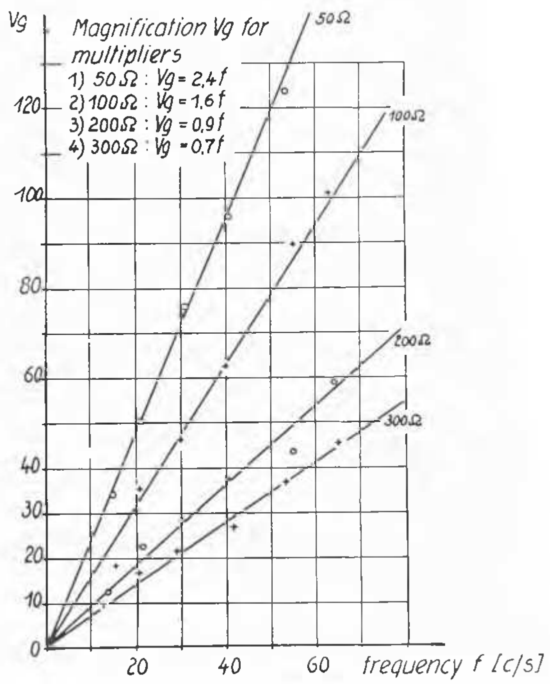

Fig. 4 Calibration curves for the vibration with different meter multipliers.
Courbes d'étalonnage avec différents multiplicateurs.

4. Evaluation and results of vibration measurements and soil investigations.

The measured values offer a possibility of establishing the relationships between the resonant frequencies, the amplitudes of damping, and the soil mechanics data whithout any theoretical prejudice.

They are many factors influencing the behaviour of the subsoil under forced vibrations, individual estimates of their effects being possible only with great difficulty. The complex conditions of deformation and stress under the plate of the vibrator ; the effect of grain-size distribution and grain shape ; the permeability, prestressing due to the geological processes, and other effects contribute to the lack of a clear theoretical base as a starting position. For this reason the experiment was chosen as an adequate approach. The investigations are in part based on the measured values that were obtained on the natural soil by the use of the vibrator. On the other hand, the work depends on the comparison of test results that were obtained from undisturbed soil samples taken from test pits. In all these cases sampling was always performed with the utmost care.

More than 400 measurements were made at 40 measuring points on four different types of subsoils. These soils had different compaction values, their grading being illustrated in Fig. 5. The measurements comprised various eccentricities of the revolving masses as different contact pressures, with different areas and shapes of the base plates of the vibrator used.

The amplitudes have been measured as a function of the frequency at the vibrator, whereas the measurements of the phase displacements were carried out at definite distances for determining the propagation velocities of the waves. Fig. 6 shows these records as schematic cutouts of the film. The frequency can be determined by means of the time mark, while the values of the amplitude is obtained from the calibration curve. Plotting of the frequency/amplitude curves

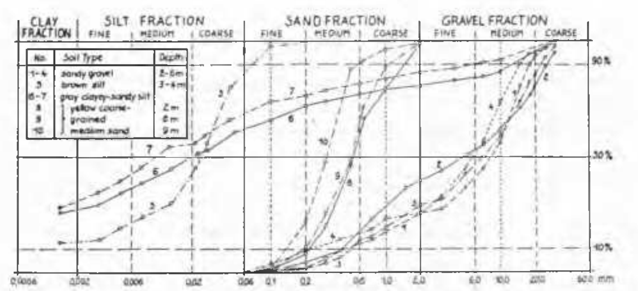

Fig. 5 Grain size distribution curves of the soil types investigated
Courbes granulométriques des sols étudiés.

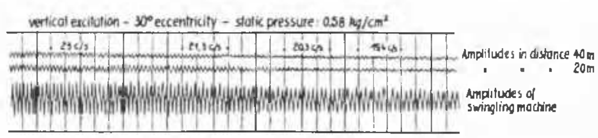

Fig. 6 Schematic cutout from an oscillograph film for medium sand.
Vibrations enregistrées dans le cas d'un sable moyen.

(Fig. 7) is effected by using the film recordings which indicate the resonance points for the system of vibration. The result is that the resonant frequency increases with the increase of the static pressure of the subsoil. After exceeding a definite value, the relative magnitude of which depends upon the surface area, the resonant frequency again decreases. The resonance points marked clearly will recede when the static pressure decreases, while several amplitude maxima frequently develop. When measuring the vibrations of the soil it is practical, therefore, to establish the same ratio between the centrifugal force and the surface area as later will occur in the construction of a machine foundation.

The examination of the soil also covered the determination of bulk density, moisture content, compactness, consolidation modulus and swelling modulus, as well as shear strength of the subsoil in question. These values were determined at different depth applying the prevailing methods of soil mechanics. The soil tested was very homogeneous at all sites, and samples were taken from the outcrop down to a depth of about 10 metres.

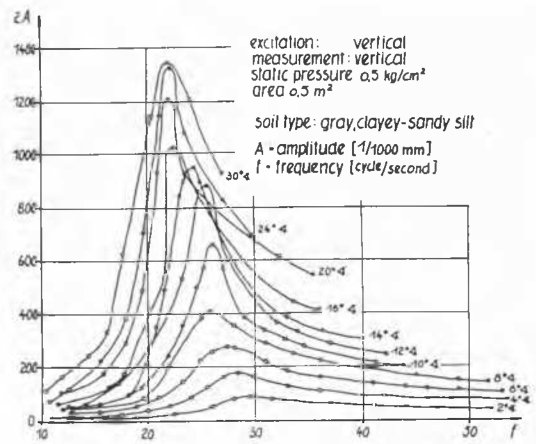

Fig. 7 Frequency-amplitude curves for medium sand.

Courbes fréquence-amplitude (cas d'un sable moyen).

4.1. *Correlation between resonant frequency and swelling modulus.* — The natural frequency decreases with the increase of the exciting force but, as LORENZ [3] already stated, not in the same measure as was formerly supposed. This fact could be explained by the increase of the resonant soil mass that is enlarged with the increase of the exciting force. This assumption being correct, the natural frequency should not increase when the area is enlarged, while the soil pressure is maintained constant. This phenomenon may serve as an indication of the falsely harmonic nature of the vibrations.

The resonance amplitudes, however, show a decrease with the same static pressure and an increase in area.

The correlation shown in Fig. 8 was, therefore, established from the measurements by comparison of the resonant frequencies and the swelling modulus V_s. For plotting the values of the natural frequencies were related to the standard apparatus of 3·0 ton having a baseplate area of 1·0 square meter and an eccentricity of unbalance of 20°. The swelling modulus was determined in the same way for a static pressure of 0·3 kg/cm². The relief and the re-loading was rated for the range of the centrifugal force. This curve indicates initially a nearly linear relationship between swelling modulus and natural frequency. When the strength of the subsoil increases, this curve approaches an asymptotic limiting value.

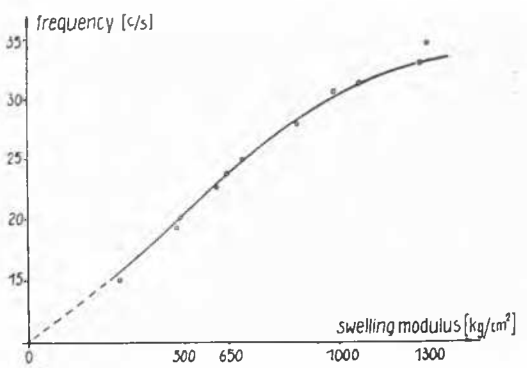

Fig. 8 Variation of resonant frequency with the swelling modulus.

Fréquence de résonance en fonction du module de gonflement.

4.2. *Damping effect.* — With a constant static soil pressure p_{st} the resonance amplitudes decrease with increasing baseplate area. This statement is an indication of increasing damping resistance as a function of the increasing area. Moreover,t he damping is influenced by the coefficient of swelling of the subsoil. This relationship is shown in Fig. 9.

The result of these factors is a proportional increase of damping resistance as a function of the area. For the Same area, the consolidation modulus and the damping resistance have a combined effect.

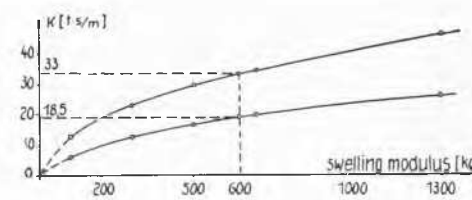

Fig. 9 Damping resistance as a function of the area and the swelling modulus.

Amortissement en fonction de la surface et du module de gonflement.

4.3. *Waves velocities.* — The waves produced in the subsoil are propagated at a definite velocity. It has been known for a long period that this velocity largely depends upon the compactness and the consistency of the soil [1]. These characteristics of the soil in turn being influenced by swelling modulus of the subsoil (Fig. 10) the variation of this value with the

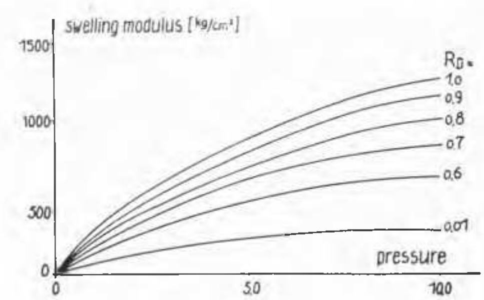

Fig. 10 Effect of relative density on swelling modulus.

Influence de la densité relative sur le module de gonflement.

velocity is illustrated in Fig. 11 based on the measured values of the velocity for a frequency of 20 c/s.

This curve is a parabola, approaching asymptotically a limiting value with increasing consolidation modulus. At this point an attempt can be made to explain the dependance of the wave velocity on the frequency by the consolidation modulus that is a function of the stress.

5. Application to the generally known equation of oscillation.

Assuming a direct propagation of pressure and considering only one degree of freedom of motion, we introduce these experimentally defined correlations into the well-known equation of oscillation. The following differential equation is obtained as a result of the equilibrium between the inertia resistance of the wave area in the soil and the difference of the restoring forces acting on it from the top and the bottom. These restoring forces for the vibrations in the interior of the soil are elastic when a force P which changes periodically acts on the (See 7).

$$\frac{\partial^2 u}{\partial t^2} = \frac{V_s}{\mu}\left(\frac{1}{x} \cdot \frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2}\right) \quad \ldots . \quad (4)$$

In contrast to this, the following equation yields for the vibration in the base of the foundation as a solution for the equation of the forced oscillations :

$$u = \frac{A}{2\tan\varphi} \cdot \sin\omega\left(t - \frac{\sqrt{A_0}}{2\tan\varphi \cdot v}\right) \quad \ldots . \quad (5)$$

where :

- u = is the absolute displacement of any area element in the subsoil – [centimeters]
- A = is the amplitude of the vibration of the area A_0 — [centimeters]
- ω = is $2\pi f$, i.e. the frequency of the excited vibration – $[s^{-1}]$
- f = is the frequency per second – $[s^{-1}]$
- φ = is the angle of internal friction
- V_s = is the swelling modulus assumed to be constant within the range of small stresses – [kgs per square centimeter]
- $\mu = \frac{\gamma}{g}\left[\frac{\text{kg} \cdot s^2}{\text{cm}^4}\right]$ i.e. the density of the subsoil $\frac{\text{unit weight}}{\text{gravity}}$
- t = is the time – [seconds]
- v = is the propagation velocity of the wavest – [centimeters per second]
- α = angle of phase difference
- m = mass of swinging machine or mass of vibrating foundation

An equilibrium must exist within the area A_0 between the inertia resistance of the mass $m\frac{\partial^2 u}{\partial t^2}$, the elastic restoring force $V_s \cdot A_0 \cdot \frac{\partial u}{\partial x}$ and the exciting force which acts periodically

$$P = P_1 \cdot \sin\omega \cdot \left[\left(t - \frac{\sqrt{A_0}}{2\tan\varphi \cdot v} + \alpha\right)\right] \quad (6)$$

From this condition of equilibrium the amplitude A is obtained

$$A = \frac{P_1 \cdot \sin\alpha \cdot 2\tan\varphi}{\frac{V_s \cdot A_0}{v} \cdot \omega} \quad (7)$$

as well as the phase displacement

$$\tan\alpha = \frac{\frac{V_s \cdot A_0}{v} \cdot \omega}{V_s\sqrt{A_0}\ 2\tan\varphi - m \cdot \omega^2} \quad (8)$$

These values can be used for the point-by-point calculation of the frequency amplitude curve. A reasonably good agreement with the curve obteined from experiments has been observed in all the cases investigated (Fig. 12).

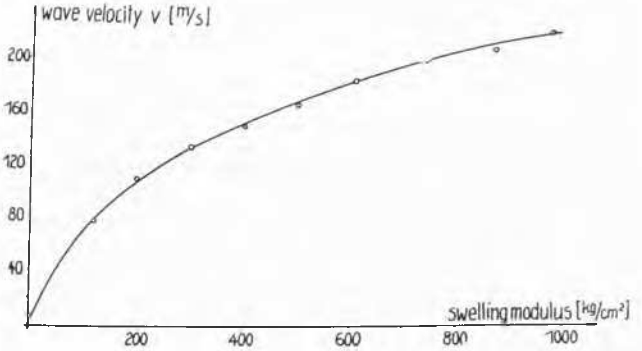

Fig. 11 Variation of wave velocity with the swelling modulus.
Vitesse de propagation en fonction du module de gonflement.

A comparison of the amplitude values and of those for the damping with the corresponding values based on the theory of the mass point results in a conformity when introducing the following terms :

for the stiffness of the spring

$$C = 2\tan\varphi \cdot V_s \cdot \sqrt{A^0} \quad [\text{kg/cm}] \quad \ldots . \quad (9)$$

and for the damping

$$k = A_0\sqrt{V_s \cdot \mu}\left[\frac{\text{kg} \cdot s^2}{\text{cm}^{\cdot}}\right] \quad \ldots . \quad (10)$$

These results can be also transferred to the resolved table foundation by considering the effects of spring action and of coupling between the base plate and the table plate.

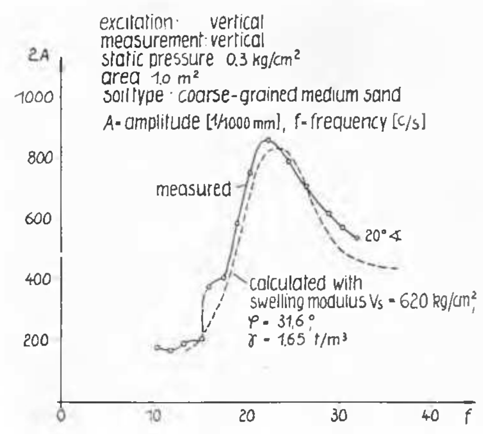

Fig. 12 Measured and calculated frequency-amplitude curves for the soil.
Courbes fréquence-amplitude mesurées et calculées pour le sol.

6. Vibration measurement at a monolithic foundation and check by the results of calculation

For the analysis of the relevant phenomena the frequencies amptitudes, and propagation velocities of the waves were measured at six points of a foundation for a blower (Fig. 13).

The excitation was by means of the vibrator described using different eccentricities. The oscillations of the vibrator described using different eccentricities. The oscillations of this foundation were very uniform. The frequency-amplitude curve obtained by calculation for this arrangement agreed closely with the measured values. The following data determined in the laboratory were introduced for this calculation :

$$\gamma = 1{\cdot}90 \ [\mathrm{t/m^3}]$$
$$\tan \varphi = 0{\cdot}71 \, ; \varphi = 31°$$
$$V_s = 515 \, p_{st} \ [\mathrm{kg/cm^2}]$$

The same type of comparison between the measured values and the calculation is made in all the evaluations of the vibration measurements. Except in a few cases, there has always been close agreement between the relevant curves.

An essential premise is, of course, that the properties of the soil up to a greater depth are essentially the same when compared with the dimensions of the foundation, or that the change of these properties is continuous.

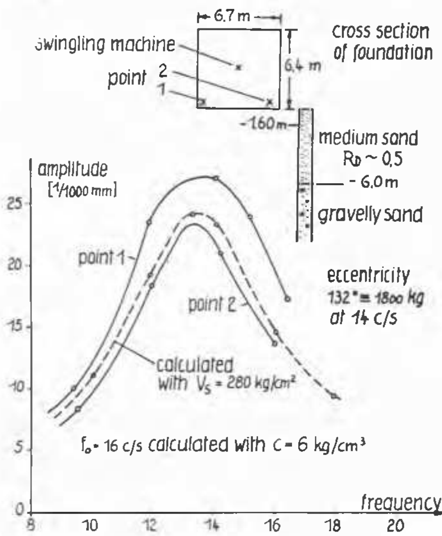

Fig. 13 Measured and calculated frequency-amplitude curves for a monolithic foundation.

Courbes fréquence-amplitude mesurées et calculées pour une fondation monolithe.

7. Influence of vibration on compactness

Investigating the compaction of non-cohesive subsoil under the action of vibrations demonstrates that the effect of acceleration on compaction depends substantially on the static pressure (Fig. 14 for medium sand). Up to a certain value of acceleration, the loaded soil sample does not consolidate at all retaining the same degree of compaction. The acceleration exceeding the critical value, the process of compaction is initiated. The region in which compaction can be determined is a comparatively small interval of the acceleration values. When the acceleration increases stability of compaction is attained. The limiting value of the relative compaction is between 0·7 and 0·8. The volume of the voids in the subsoil remaining constant, the value of the critical acceleration is the greater the higher the static pressure put on the sample. The magnitude depends, moreover, upon the size and the shape of the grains as well as upon the moisture content of the soil. The values of the critical acceleration increase up to a limiting value with the increase of moisture content, and subsequently decrease slightly when a complete saturation is reached. Then additional settlements due to the influence of vibrations can occur only when the acceleration of the vibrations for the subsoil exceeds a critical value.

This relationship enables the determination of the depth of the soil layer to be compacted, or of a cushion below a foundation, respectively. This thickness yields the distance from the surface to the point of intersection of the curves which are obtained from the expected and the critical acceleration of the vibration. (Fig. 15).

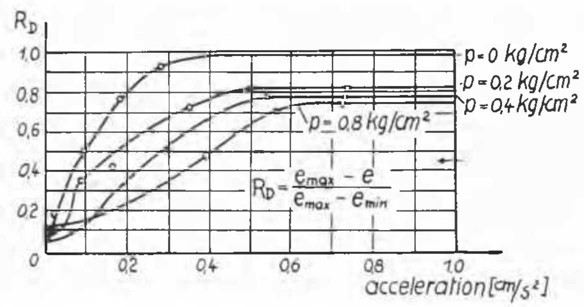

Fig. 14 Variation of the relative density for medium sand with the vibration acceleration when the load is variable.

Variation de la densité relative d'un sable moyen avec l'accélération lorsque la charge est variable.

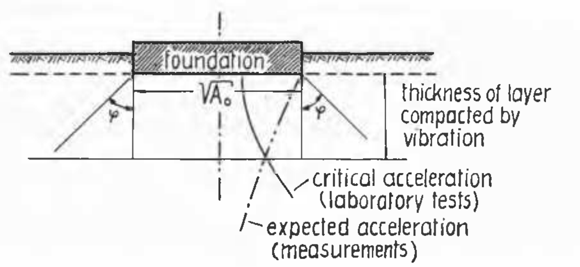

Fig. 15 Determination of the thickness of layer of subsoil to be compacted.

Détermination de l'épaisseur de la couche à comptacter.

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