The design of Machine Foundations related to the Bulb of Pressure

La répartition des pressions et le calcul des fondations de machines

by H. A. Balakrishna Rao, B. E., M. E., Senior Scientific Officer, Civil & Hydraulic Engineering, Indian Institute of Science, Bangalore-12

Summary

The first part of the paper gives the experimental procedure for determining the static pressure increase in the soil as a function of time due to the operation of a (Degebo type) surface vibrator. It has been experimentally established that a peak pressure within the sub-soil is reached within a period of five minutes as distinct from no such pressure development under free vibrations. The finding has practical use in evolving a mathematical procedure in the technique of machine foundation design.

The second part of the paper assumes the machine foundation as a single mass-spring system with single degree freedom without damping. Among the three unknowns in this type of problem (Viz., the spring constant, the mass of soil participating in vibrations and the natural frequency), the spring constant is determined by Pauw's method, assuming a trapezoidal distribution of the surface load. Assigning a value for the boundary of the oscillating active mass of soil based on the pressure bulbs and the density at the site this presents a new method for predicting the natural frequency of the foundation-soil system. The validity of this method is established by comparing its results with the experimental observations of many investigators which exhibits very little difference. Also a rational method is suggested for the reciprocating type of machines, in general, whose foundations must have amplitudes of vibration within the prescribed limits.

The third part of the paper covers the experimental investigations regarding the determination of natural frequency by resonant frequency methods on silty soils and on sands. The paper also presents the limitations and the validity of the new methods, evolved at the Indian Institute of Science.

Sommaire

La première partie de cet article décrit la méthode expérimentale employée pour déterminer l'augmentation de la pression interne du sol en fonction du temps, due à l'action d'un vibrateur de surface (modèle Degebo). On a établi expérimentalement qu'un maximum de pression était atteint en 5 minutes, ce qui n'arrive pas avec des vibrations non assujetties. Cette constatation a un côté pratique : elle a permis la mise au point d'une méthode mathématique de calcul des fondations de machines.

Dans la seconde partie du rapport l'auteur assimile la fondation d'une machine à un système à ressort à un seul degré de liberté sans amortissement. Des 3 inconnues dans ce genre de problème, savoir : la constante du ressort, la masse de terrain intéressée par les vibrations et la fréquence naturelle de la fondation, la constante du ressort est déterminée par la méthode de Pauw, basée sur une répartition trapézoïdale de la charge superficielle. Si l'on se donne la limite de la région du sol en vibration, en se basant sur la répartition de la charge superficielle et la densité en place, on peut en déduire un nouveau mode de détermination de la fréquence propre du système fondation-terrain naturel. La validité de cette méthode résulte de la concordance entre les résultats qui en découlent et les valeurs expérimentales très voisines obtenues par de nombreux expérimentateurs. L'auteur propose également une méthode rationnelle pour le cas des machines à double effet dont les fondations doivent avoir une amplitude de vibrations comprises entre des valeurs déterminées. La troisième partie du rapport décrit des recherches expérimentales concernant la détermination de fréquence propre par les méthodes de résonnance dans le cas de terrains limoneux, et de sables. L'article montre aussi les limitations et la validité des méthodes nouvelles, élaborées à l'Institut Scientifique Indien de Bangalore.

Introduction

Experimental investigations were conducted to study qualitatively the nature of pressure increase in sand due to a vibrator operating on the surface of a sand bed. While no attempts were made to isolate the contribution of dynamic load, the nearness of the vibrator and possible plastic deformation, the author discovered that pressure increases asymptotically with time. This observation has been used in developing a theoretical method of predicting natural frequency.

This paper also gives the investigations on the peak amplitudes at a point, using a «Degebo» type vibration generator. Relationship between natural frequency and total load as well as many other useful factors have been established. The investigations show that the contribution of total load to natural frequency is greater than the influence of area. Some field observations are also explained.

Experimental investigations on pressure increase under dynamic loads

The Fig. 1, shows a tank of 4 ft. 6 in. by 4 ft. 6 in. by 4 ft. lined internally with bituminous material and filled with medium to coarse sand. The overall density was found to be 98 lb. per cub. ft. A strain gauge pressure pick-up [1] was placed at a known level inside the sand. This pick-up has a thin aluminium disc with a strain gauge attached to the underside of the disc, the disc itself being simply supported on the periphery. Arrangements were made to obtain a calibration chart by subjecting the disc to the air pressure, the latter being measured by a pressure gauge. A "Degebo" type vibration generator is placed on a mild steel plate $\frac{1}{4}$ in. thick, this plate corresponding to the rigid base of a foundation block. A jack and proving ring provided an accurate static load, and the reaction was derived from a cross bar.

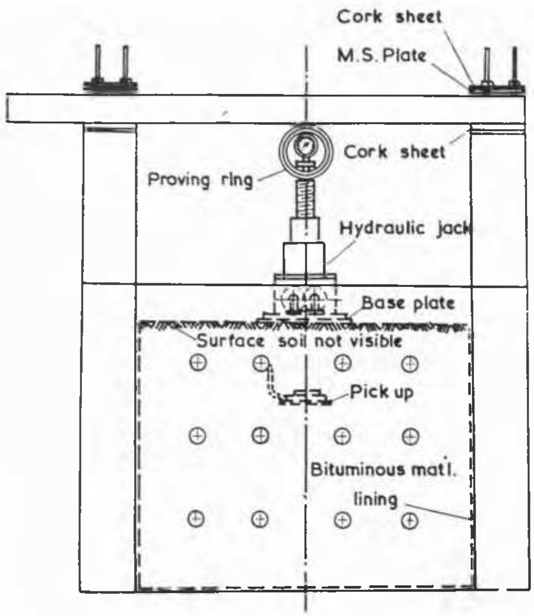

Fig. 1 Experimental set-up for studies on pressure increase. Montage expérimental pour les études de variation de charge.

A direct current electric motor with a flexible shaft was used to drive the vibrator and a stroboscope to measure and to maintain the speed at a definite value. By operating the motor at 2 000 r.p.m, the pressure generated in the soil was recorded as a function of time. Fig. 2, is a typical record for three combinations of static load and one value of dynamic load. The static load was kept constant by continuous operation of the jack. After each experiment the sand was disturbed sufficiently to dissipate the pressure recorded by the pick-up and then recompacted for the next experiment.

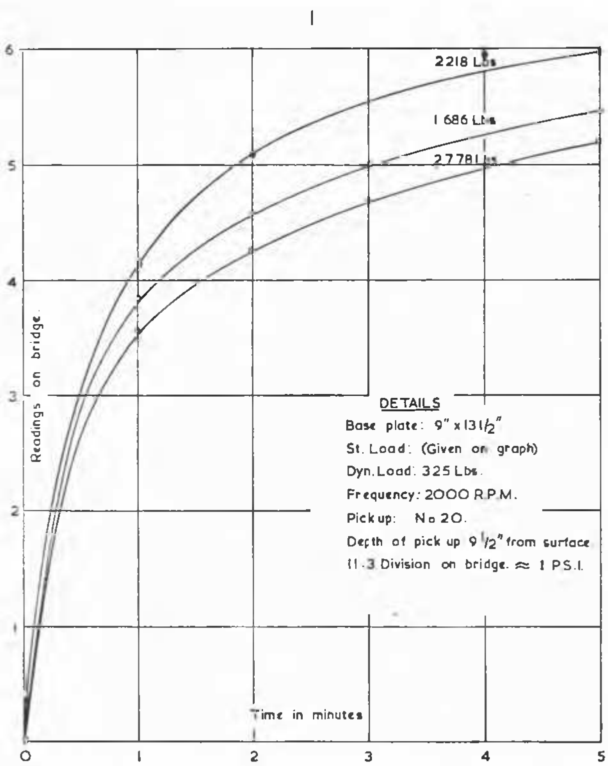

Fig. 2 Graph showing pressure increase as a function of time. Courbes de variation de charge en fonction du temps.

Discussion of test results

While the dynamic load is an oscillating sine wave type of load varying over a constant value of the static load, the static pressure increase occurs because of three conditions (2) :

(a) The settlement of the vibrator due to the close proximity of the pick-up (neglecting the displacement of the pick-up itself).

(b) Compaction of the soil due to the vibrator operating on the surface.

(c) The nature of the soil which does not allow the pressure built up in one cycle to dissipate immediately when the load reverses itself, due to the upward centrifugal force generated by the vibrator, in the succeeding cycles of load repetition.

Conclusions

1. An optimum combination of static and dynamic load gives peak pressure increase.

2. The peak pressure is reached about five minutes after starting the vibrator while in the first minute about 60 per cent of the pressure increase will have occurred.

3. The dynamic load has the effect of an additional static load, as far as the static pressure increase is concerned. This observation has been made use of in developing the theoretical analysis in the following pages.

Natural frequency of foundation soil systems

Any design of machine foundation must be safe under both static and dynamic conditions, the latter being included here. For the purposes of designing a machine foundation for the dynamic condition, the author assumes that the soil is equivalent to a spring supporting the weight W_v, being the sum of the weight of the machine and the concrete foundation block. The design principally aims at the elimination of resonance that may set in when the machine operates. The differential equation governing such a case is:

$$\frac{W_v}{g}\ddot{X} - KX = 0 \qquad (1)$$

$$f_n = \frac{1}{2\pi}\sqrt{\left(\frac{Kg}{W_v}\right)} \qquad (2)$$

where K is the spring constant, X is the displacement, $\ddot{X}$ is the acceleration, and f_n is the natural frequency, but it has been found that a considerable volume of soil participates in vibrations. Hence this mass is considered to be a part of the system, which in turn is supported on weightless springs. Hence the natural frequency is given by :

$$f_n = \frac{1}{2\pi}\sqrt{\left(\frac{Kg}{W_v + W_s}\right)} \qquad (3)$$

Hitherto no analytical work has been developed regarding the mass of soil oscillating. Hence to determine the natural frequency, it is necessary to know the spring constant K, and the weight of soil oscillating, W_s. The determination of K itself is not very easy in the case of soils. It has been found to depend on a number of factors like :

(a) Density.
(b) Shear modulus.
(c) The dynamic stress intensity.
(d) The static stress intensity.

(e) The size and minimum width of the base of the foundation block and so on. It has not been possible to determine the spring constant comprising all these factors. However, a method has been given by Pauw for the determination of this spring constant (3).

Pauw's method for finding the spring constant

Under certain assumptions and by considering surface deformation due to a trapezoidal load distribution inside the soil mass Pauw has given the expressions for spring constant as :

$$K_z^{xy} = \beta b^2 r_z^{xy}$$

for cohesionless soil with rectangular surface load, where:

$$\frac{1}{r_z^{xy}} = \int_0^\infty \frac{dw}{(r+w)\cdot(s+w)\cdot(1+w)} \quad (4)$$

in which $r = a/b$, $s = \alpha h/b$, $w = \alpha z/b$, b being the minimum width of the foundation block, a is the length of the block, z is the depth of an elemental area below the surface of the soil, $(h + z)$ is the effective depth, and $\alpha/2$ is the angle of dispersion, β is a constant obtained by assuming that the modulus of Elasticity varies linearly with effective depth in the case of cohesionless soils and is a constant irrespective of depth in cohesive soils. A similar expression is also given by Pauw for cohesive soils, which is not included here.

The concept of density pressure bulb

It has been observed by many investigators that the natural frequency for the vibration system decreases because of the increase of static or dynamic load. CROCKETT [5] suggests that a certain mass of soil may oscillate as an active mass in the form of a bulb. However, there is no experimental evidence to support this statement. Hence it was proposed to investigate whether the mass of soil oscillating could be correlated with the mass enclosed in a static pressure bulb, given by Boussinesq's equation. For simplicity's sake the load was assumed to be a point load, and consequently the pressure bulbs are spherical.

Since the natural frequency decreases due to an additional dynamic load, it was first assumed that the total forces acting should be the mass of the foundation block, the machine and also the maximum positive dynamic load. The value of dynamic load is taken as zero for free vibration problems. This assumption is reasonably correct according to the experimental observations made previously. Hence the formula is :

$$f_n = \frac{1}{2\pi}\sqrt{\left(\frac{Kg}{W_s + W_{vs} + W_{vd}}\right)} \quad (5)$$

where, W_{vs} is the weight of foundation block, W_{vd} is the maximum positive dynamic load, and W_s is the weight of soil participating in vibrations. By this equation it is obvious that a change in natural frequency can be brought about by changing the static or dynamic loads, while a change in the value of the spring constant due to a change in the dynamic load is very small and hence neglected. Now the value of the intensity of stress is chosen as the guiding factor to find out the mass of soil oscillating. Since it is well known that density is one of the factors which changes the natural frequency, many trials established that it is possible to connect the value of density with the value of stress intensity, thereby setting a boundary to the active mass of soil oscillating. In other words, if ρ is the density of soil in pounds per cubic foot, then the mass of soil oscillating is the one enclosed in the pressure bulb of intensity ($\sigma = \rho$) pounds per square foot. The deepest point of the pressure bulb is obtained by Boussinesq's equation:

$$Z^2 = 0{\cdot}4775\, Q/\sigma = 0{\cdot}4775\, Q/\rho \quad (6)$$

The mass of soil assumed to be oscillating

$$4/3\left(\frac{0{\cdot}4775}{4} Q/\rho\right)^{3/2} \cdot\pi\cdot\rho$$

where Q is the sum of the static and dynamic load acting as a concentrated load. Therefore the natural frequency is given by the equation :

$$f_n = 1/2\pi \times \sqrt{\left(\frac{\beta b^2 r_z^{xy} g}{\frac{4}{3}\left(\frac{Q}{\rho} \times \frac{0{\cdot}4775}{4}\right)^{3/2}\cdot\pi\rho + W_{vd} + W_{vs}}\right)} \quad (7)$$

This was applied to the experimental observations of CONVERSE [6], PAUW [3] and EASTWOOD [7]. There is excellent agreement between the calculated and experimental values except in cases where the total load on the base plate is quite small, causing a relatively small load to be transferred to the soil. This can be explained by the fact that for plates sustaining small loads, the natural frequency is independent of factors like, minimum width and area, a contention proved by experimental results.

Mass of machine foundation block by the new concept

HOOL and KINNIE [8] have given a method for designing a rigid foundation block for reciprocating machine. Equating the disturbing forces created by the reciprocating and rotating parts to the weights of the machine with the foundation block, and part of the soil which is assumed to be C times the mass of the foundation block only, the following equation is obtained:

$$\frac{W_3 + W_4 + CW_4}{g}\cdot a = W_1\omega^2 r/g + W_2\cdot\frac{\omega^2 r}{g}\left(1 + \frac{1}{q}\right) \quad \ldots\ldots \quad (8)$$

where, W_3 is the weight of the machine, W_4 is the weight of foundation block, CW_4 is the weight of soil oscillating (by assumption), a is the acceleration given by $b\omega^2$, W_1 is the weight of rotating parts, W_2 the weight of reciprocating parts, ω is the angular velocity, r is the radius of the crank, q is the l/r ratio, and l is the length of the connecting rod. By this method the value of C has to be decided on judgement. Two methods for evaluating the value of C has been found, but here one method has been suggested :

Assuming $W_4 = A/1 + C$, where A can be found for the given machine by the equation,

$$A = \frac{W_1 r}{b} + \frac{W_2 r}{b}\left(1 + \frac{1}{q}\right) - W_3 \quad (9)$$

where b is the maximum permissible amplitude, it is possible to find out the mass of soil enclosed in the density pressure bulb assuming that the sum of W_4, W_3 and also the dynamic load (this can be evaluated for the given machine) act together as a concentrated load. Equating this mass of soil to CM_4, (because of the definition of C), which is the same as $AC/1 + C$ a cubic equation of the form

$$pC^3 + qC^2 - rC - s = 0,$$

can be obtained where p, q, r, and s are all positive and none are zero. This equation can have only one positive real root and hence gives a unique value of C. This method has been found to give fairly conservative values.

Experimental investigations on the behaviour of soils at resonant frequency

A tank of 4 ft. 6 in. by 4 ft. 6 in. by 4 ft. was constructed below ground level inside the laboratory and the top 8 in. of the tank was plastered; the remaining portion had no protection other than the soil, which reduced the reflection of the waves considerably. A silty soil was transferred into the tank, after removing the very coarse material. The overall density of this silty soil was 92 lb. per cub. ft. A mild steel base plate $\frac{1}{4}$ in. thick was placed on the carefully levelled soil surface and excited by means of a Degebo type vibrator carrying different weights of sand bags which provided the static load. An electrodynamic pick-up was placed exactly at 4 in. from the edge of the base plate as shown in Fig. 3. This pick-up is an absolute velocity pick-up connected in series with an integrating differentiating device, capable of measuring displacement as well as acceleration. The peak value of amplitude was found by increasing speed gradually for the given static load with the help of an oscilloscope and the corresponding frequency was measured by means of a stroboscope. The amplitudes of displacement were measured at different frequencies very close to the natural frequencies. The experiments were conducted for three different eccentric masses, eight different static loads and thirteen different base plates. Later the experiments were repeated on sand of an overall density of 91 lb. per cub. ft., for the same three eccentric masses and static loads on seven different baseplates. In all these experiments the total load on the baseplates were gradually varied at increments of 2 cwts. to a maximum load of 1 ton.

Fig. 3 Experimental set-up for studies on natural frequency.
Montage expérimental pour les études de fréquence propre.

The test results

Figs. 4 gives a typical result of the amplitude-frequency relationship for three different eccentric masses, for one static load and for a particular base plate. For higher static load values these graphs had well defined peaks at resonance, but for smaller values of total loads the graphs were not well defined even when the intensities were high. Fig. 5 showing the relationship between peak amplitude at resonance for base plates of different areas while the total load on all the base plates was the same, but the intensities were different. This shows that the peak amplitude at resonance is a constant after a certain area as long as the total load is a constant; this also applies to different dynamic load. This appears to establish the importance of total load, rather than the intensity of loading. Fig. 6, shows the typical rela-

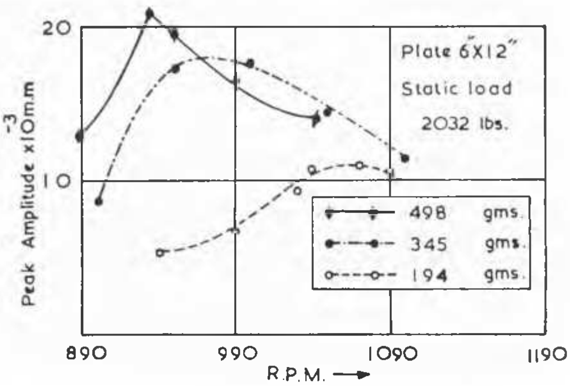

Fig. 4 Amplitude-frequency relationship for different dynamic loads on silty soil.
Relation entre l'amplitude et la fréquence pour un so limoneux et des charges dynamiques différentes.

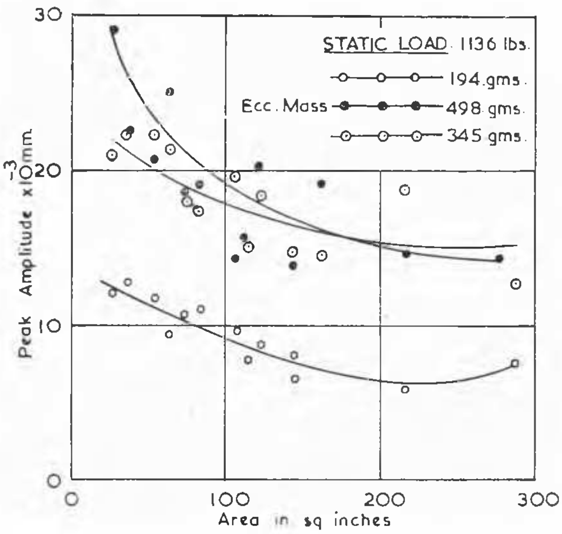

Fig. 5 Graph of peak amplitude at resonance plotted against area on silty soil.
Courbes donnant l'amplitude maximale quand il y a résonnance en fonction de l'aire chargée dans le cas d'un terrain limoneux.

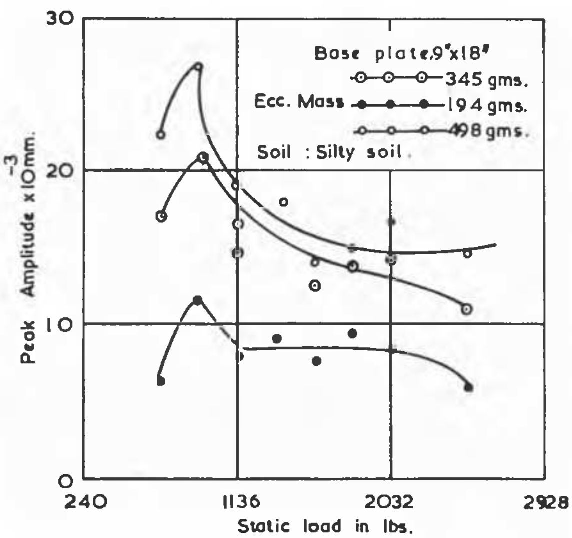

Fig. 6 Graph of peak amplitude at resonance against static load.
Courbes d'amplitude maximale quand il y a résonnance en fonction de la charge statique.

tionship between peak amplitude at resonance for different total loads for one particular base plate. This shows that there is a peak-peak amplitude at a certain total load and this load appears to be the same for all the base plates. Fig. 7 shows a typical relationship between natural frequency and total load on the base plate. The scattering in the values appears to be predominant at lower total loads, while at higher loads there seems to be a definite relationship between natural frequency and total loads. It has been found

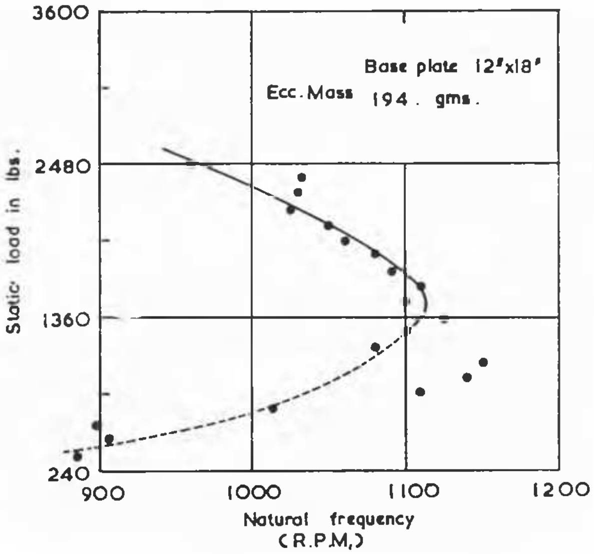

Fig. 7 Variation of natural frequency against static load (data cannot be concluded in the range of dotted line).

Variation de la fréquence propre en fonction de la charge statique (les valeurs indiquées dans la région où la courbe est pointillée ne permettent de tirer aucune conclusion).

Table 1

Type of Soil : Silty

Size of base plate	*Resonant Frequency in R.P.M.* (1)	(2)	(3)	(4)	(5)	*Ecc. Mass in* gms
12" × 24"	1 025	990	1 050	965	900	345
12" × 18"	1 050	1 000	1 050	1 000	920	345
12" × 12"	1 250	1 010	1 050	920	880	345
12" × 24"	1 100	1 000	1 050	990	920	498
12" × 18"	980	1 100	1 060	1 010	940	498
12" × 12"	1 200	930*	1 040	940	870	498
12" × 24"	910*	950*	995*	940	910	194
12" × 18"	1 270	1 100	1 125	1 100	990	194
12" × 12"	1 010	1 100	1 140	1 050	970	194

Remarks : 1. * Indicates that the results do not fit into the Gen. Pattern.
2. — indicates that the readings could not be obtained.
3. Static load conditions : (1) 688 lbs ; (2) 1136 lbs ; (3) 1584 lbs (4) 2032 lbs ; (5) 2480 lbs.

Table 2

Type of soil : Sand

Size of base plate	*Resonant Frequency in R.P.M.* (1)	(2)	(3)	(4)	(5)	*Ecc. Mass in* gms
12" × 24"	1 220	1 150	1 140	1 025	940	345
12" × 18"	1 250	1 050	1 125	1 025	935	345
12" × 24"	—	1 175	1 125	1 025	940	498
12" × 18"	—	1 050	1 150	1 025	960	498
12" × 24"	1 250	1 150	1 100	1 000	940	194
12" × 18"	1 300	1 125	1 125	1 025	965	194

Remarks : 1. * Indicates that the results do not fit into the Gen. Pattern.
2. — indicates that the readings could not be obtained.
3. Static load conditions : (1) 688 lbs ; (2) 1136 lbs ; (3) 1584 lbs (4) 2032 lbs ; (5) 2480 lbs.

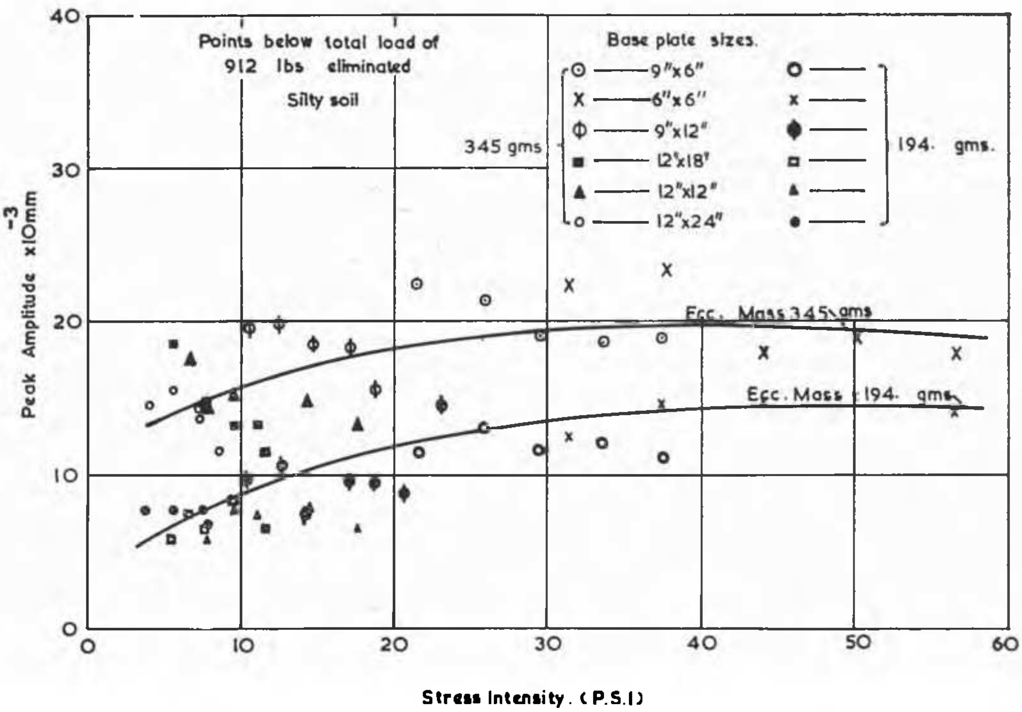

Fig. 8 Graph showing peak amplitude at resonance plotted against stress intensity in lb. per sq. in.

Courbe de l'amplitude maximale à la résonnance en fonction de la contrainte en livres par pouce carré.

that about 20 per cent variation in natural frequency value is possible due to a variation of total load from $\frac{1}{4}$ ton to 1 ton. This has been confirmed by earlier investigators. This observation was also noted from the published graphs of Eastwood i.e., when extrapolated for the same total load for different base plates (from the graphs), the variation was only 5 per cent. Fig. 8 shows peak amplitude at resonance plotted against intensity of load, which appears to be a constant beyond a certain intensity of load. But the values for higher dynamic loads, the smaller areas, as well as plates with the smaller total loads, showed considerable scattering, thereby proving the edge effects on settlement, which become dominant at higher dynamic loads. In fact this statement has been made by several investigators, who suggest taking larger areas for studies.

Table 1 gives natural frequency on silty soil for three different base plates for one dynamic load while the intensity of load has varied considerably, because of maintaining the total load as a constant. Table 2 gives a typical result on sandy soils. It may be observed that the value of natural frequency at higher load ranges for different base plates is constant. At lower loads there are variations but this may be because of the difficulties encountered in locating resonant frequency at low static loads. This observation also suggests the importance of total load.

Conclusions

Since the method of predicting natural frequency gives very close correlation with the experimental results for different conditions this can be taken as reasonably accurate. This method could be used for evaluating an empirical constant. In addition some useful observations on the behavior at resonant frequency has been outlined.

Hitherto the approach for the design of machine foundations has been to allow a lower static stress intensity. The observations enumerated indicate the importance of the total load oscillating. But a lower stress intensity gives a greater area and thus a more stable system. It should be noted that the equation for natural frequency contains two important factors, k and W_s. If an area is chosen for the base of the foundation block the variation of k will not be much for different static loads. But since the mass of soil oscillating varies, the natural frequency varies. The variation of the mass of oscillating soil appears to be a function of total load rather than of the intensity of the load. This can also explain why some machine foundations resonate at very low frequencies. It is implied that the natural frequency of a foundation-soil system is much more important than the natural frequency of the soil itself. But the limitations to be borne in mind while applying the theory are that the fundamental assumption of a single mass spring system may not always be correct. For want of better assumptions and in order to simplify the problem, this assumption has been made. In addition, the foundation block is supposed to be laid on the surface of the soil, which is not normally done in practice. This is the main reason why the method gives fairly conservative values for C of Hool and Kinnie's method.

Acknowledgements

The author thanks Professor N. S. Govinda Rao for his suggestions, comments and the keen interest taken during the investigations. The facilities provided by the authorities of the Indian Institute of Science, The Central Board of Irrigation and Power are gratefully acknowledged.

References

[1] Nagaraj, C. N. and Balakrishna Rao, H. A. (1959). The development of a strain gauge pressure pick up for use with soils, *Journal of Irrigation and Power*, Volume 16, July, pp. 334-337.

[2] — and — (1959). Stress at a point within a semi-infinite elastic medium subjected to a surface vibratory load, *Journal of Indian Roads Congress*, Vol. XXIV-2, Nov., pp. 461-473.

[3] Pauw, A. (1953). A dynamic analogy for foundation soil systems, A.S.T.M., Special publication, No. 156, pp. 90-112.

[4] Balakrishna Rao, H. A. and Nagaraj, C. N. (1960). A New method for predicting the natural frequency of foundation-soil systems, *The Structural Engineer*, October, pp. 310-316.

[5] Crockett, J. H. A. and Hammond, R. E. R. The dynamic principles of Machine foundations and ground, Institution of Mech. Engineers (Adv. Copy).

[6] Converse, F. J. (1953). Compaction of sand at resonant Frequency, *A.S.T.M. Special publication*, No. 156, pp. 124-136.

[7] Eastwood, W. (1953). Vibrations in foundations, *The Structural Engineer*, March, pp. 82-98.

[8] Hool and Kinnie (1943). Foundations, Abutments and Footings, Mcgraw Hill Book Co., pp. 317-320.

[9] Degebo Publications :

(a) Hertwig, A., Fruh, G., Lorenz, H. Determination by means of Forced Vibrations of Soil properties of Particular importance for Construction Work. Pub. 1, Berlin, 1933.

(b) Ramspeck, A. and Schulzw, G. A. The dispersion of elastic waves in the Ground. Vol. 4, Berlin, 1933.