Three-Dimensional Consolidation of Saturated Clay

Étude expérimentale sur la consolidation à trois dimensions de l'argile saturée

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Summary

A study of three-dimensional consolidation of clay has already been undertaken by M. A. Biot during the last twenty years but the knowledge gained was not put to practical use until recently mainly because of the mathematical complexity of the theories advanced.

The authors propose an experimental method for estimating total settlement and the time-settlement relationship of the three-dimensional consolidation of saturated clay, using the results obtained from a series of laboratory model tests.

In estimating total settlement, it has been suggested that, by introducing the concept of the « Equivalent Stress Ratio » or « Equivalent *K* value » *Ke*, the three-dimensional consolidation of a model can be replaced by a triaxial consolidation test having an actual stress ratio *Ke*.

It has also been proposed to use the concept of the « Equivalent Drainage Diameter » Be in order to approximate more closely to the three-dimensional process of consolidation, based on the fact shown by N. Carrillo that three-dimensional radial flow can be resolved into a horizontal radial flow and a vertical linear flow.

Sommaire

L'étude de la consolidation à 3 dimensions de l'argile a été déjà faite depuis 1940 par M. A. Biot et d'autres. Mais, l'application n'en a pas encore été pleinement faite jusqu'à présent, en raison des complexités mathématiques qu'on rencontre dans ces études.

Le présent essai expose, en se servant des résultats obtenus dans plusieurs séries d'essais sur modèle faits au laboratoire, une méthode expérimentale pour estimer le tassement total et le tassement en fonction du temps, lors de la consolidation à 3 dimensions de l'argile saturée.

Dans l'évaluation du tassement total, il a été suggéré que, en introduisant la conception de « rapport des contraintes équivalentes » ou celle de « valeur équivalente de *K* », *Ke*, on pourrait remplacer la consolidation à 3 dimensions sur le modèle par la consolidation triaxiale ayant un rapport des contraintes, *Ke* propre.

Il a également été proposé d'appliquer la notion « diamètre de drainage équivalent — *Be* » pour obtenir une meilleure approximation du processus de consolidation à 3 dimensions, en se basant sur le fait démontré par N. Carrillo qu'un écoulement radial à 3 dimensions peut-être décomposé en un écoulement radial et un écoulement linéaire vertical.

Introduction

The study of twoand three-dimensional consolidation of clay has been undertaken by many investigators such as L. Rendulic [1], N. Carrillo [2], M.A. Biot [3], S. Takagi [4], etc... during the past thirty years, but their theories have not yet been applied to the solution of practical problems mainly because of the complexity of their treatments. Consequently, in the analysis of the consolidation settlement of clay, it has been necessary to use Terzaghi's one-dimensional consolidation theory, even in those cases, where a three-dimensional treatment may be expected. The only remedy for such a case is to consider the stress distribution by Boussinesq's theory or by some simplified routine method in estimating total settlement. The influence of the vertical as well as the horizontal stress on the consolidation settlement is not clear in the present theory, particularly in the case where there is a soft clay stratum at relatively shallow depth and where lateral flow may influence settlement of a foundation.

A large scale model experiment was started to clarify the characteristics of the three-dimensional consolidation of saturated sedimentary clay by one of the authors [5] in 1953 and it has been continued up to the present, using much smaller models.

Construction of a new triaxial consolidation machine enabled the authors to establish the experimental procedure by which three-dimensional consolidation phenomena of the model tests can be evaluated, using the concept of the "Equivalent Stress Ratio" Ke and the " Equivalent Drainage Diameter" Be.

Basic Data on Soil used

The soil specimen used in this study was taken from 1·5 — 2·0 m beneath ground level on the coast of the Inland Sea, Okayama Prefecture, Japan. Several specimens were extracted from an excavated trench, using several special thin-walled samplers of about 40 cm diameter, carefully transported to the laboratory and preserved for a long time without any change of moisture content.

All tests, except those on a large scale, were performed on undisturbed samples taken from a single trial pit.

Table 1 shows the physical and mechanical properties of the samples tested :

Table 1

Specific gravity .	2·65	Liquid limit	73·9 %
Grain size clay . .	37 %	Plastic limit	27·5 %
silt	58 %	Plasticit index	46·4 %
sand	5 %	Void ratio	1·8-2·1
Natural water content	65- 71 %	qu	2-3 t/m²
Saturation ratio	100 %	C_c	0·75-0·85

Apparatus and Test Procedure

The triaxial consolidation machine is shown in Fig. 1; Fig. 2 and 3 are photographs of the three-dimensional consolidation models. This triaxial machine works on the same principle as the ordinary triaxial compression tester, but to avoid friction of the piston and leakage of water, each of σ_1 and σ_3 is charged by the hydrostatic pressure of a mercury column suspended by a spring balance, its head being kept constant by the spring automatically throughout the whole period of consolidation.

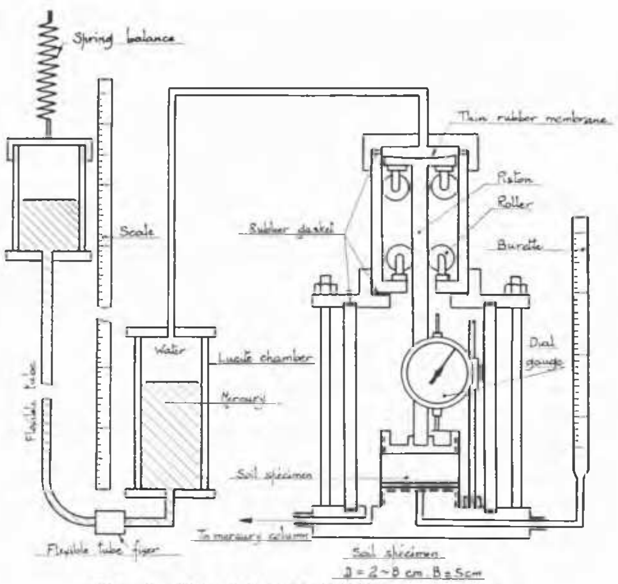

Fig. 1 Triaxial consolidation apparatus.
Appareil de consolidation triaxial.

In the triaxial tests, samples are always consolidated under a small ambient stress at first, in order to make their initial conditions as similar as possible and to squeeze out the air from the voids, followed by consolidation under increased load, keeping the stress ratio $K = \frac{\sigma_3}{\sigma_1}$ constant. In the three-dimensional models, the stress is always kept to definite small value on both piston and surcharging plate at the first step, and after settlement the central piston is loaded and the three-dimensional consolidation is measured.

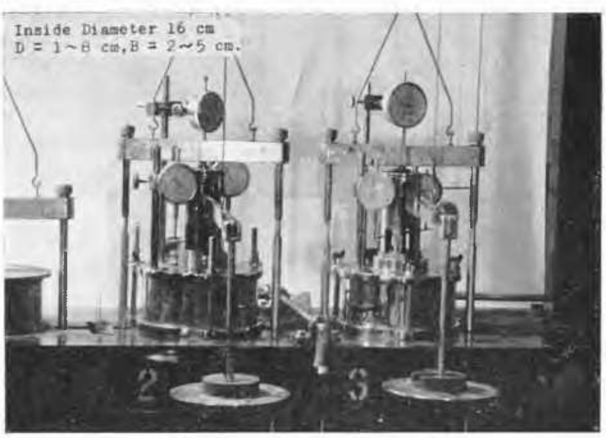

Fig. 2 Three-dimensional consolidation apparatus.
Appareil de consolidation à 3 dimensions.

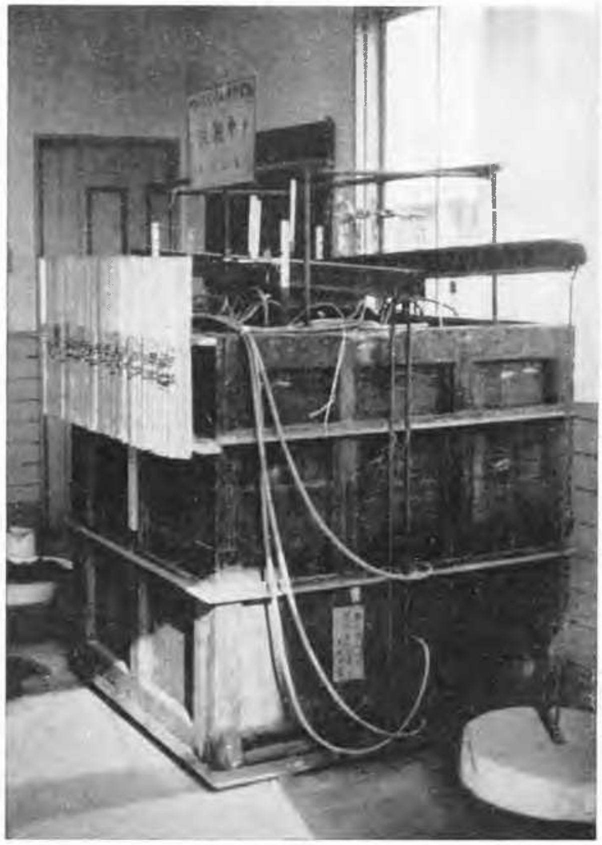

Fig. 3 A large scale model test of three-dimensional consolidation.
Essai sur modèle à grande échelle de consolidation à 3 dimensions.

Estimation of Total Settlement

Considers an element in a mass of soil subjected to increments of principal effective stress $\Delta\sigma_1$, and $\Delta\sigma_3$ ($\Delta\sigma_2 = \Delta\sigma_3$). The volume change expressed by A.W. Skempton [6] is

$$\frac{\Delta V}{V} = -C_c[\Delta\sigma_3 + S_p(\Delta\sigma_1 - \Delta\sigma_3)] \qquad (1)$$

where C_c is the average volume compressibility, and S_p structural parameter. S_p is primarily a function of " dilatancy " and is considered to be a constant for a certain soil in the case of normally consolidated clays. In this case it is equal to 0·88, measured by the triaxial consolidation test. As C_c can easily be measured by a triaxial test in ambient stress condition ($K = 1{\cdot}0$), and the average value of the stress increment in vertical direction $\overline{\Delta\sigma_1}$ in a model test can be calculated, if it is assumed that the normal stress distribution in the soil obeys Boussinesq's theory, so the average stress increment in horizontal direction $\overline{\Delta\sigma_3}$ during the whole process of three-dimensional consolidation, or its ratio $K_e = \frac{\overline{\Delta\sigma_3}}{\overline{\Delta\sigma_1}}$ can be obtained from the following equation

$$\frac{\left(\frac{\Delta V}{V}\right)_{T.D.}}{\left(\frac{\Delta V}{V}\right)_{T.A.}} = K_e + S_p\,(1 - K_e) \qquad (2)$$

where $\left(\frac{\Delta V}{V}\right)_{T \cdot D \cdot}$ is the rate of volume change in three-dimensional consolidation and is obtained by measuring the final moisture content after the three-dimensional consolidation, and $\left(\frac{\Delta V}{V}\right)_{T \cdot A \cdot}$ shows the volume change by the stress increment $\Delta\overline{\sigma_1}$, in the triaxial consolidation of ambient stress condition ($K = 1{\cdot}0$).

The plots of K_e against D/B appears to fall on a curve shown in Fig. 4 and K_e does not exceed the K_0 value of the œdometer consolidation test. (K_0 equals 0·73 in this case). The vertical strains ε_1 of the triaxial consolidation for different K values are plotted in Fig. 5 (a) and those of the model test in Fig. 5 (b). The gradients of the curves in the former cases give a smooth line when plotted against K values as shown in Fig. 6, and when the gradients of settlement ratio in the

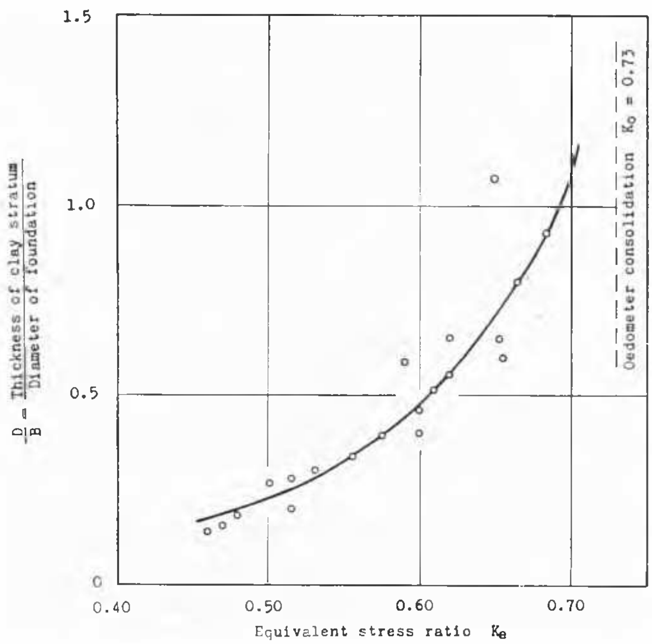

Fig. 4 Relationship between the size of a foundation and equivalent stress ratio.
Relation entre la dimension de la fondation et le rapport des contraintes équivalentes.

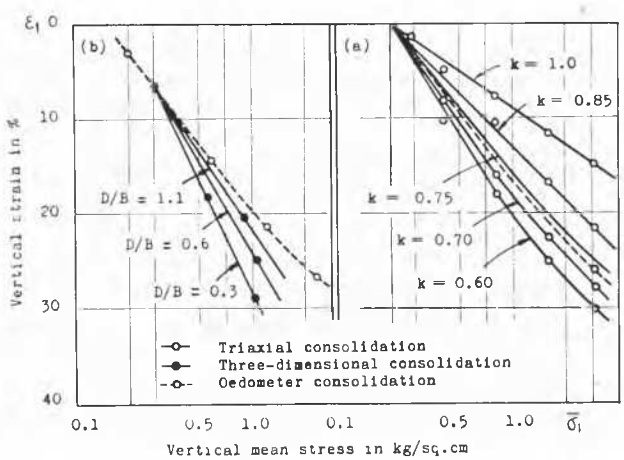

Fig. 5 Vertical strain plotted against average vertical stress in triaxial and three dimensional consolidation.
Déformation verticale en fonction des contraintes verticales moyennes dans la consolidation à 3 dimensions.

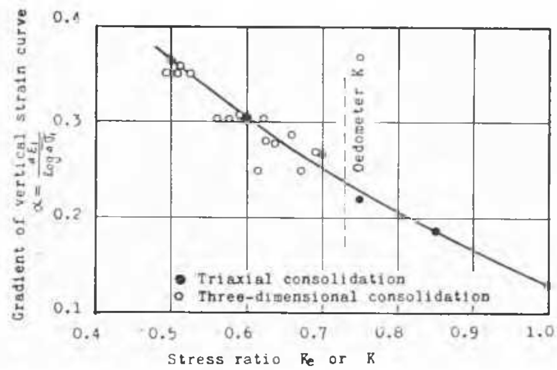

Fig. 6 Gradient of vertical strain plotted against the stress ratio.
Relation entre le gradient des contraintes verticales et le rapport des contraintes.

three-dimensional model test are plotted with K_e value on the same graph, it becomes clear that both curves coincide.

Thus three-dimensional consolidation, with a stress ratio *Ke*, can be represented by or replaced by the normal strain of a triaxial consolidation having the same principal stress ratio *Ke*. The authors propose to designate *Ke* as the "Equivalent Stress Ratio" or "Equivalent *K* Value".

A very interesting fact about the *Ke* value has been discovered. The product of the average influence value $I\sigma_m$ — calculated by the Boussinesq's equation — at a certain point on the curve in Fig. 4 and the value $\frac{1 + 2\,Ke}{3}$ at the same point, is almost equal to the one at any other point on the same curve. As

$$\Delta\sigma_1 \cdot I\sigma_m \cdot \frac{I + 2\,Ke}{3} = \frac{\overline{\Delta\sigma_1} + 2\overline{\Delta\sigma_3}}{3} = \text{const.} \quad \ldots . \quad (3)$$

it means that in the three-dimensional consolidation of a clay stratum, the average of the increment of the three principal stresses becomes constant independent of the dimensions of the clay stratum and the diameter of the foundation; in other words, the horizontal stress in three-dimensional consolidation is induced so that the average stress becomes equal.

Time-Settlement Analysis

The fundamental equation for three-dimensional process of consolidation is as follows;

$$\frac{\partial U}{\partial t} = C_h\left(\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r}\right) + C_v\frac{\partial^2 U}{\partial Z^2} \qquad (4)$$

The average consolidation ratio U can be obtained from the equation

$$U = I - (I - U_v)(I - U_h) \qquad (5)$$

where U_v and U_h are the average consolidation ratios of the vertical and horizontal flow respectively,

$$\frac{\partial U}{\partial t} = C_v\frac{\partial^2 U}{\partial Z^2} \qquad (6)$$

$$\frac{\partial U}{\partial t} = C_h\left(\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r}\right) \qquad (7)$$

In estimating the time-settlement relationship in three-dimensional consolidation, the effect of the horizontal radial flow on consolidation process must not be neglected. The solution of consolidation by the vertical linear flow 6 (is) given by K. Terzaghi [7] in his famous theory, and if this is the case where the clay stratum is enclosed by a drain in the form of a vertical cylinder, the solution of the horizontal radial flow is also given by I. da Silveira [8] and so equation (4) can readily be solved by (5). Many experiences on the three-dimensional model test have proved that three-dimensional consolidation can be approximated by assuming that there exists a cylindrical drain of appropriate diameter. Calculated values derived from this simple assumption coincide with the time-settlement curve of a model test is shown in Fig. 7. The authors refer to this assumed drainage diameter as B_e, or the "Equivalent Drainage Diameter". It is the function of D/B and its relationship is shown by the curve in Fig. 8. This curve is hyperbolic and so the product of D/B and Be/B becomes a constant.

The data obtained from large scale tests carried out in 1954 [5] are on the same curve.

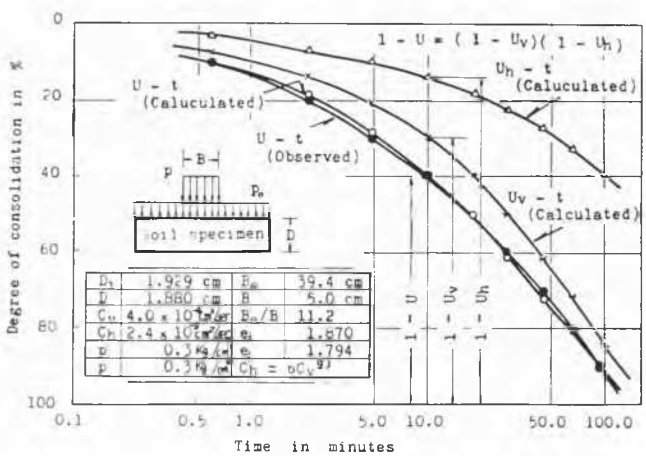

Fig. 7 Three-dimensional consolidation curve and its analysis
Courbe de consolidation à 3 dimensions et sa justification.

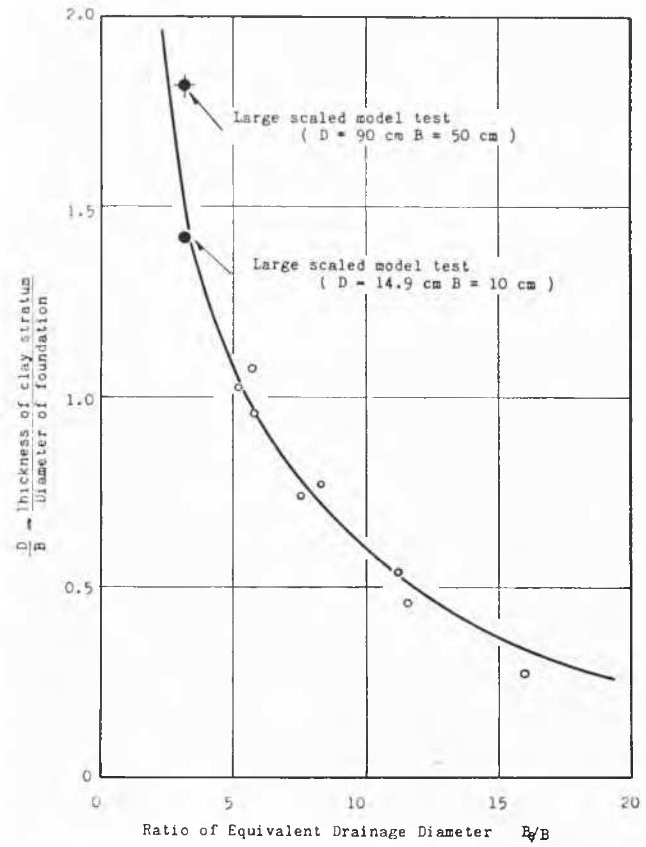

Fig. 8 Ratio of equivalent drainage diameter plotted against size of foundation.
Relation entre les dimensions de la fondation et le diamètre équivalent de drainage.

Conclusions

The three-dimensional process of consolidation where a partial load is applied to a clay stratum of finite thickness can be approximated by an experimental method, using the concept of the "Equivalent *K Value*" *Ke* and the "Equivalent Drainage Diameter" *Be*. The procedure for estimating total settlement in three-dimensional consolidation is the determination of the "Equivalent *K* value" *Ke* from Fig. 4 and to perform the triaxial consolidation test at the stress ratio *Ke*.

The following table shows, for example, how the ratio of the settlement of a three-dimensional case to that of one-dimensional treatment may change with the ratio D/B in the present case, calculated from Figs. 4 and 6.

Table 2

The effect of the thickness of clay layer on three-dimensional settlement

D/B	0·3	0·5	0·7	1·0
w_T/w_0	1·40	1·24	1·18	1·08

When the thickness of a clay stratum becomes equal to the width of a foundation, the error becomes less than 10 per cent.

In estimating the time-settlement relation more precisely, the horizontal drainage should be modified, using the "Equivalent Drainage Diameter" shown in Fig. 8. In this case, contrary to estimation of total settlement, the thicker a consolidating stratum, the greater becomes the effect of horizontal drainage on consolidation. The authors wish to express appreciation to Dr. T. Mogami, Professor of the University of Tokyo, for suggestions and to the Members of their laboratory for performing the necessary experiments.

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