Design of Structures on Elastic Foundations

Calcul des ouvrages sur appui élastique

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Summary

The authors describe a method useb by Soviet engineers for designing beams and slabs on elastic foundations.

M. I. Gorbunov-Possadov has derived a formula of linear connections between the coefficients of a double power series, expressing the distribution of normal pressures on the rectangular area of an elastic half-space surface, and the coefficients of the same series expressing the vertical displacement of points on this surface. On this basis the solution of the problems of settlement of rigid foundations with central and eccentric loads and the bending of flexible rectangular slabs, under the action of uniform and concentrated loads, are calculated. A solution is derived for slabs of great length, of the airfield slab type with loads located near the free edge.

For continuous foundations, double power series pass into single ones. The methods of calculation of rigid, short, infinite and semi-infinite beams are given on elastic foundations with any loads.

R. V. Serebrjanyi has solved the problem of bending under a concentrated load of rectangular slabs of great length hinged to adjoining ones and resting on an elastic foundation. The solution is obtained by integral transformations. Tables of slab deflections and bending moments are given.

Sommaire

Exposé sommaire du développement en U.R.S.S. des méthodes de calcul des poutres et dalles sur appuis élastique d'après différents modèles de propriétés mécaniques du sol.

Le rapport est essentiellement consacré à la solution de problèmes à trois dimensions que les auteurs ont pu résoudre en partant d'un modèle élastique semi-infini.

M. Gorbounov-Passadov a traité les problèmes de calcul des poutres et dalles de fondations d'une rigidité et de dimensions quelconques, ayant une semelle de forme rectangulaire. Il a trouvé l'expression de la relation linéaire qui existe entre les coefficients de la série exponentielle double exprimant la distribution des pressions normales sur une aire rectangulaire en surface d'un demi-espace élastique ainsi que l'expression des coefficients d'une série identique exprimant le déplacement vertical des points sur cette aire. C'est sur cette base qu'ont été résolus les problèmes du tassement des fondations rigides soumises à des charges centrées ou excentrées et de la flexion des dalles rectangulaires flexibles soumises à des charges uniformément réparties et concentrées.

Une solution a été donnée pour les dalles de grande longueur du type employé pour les revêtements des aérodromes, chargé sur le bord extérieur.

Pour les fondations continues, les séries exponentielles doubles se transforment en séries simples. Sont également exposés les procédés de calcul de poutres rigides, courtes, infinies et semi-infinies sur une fondation élastique, sollicitées par une charge quelconque.

R. Serebriany a trouvé la solution du problème de la flexion de dalles rectangulaires de grande longueur unies les unes aux autres par des articulations et reposant sur une couche élastique, sollicitées par une charge concentrée. La solution a été obtenue à l'aide de transformations intégrales. Des tables donnant les déflexions des dalles et les moments de flexion ont été calculées.

In the Soviet Union, the foundation bed is generally considered as an elastic half-space for practical calculations of the strength of reinforced concrete foundations. It is estimated, that even though this hypothesis does not make adequate allowance for the properties of the soil, it is nevertheless nearer to actual conditions than the Winkler hypothesis.

The pioneers of this new method of calculation in the U.S.S.R. were G. E. Proctor and N. M. Gersevanov. Considerable contributions were also made by V. A. Florin, B. N. Zhemochkin, O. J. Shekhter, P. I. Klubin and others, who considered the plane problem and the axial symmetrical problem.

M. M. Filonenko-Borodich, V. Z. Vlasov, P. L. Pasternak have developed design methods in which the foundation is represented by two bed coefficients representing compression and shear resistance. A. P. Sinitzin, I. J. Stajerman, and I. I. Cherkasov used combinations of models of elastic half-spaces and the Winkler model. G. K. Klein introduced a calculations method for foundation beds with the modulus of elasticity changing with depth. O. J. Shekter, V. N. Avramenko, L. I. Lokkenberg-Fedulova, I. K. Samarin, K. E. Yegorov and others worked out calculation methods for the case of an elastic layer of finite thickness. B. G. Korenev derived a solution for most of these models simultaneously, and the same author considered the problem of estimating plastic hinges in slabs on elastic foundations.

Reviews of these works are given in several works (M. I. Gorbunov-Possadov, 1949, B. J. Korenev, 1954, N. A. Zytovitch, 1956).

The authors consider the design of beams and rectangular slabs with free edges resting on an leastic half-space (M. I. Gorbunov-Possadov), as well as slabs hinged with adjoining ones along common rectilinear lines (R. V. Serebrjanyi).

For a rectangular supporting foundation area. the connection between vertical displacement $W(x.y)$ of the half-space surface and normal pressures $p(x.y)$, transmitted to the soil by the foundation according to Boussinesq is determined by formula :

$$W(x, y) = \frac{(1 - \nu_0^2)\, ab}{\pi E_0} \int_{-1}^{1} \int_{-1}^{1} \frac{p(\bar{x}, \bar{y})\, d\bar{x}\, d\bar{y}}{\sqrt{a^2(\bar{x} - x)^2 + b(\bar{y} - y)^2}} \tag{1}$$

(friction of the foundation against the soil is neglected).

In formula (1) ν_0 and E_0 are respectively Poisson's ratio and the modulas of deformation of the foundation bed, $x = \frac{x'}{a}$ and $y = \frac{y'}{b}$ — reduced coordinates of displaced point (while axis x is directed parallel with the big side and x' and y' — actual coordinates) a and b — correspondingly half-length and half-width of supporting area, $\bar{x}$ and $\bar{y}$ — reduced coordinates of pressure element.

It is assumed that the unknown distribution of reactive pressures is expressed by a double power series :

$$p(x, y) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} a_{ij}\, x^i y^j \tag{2}$$

Thus, settling may also be expressed by a double power series (Gorbunov-Possadov, 1939).

$$W(x,y) = \frac{(1-\nu_0^2)a}{\pi E_0} \sum_{v=0}^{\infty} \sum_{u=0}^{\infty} \left(\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} C_{u,v,i,j}\, a_{ij} \right) x^u y^v$$

where (3)

$$C_{u,v,i,j} = i!j! \sum_{m=M}^{i} \sum_{n=N}^{j} \times \frac{\beta^{v-j}\,[(-1)^m + (-1)^{u+m-i}]\,[(-1)^n + (-1)^{v+n-j}]}{(i-m)!(j-n)!m!n!(u+m-i)!(v+n-j)!}$$
$$\times \frac{d^{v+n-j}}{d\beta^{v+n-j}} \int_o^{\beta} \beta^{n+i-u} \int_o^{1/\beta} \frac{d^{u+m-i}}{dt^{u+m-i}} \frac{t^m dt}{\sqrt{1+t^2}}\, d\beta \quad \ldots \tag{4}$$

and $M = 0$ at $i \leqslant u$, $M = i - u$ at $i \geqslant u$, $N = 0$ at $j \leqslant v$, $N = j - v$ at $j \geqslant v$, $\beta = \frac{b}{a} \leqslant 1$.

For values of u, v, i, j from 0 to 6, algebraic expression of integral equations (4) have been found for $\alpha = \frac{1}{\beta} = 1, 1{\cdot}5, 2, 3, 5, 7, 10$ with their numerical values.

For rigid rectangular slabs with central load P, unknown coefficients $a_{2i,\,2j}$ in equation (2) are determined on condition of equilibrium :

$$4ab \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{a_{2i,\,2j}}{(2i+1)\,(2j+1)} = P \tag{5}$$

and constant settling conditions. On this basis, the coefficients of all derivatives $x^{2u}\, y^{2v}$ are taken equal to zero, except for $x^o y^o$. The obtained infinite system of equations in relation to $a_{2i,2j}$, is substituted by a shorter one. A similar method for problems solved in single series was used by H. Borovicka (1936, 1939), V. A. Florin (1937) and M. I. Gorbunov-Possadov (1937, 1941).

As a result, for example, for a rigid square stamp ($\alpha = 1$) at $2n = 8$ the following equation was obtained :

$$p(x,y) = [0{,}541 + 0{,}248(x^2+y^2) + 0{,}251(x^4+y^4) - 0{,}008x^2y^2 + 0{,}280(x^6+y^6) + 0{,}042(x^4y^2+x^2y^4) + 0{,}499(x^8+y^8) + 0{,}079(x^6y^2+x^2y^6) - 0{,}186x^4y^4]\,\frac{P}{F} \tag{6}$$

$$W_p = \frac{1-\nu_0^2}{E_0}\, K_0\, \frac{P}{\sqrt{F}}, \quad K_0 = 0{,}913 \quad (F = 4ab) \quad \ldots \tag{7}$$

Similar solutions were received for eccentrically loaded slabs. For moments $\mathfrak{M}_x$ and $\mathfrak{M}_y$, acting correspondingly along axes x and y, the angles of turning of the foundations were :

$$\mathrm{tg}\,\varphi_x = \frac{1-\nu_0^2}{E_0}\, K_1\, \frac{\mathfrak{M}_x}{a^3}, \quad \mathrm{tg}\,\varphi_y = \frac{1-\nu^2}{E_0}\, K_2\, \frac{\mathfrak{M}}{f^3} \quad \ldots \tag{8}$$

On the basis of the received data, taking into account the decreasing character of values K_0, K_1, K_2 with increasing power of pressure polynomials, graphs were plotted of more accurate values (Fig. 1).

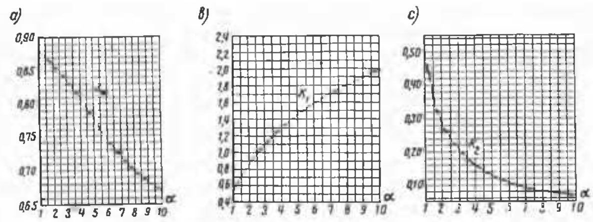

Fig. 1 Graphs for determination :

(a) by formula (7) of settlement of rigid rectangular plate with ratio of sides α under action of central force P ;
(b) by the first formula (8) of angles of turning of a rigid rectangular plate under the action of moment $\mathfrak{M}_x$;
(c) according to second formula (8) under action of moment $\mathfrak{M}_y$.

Graphiques servant à déterminer :

(a) le tassement d'une dalle rectangulaire rigide sollicitée par une charge concentrée P en fonction du rapport des côtés d'après la formule (7) ;
(b) les angles d'inclinaison d'une dalle rectangulaire rigide sollicitée par un moment $\mathfrak{M}_x$, d'après la première formule (8) ;
(c) sollicitée par un moment $\mathfrak{M}_y$, d'après la seconde formule (8).

For flexible rectangular slabs the unknown bending equation with continuous loads were written in the form of a double series :

$$Y(x,y) = \sum_{u=0}^{\infty} \sum_{v=0}^{\infty} B_{u,v} x^u y^v \tag{9}$$

In case concentrated load P is placed at any point of the slab, series (9) has the following member added :

$$\frac{Pa^2}{8\pi D}\, \rho^2\, ln\rho \quad \ldots \tag{10}$$

where

ρ — distance from point of load application reduced to a,
D — cylindrical rigidity of slab.

On the basis of the biharmonic equation of slab bending and edge conditions, a linear function is determined between the unknown coefficients in series (2) and the coefficients in series (9). Consequently, comparison with similar powers in series (3) and (9) results in an infinite linear system of equations in relation to unknowns a_{ij}, which are substituted by shorter ones. During calculations irregular function (10), as well as the derivatives of this function are substituted by power interpolation polynomials.

Fig. 2 is a graph of lines of equal non-dimensional calculation values, when the slab has a concentrated load at the centre. Lines of reactive pressures $\bar{p}$ are given for each of the four quadrants, bending moments $\bar{M}_x$, transversal forces $\bar{N}_x$ and torsional moments $\bar{H}_x$ for the case when the slab is

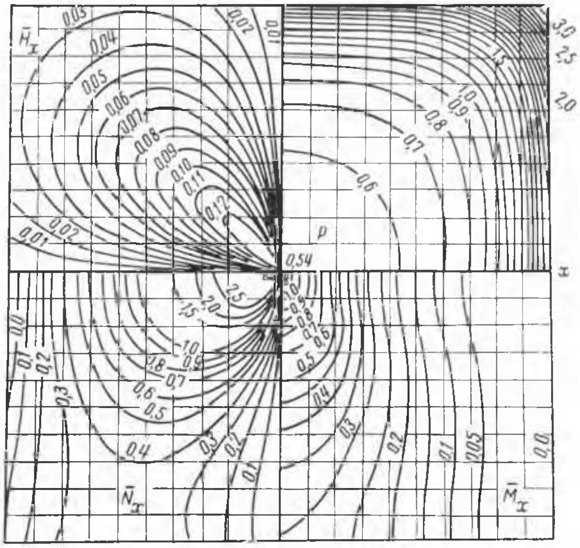

Fig. 2 Graph. of non dimensional reactions $\bar{p}$, bending moments $\bar{M}_x$, torsional moments $\bar{H}_{xy}$ and transversal forces $\bar{N}_x$ for calculating by formulas (12) of rigid square plate under the action of a concentrated load P.

Graphiques exprimés en unités sans dimensions des réactions $\bar{p}$, des moments de flexion $\bar{M}_x$, des moments de torsion $\bar{H}_x$ et des forces transversales $\bar{N}_x$ pour le calcul d'une dalle carrée rigide sollicitée par une charge concentrée P, d'après les formules (12).

so rigid that the reaction pressures actually do not differ from the pressures described in equation (6). This is assumed when the power of flexibility of the rectangular slab :

$$r = \frac{\pi a^2 b E_0}{D(1 - \nu^2)} \leqslant \frac{8}{\sqrt{\alpha}} \tag{11}$$

The actual design values are determined by the following formulae :

$$p = \bar{p}\,\frac{P}{F},\ M_x = \bar{M}_x a^2 \frac{P}{F},\ N_x = \bar{N}_x a \frac{P}{F},\ H_x = H_x a^2 \frac{P}{F} \tag{12}$$

The solution for a flexible square slab of very large dimensions ($r \geqslant 50$) with a concentrated load near the edge based on the Saint-Venan principle and the theory of similarity to compose the tables (Gorbunov-Possadov, 1959) for calculating thin slabs. These tables are employed for designing airfield pavements, concrete floors, and slabs below columns.

Fig. 3 shows curves calculated for loads at the middle of the edge of square slabs with cylindrical rigidity D of a concentrated load $_b = 50$. Calculation is performed by the following formula :

$$p = \bar{p}\,\frac{P}{L^2},\ W = \bar{W}\,\frac{(1-\nu_0^2)}{E_0}\frac{P}{L},\ M_x = M_x P,\ \bar{M}_y = \bar{M}_y P,$$

$$L = \sqrt{\frac{2D(1-\nu_0^2)}{E_0}},\ \xi = x'/L,\ \eta = y'/L \tag{12'}$$

For beams with $\alpha \geqslant 10$ calculation is simplified by ignoring the squares and higher powers of β in the algebraic form of equations (4). As a result, for example, for symmetrical loads the following is obtained :

when

$$V \neq 0,\ i \neq u\,;\ C_{2u,\ 2v,\ 2i,\ 2j} = 0$$

when

$$V \neq 0,\ i = u\,;\ C_{2u,\ 2v,\ 2i,\ 2j} = \frac{4}{2v(2j - 2v + 1)}$$

when

$$V = 0,\ 1 \neq u\,;\ C_{2u,\ 2v,\ 2i,\ 2j} = \frac{4}{(2i - 2u)(2j + 1)}$$

when

$$V = 0, i = u\,; C_{2u,\ 2v,\ 2i,\ 2j} = \frac{4}{2j+1}\left(ln 2\alpha - d_{2u} + \frac{1}{2j+1}\right)$$

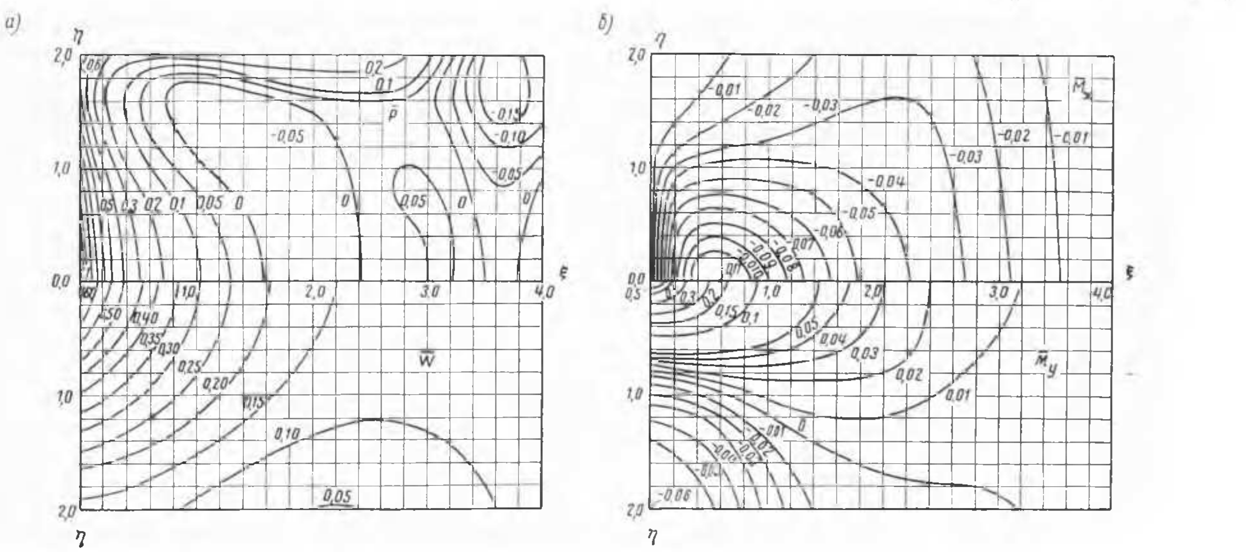

Fig. 3 Graphs of non dimensional values for calculation by formula (12′) of flexible square plate of great length.

Graphiques exprimés en unités sans dimensions pour le calcul d'une dalle carrée flexible d'une grande longueur d'après les formules (12′).

where

$$d_{2u} = \sum_{s=1}^{2u} \frac{1}{S}, \quad d_0 = 0 \tag{13}$$

Further, on the understanding that, the beam does not bend transversely and, that the coefficients for all multiplications x^{2u} y^{2v} in series (3) at $v > 0$ should equal zero, the following equations apply to symmetrical loading :

$$\sum_{j=0}^{\infty} \frac{a_{u,2j}}{2j - 2v + 1} = 0 \qquad \begin{array}{l} u = 0,1,2,3,. \\ v = 1,2,3, \end{array} \tag{14}$$

the accurate solution being :

$$a_{u,2j} = \frac{1{\cdot}3{\cdot}5\ldots(2j-1)}{2{\cdot}4{\cdot}6\ldots 2j} a_{u,0} \tag{15}$$

leading to the following equation for reactive pressures :

$$p(x,y) = \sum_{i=0}^{\infty} \frac{a_{i,0} x^i}{\sqrt{1-y^2}} \tag{16}$$

Consequently, the transverse reactions are distributed in accordance with the formula for rigid strips of M. SADOWSKY (1928).

A similar solution is also correct for asymmetrical loads.

Formula (16) may be used to determine the distribution of average transverse pressure in the form of a single series :

$$p(x) = \sum_{i=0}^{\infty} a_i x^i \qquad \text{where} \qquad a_i = \frac{\pi}{2} a_{i,0} \tag{17}$$

Displacement of the foundation under the beam with a symmetrical load is expressed by the following formula :

$$W(x) = \frac{4(1-\nu_0^2)}{\pi E_0} \sum_{u=0}^{\infty} \left[\sum_{\substack{i=0 \\ i \neq u}}^{\infty} \frac{a_{2i}}{2i - 2u} + (ln 4\alpha - d_{2u}) a_{2u} \right] x^{2u} \quad \ldots . \tag{18}$$

The equation of bending of beam $Y(x)$ in case of a continuous loas or concentrated load placed at the ends, after integration of a common differential equation of beam bending, is also expressed in the form of a power series, the coefficients of which are directly proportional to a_{2i}. In case of intermittent loads or of a concentrated load placed at the inner sections of the beam, irregular functions in the bending equations may be replaced by interpolation power polynomials on the basis of minimum squares. Therefore, to ensure accurate or approximate execution of identity $w(x) \equiv Y(x)$ in this case, equalize the coefficients at similar powers x in series $w(x)$ and $Y(x)$. The system thereby obtained with the addition of equations expressing equilibrium conditions is approximately solved by substituting it in abbreviated form with the (6) or (5) equations.

Fig. 4a illustrates the changing of non-dimensional diagrams of calculated values for uniform load q in the function of α and beam flexibility :

$$t = \frac{\pi E_0 a^3 b}{2(1-\nu_0^2) E_1 J} \tag{19}$$

($E_1 J$ — beam rigidity)

To obtain the actual values of the reactions p, bending moment M and deflections (settling) Y employ the following formulae :

$$p = \bar{p}_q; \quad \bar{M} = Mqa^2; \quad Y = \bar{Y}\, \frac{1-\nu_0^2}{E_0} q \ldots . \tag{20}$$

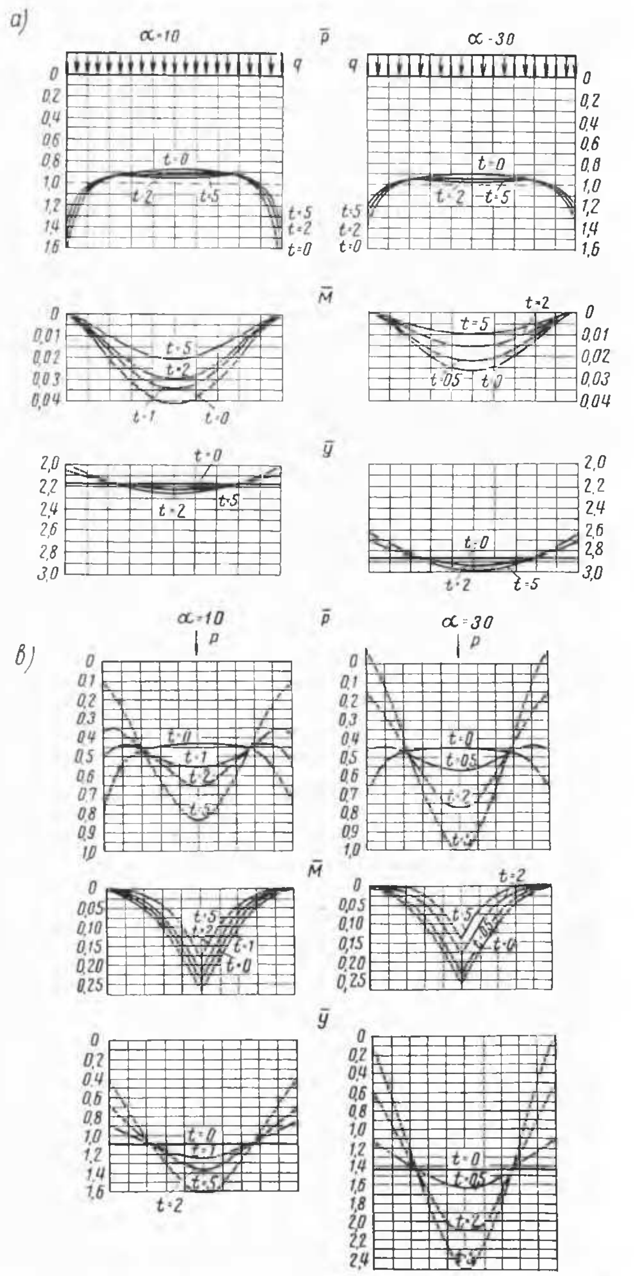

Fig. 4 Graphs of non dimensional diagrams of reactions $\bar{p}$, moments $\bar{M}$ and deflections (settlements) $\bar{Y}$ for calculating a beam of finite rigidity and length by formulae (20) for different values of flexibility of beam t and ratio of sides α :
(a) for uniform load q and
(b) for concentrated load P, placed at the centre of the beam.

Graphiques exprimés en unités sans dimensions des réactions p, des moments $\bar{M}$ et des déflexions (tassements) $\bar{Y}$ pour le calcul des poutres d'une rigidité et longueur limitées d'après les différents indices de la flexion de la poutre t et les rapports des côtés α d'après les formules (20) :
(a) dans le cas d'une charge uniforme q ;
(b) d'une charge P concentrée, appliquée au milieu de la poutre.

Fig. 4b shows a similar graph for a concentrated load placed at the middle of the beam. The transient formulae are found with substitution of q by $\frac{P}{a}$ in (20).

When $t < 0{\cdot}5$, the diagram of calculated values practically coincides with the diagram for an absolutely rigid beam ($t = 0$). For this case, tables have been composed of non-dimensional values $\overline{p}$, $\overline{Q}$, $\overline{M}$, $\overline{Y}$ (Q — transverse forces) for beam design at values α from 10 to 100 and any kinds of loads.

On the other hand, when $t \geqslant 10$ and passing from reduced abscissa $x = \frac{x'}{a}$ to abscissa $\xi = \frac{x'}{L}$ where

$$L = \sqrt[3]{\frac{E_1 J(1 - \nu_0^2)}{bE_0}} \tag{21}$$

the diagrams also do not depend on t and change into diagrams for an infinite beam (in the case of loads remote from both ends), of for the diagrams of half-infinite beams (when loaded near one of the ends). For these cases, employed for designing continuous foundations, detailed tables have been prepared of ordinates of non-dimensional diagrams for the values of $\overline{p}$, $\overline{Q}$, $\overline{M}$, $\overline{Y}$ in any section ξ for various reduced half-widths of the beam $\beta = \frac{b}{L}$ and concentrated load placed at any reduced distance $\delta = \frac{d}{L}$ from the beam end.

The problem of designing a thin rectangulat slab of great length resting on an elastic foundation and with a concentrated load, where the edge his hinged with the edges of the adjoining slab is important, mainly in connection with pavement design. This problem, depending on whether the load is at the centre of the slab, near its edge or corner, can be solved with sufficient accuracy if the following are assumed :

(a) an infinite horizontal slab,
(b) two slabs in the form of half-spaces hinged along a common line,
(c) four slabs in the form of a quarter of a plane hinged along two mutually perpendicular lines.

The first problem has already been well investigated. The solution of the second and third problems is found by means of integral transformations of Fourier.

A hinged joint is such when the values of deflections $W(x,y)$ and the total tangential force $N_n + (1 - \nu)\frac{\partial H_{nt}}{\partial t}$, where ν- Poisson's ratio of slab material n and t directions perpendicular and parallel with the common edge of two slabs, remain continuous when passing from one slab to another, and the value of normal bending moment M_n changes to zero at this edge.

It is considered that all linear dimensions are expressed in fraction of $L = \sqrt[3]{\frac{2D(1 - \nu_0^2)}{E_0}}$ (m). Let us assume that the total external load is P and that it acts on all four slabs hinged along axes x and y, resting on an elastic layer of thickness h, below which is an unloaded layer. Then, multiplying the bending equation of the slab and the equation connecting the deflection of the surface layer with the external load by $\frac{1}{2\pi} e^{i\eta x + i\xi y}$, after integration for all values of x and y the following expression is found for slab deflexion :

$$W(x,y) = \frac{E_0 L}{2\pi P(1 - \nu_0^2)} \times \iint_{-\infty}^{+\infty} \frac{Q(\eta,\xi) + (\eta^2 + \nu\xi^2)\, B_1(\xi) + (\nu\eta^2 + \xi^2)\, B_2(\eta)}{(\eta^2 + \xi^2)^2\, F(\eta,\xi,h) + 1} \times F(\eta,\xi,h)\, e^{-i\eta x - i\xi y}\, d\eta\, d\xi \tag{22}$$

Here $Q(\eta, \xi)$ — Fourier's transformant obtained by multiplying by $\frac{L^2}{2\pi P}$ the function of distribution of the external load on the slab, $F(\eta, \xi, h)$ — the same by multiplying by $\frac{E_0 L}{P(1 - \nu_0^2)}$ the function of deflexion of the surface of the elastic layer, under the action of unit concentrated forces placed at the origin of the coordinates.

$B_1(\xi)$ and $B_2(\eta)$ are determined by the system of Fredholm integral equations of the second kind obtained on condition of changing into zero at the joints of slabs under normal bending moments. In case there are two slabs connected along axis y, $B_2 = 0$ and B_1 is determined by the ratio :

$$B_1(\xi)\left\{-\pi(1-\nu)\xi + \int_{-\infty}^{+\infty} \frac{(\eta^2+\nu\xi^2)\,[(1-\nu)\xi^2(\eta^2+\xi^2)\, F+1]}{[(\eta^2+\xi^2)^2\, F+1]\,(\eta^2+\xi^2]}\, d\eta\right\} = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \frac{(\eta^2+\nu\xi^2)\, Q F}{(\eta^2+\xi^2)\, F+1}\, d\eta \quad \ldots . \tag{23}$$

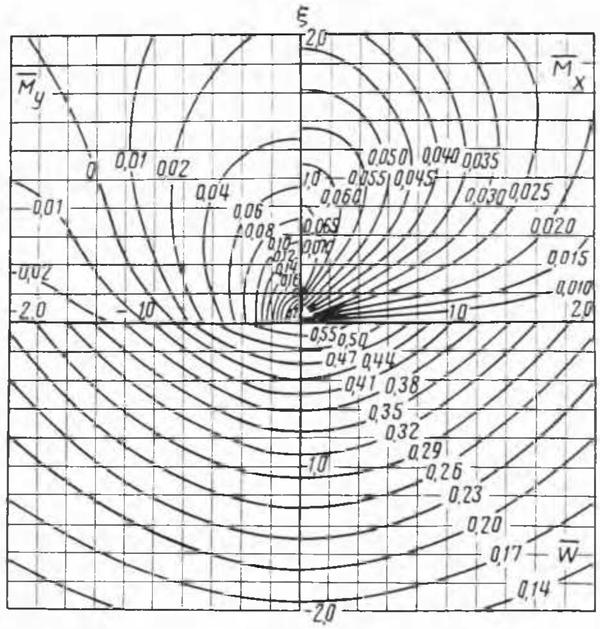

Fig. 5 Graphs of non dimensional diagrams of deflections $\overline{W}$ and bending moments $\overline{M}_x$ and $\overline{M}_y$ for calculating by formulae (12′) of two semi-infinite slabs hinged along axis x and loaded with concentrated load P at the origin of the coordinates.

Graphiques exprimés en unités sans dimensions des déflexions $\overline{W}$ et des moments de flexion $\overline{M}_x$ et $\overline{M}_y$, pour le calcul de deux dalles semi-infinies, unies par des articulations de long de l'axe x et sollicitées par des charges concentrées P à l'origine des coordonnées, d'après les formules (12).

The formula for reactions is derived from (22) when the factor before the integral is replaced by $\frac{L^2}{2\pi P}$, and the kernel expression is divided by F.

The expression for bending and torsional moments, as well as for transverse forces is obtained from (22) by differentiation at x and y as parameters.

The asymptotic equations were found for B_1 and B_2 with large independent variables, due to which all infinite limits of integration were replaced by finite ones and all required integrals were calculated by the Simpson formula. All tables of deflexions and bending moments were composed for the slab for cases when the concentrated load was placed at different points, located perpendicular to the edge in the case of two slabs and along the bisector for four slabs. The thickness of the foundation bed layer h is considered equal to 0·5, 1, and 2,4.

Fig. 5. showes the lines of equal non-dimensional deflexions and moments with load at the joint h of two semi-infinite slabs. Instead of x and y at this figure are written correspondingly ξ and η. The foundation bed in this case is considered as an elastic semi-space ($h = \infty$).

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