Compressibility of Sands and Settlements of Model Footings and Piles in Sand

Compressibilité des sables et tassements de maquettes de semelles et de pieux dans le sable

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Summary

Part 1 of the Paper considers the compressibility of sand in relation to intergranular contacts, and a hypothesis is put forward that the compression strain in sand of low roundness at low constant stress ratios increases as the square root of the mean applied pressure. A variety of evidence has been found to support the new hypothesis.

Part 2 gives settlement results for a series of loading tests on model footings and piles, loaded to beyond their ultimate bearing capacity after being supported during deposition of the sand. The results therefore differ from those obtained on individual piles or groups driven from the surface. Shallow footings and piles in dense sand gave a sharp peak in the load-settlement graph, unlike deeper footings. In loose sand a curved transition gave way to an almost straight section, which marked the passing of the ultimate bearing capacity. There was some evidence that at each porosity tested the ultimate settlement did not increase beyond a certain depth of embedment. The effects of submergence, upward hydraulic gradient and depressed water table were also studied.

Sommaire

La première partie de l'Étude traite de la compressibilité des sables en fonction des contacts intergranulaires et envisage une nouvelle hypothèse selon laquelle la contrainte de compression des sables ayant un faible « indice de rondeur » est, pour de faibles valeurs des forces appliquées, directement proportionnelle à la racine carrée de la pression moyenne exercée. De multiples expériences corroborent cette nouvelle hypothèse.

La deuxième partie de l'Étude présente les résultats d'essais de surcharge statique sur des maquettes de semelles et de pieux, chargées au-delà de leur capacité portante limite. Elles ont été soutenues avant l'application de la surcharge, tandis que l'on procédait au dépôt du sable. Les résultats diffèrent donc de ceux obtenus avec des pieux isolés ou groupés, battus à partir de la surface.

Dans le cas de pieux et de semelles peu profonds la courbe « surcharge-tassement » a accusé une pointe caractéristique, inexistante dans le cas des semelles plus profondes. Les essais réalisés avec des sables meubles ont donné des courbes qui deviennent graduellement presque des droites illustrant ainsi que la capacité portante a été dépassée.

Il semble qu'avec des échantillons de sables ayant différentes porosités, le tassement maximum n'a pas dépassé une profondeur déterminée.

L'auteur a également étudié les effets de la submersion du sable d'un gradient hydraulique vertical, et d'un abaissement du niveau de la nappe.

Part I

Compressibility of Sands

1. Introduction

To forecast the settlement of a foundation, it is convenient to consider separately the stress-strain relations of the soil in both compression (zero lateral strain) and shear (full lateral strain permitted).

The compressibility of sand and other cohesionless soils cannot be accurately determined in the œdometer because of side friction. A better way is to fit a triaxial sample with a sensitive lateral strain indicator (MURDOCK, 1948; BISHOP and HENKEL, 1957; BISHOP, 1958) to maintain zero lateral strain; this is called a K_o consolidation test.

In sands, as in clays, the *linear* compressibility depends considerably on the principal stress ratio, even when low. At low stress ratios up to $1/K_o$ the *volume* compressibility depends only on the mean effective stress and not on the stress ratio (FRAZER, 1957). KIRKPATRICK (1954) has observed that in drained triaxial compression tests, volume expansion does not start until a particular stress ratio which does not depend on porosity, and corresponds to mobilizing a Φ of 25-30°. This compares with the Φ of about 22-27° mobilized in a K_o consolidation test, in which it depends markedly on porosity.

Sand hardly has a Young's modulus, as its elastic limit occurs in a drained test (CHEN, 1948) at a very small axial strain, which may be less than 0·01 per cent. E varies approximately as $p^{1/2}$.

A settlement estimate based on laboratory K_o compression tests will in general be a *lower* limit. Ideally settlement estimates for sand should consider foundation geometry and loading, soil compressibility, existing lateral pressures at depth, grain shape and relative porosity.

Both the ultimate bearing capacity and settlement of a foundation in sand at a given pressure depend on the size of the loaded area.

2. Particle Mechanics in Relation to Compressibility

(a) *General*—In published papers dealing with the compressibility of cohesionless soils, (e.g. WILSON and SUTTON, 1948; JAKOBSON, 1957) it has been assumed that granular particles interact like equal elastic spheres. Theoretical relations based on Hertz's work are available for such spheres in various regular packings. This approach was shown to be not generally valid for sands by the experimental observation (CHAPLIN,

1961) that in several K_0 consolidation tests on sands after a short transition, strain increased linearly with the square root of the applied stress.

The traditional Hertzian theory predicts that the compression strain should increase as the 2/3 power of the stress, which it does (FATT, 1957) for perfect spheres, but it could not be adapted, so the new hypothesis was put forward for some non-spherical grain-to-grain contacts. It links up with recent work on the indentation and hardness of brittle materials (e.g. TABOR, 1956).

(b) *Contact loads in simple packings*—Table 1 gives the number of contacts per particle for regular packings of like spherical particles. The relative porosities are theoretical, corresponding to the concepts of KOLBUSZEWSKI (1948). Fig. 1 shows how the average load at a contact is reduced at high relative porosities by the greater number of contacts per grain and the smaller amount of (solid + void) space per particle.

Table 1

Contacts and Loads in Regular Packings

Packing	*Porosity* %	*Relative Porosity* %	*Contacts per particle*	*Av. load per contact*
Cubic	47·64	0	6	$4\sigma R^2$
Orthorhombic	39·54	37·3	8	$\frac{2}{(\sqrt{3}+\sqrt{3})}\sigma R^2$
Tetragonal-sphenoidal	30·19	80·5	10	$2\sigma R^2$
Rhombohedral	25·95	100	12	$\frac{4\sqrt{2}\sigma R^2}{3}$

(c) *Some possible types of grain-to-grain contact*—The geometrically possible main types of contact between non-re-entrant grains are given in Table 2. Those marked with an asterisk would tend to deform proportionally to the square root of the contact load.

Table 2

Some Types of Grain-to-Grain Contact

Surface of first particle near point of contact	*Surface of second particle near point of contact*
Flat, spherical or ellipsoidal	Spherical or ellipsoidal
Wedge	— —
Wedge (finite)	Flat (wedge and flat parallel)
* Wedge (finite)	Flat (wedge and flat not parallel)
* Wedge	Wedge (axes crossed)
Wedge	Cylindrical
* Pyramidal or conical	Flat or large radius of curvature
Pyramidal or conical	Spherical or ellipsoidal
Cylindrical	Cylindrical

Only the cases indicated have a reasonably constant ratio between the elastic and plastic components of deformation. For others the work-hardening properties should be considered.

(d) *The new hypothesis and its application*—For grain contacts where the geometry remains similar as the load increases, the hypothesis (CHAPLIN, 1961) is that *under a constant principal stress ratio between 1 and $1/K_0$ the volumetric and major principal strains shouls increase after an initial transition in proportion to the square root of the mean applied stress.*

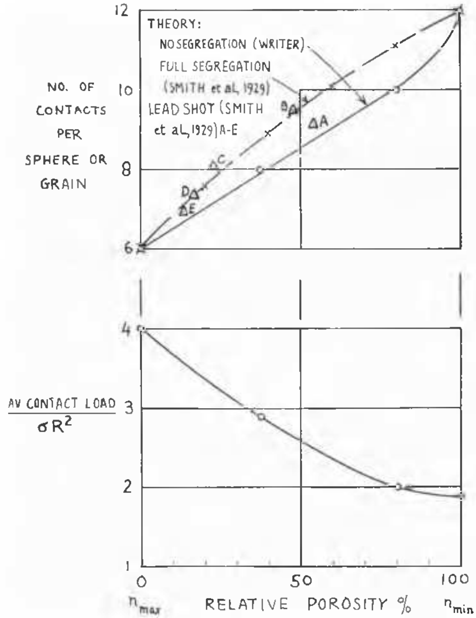

Fig. 1 Influence of Relative Porosity on Grain Contacts and Contact Load for Equal Spherical Particles.

Influence de la porosité sur les contacts intergranulaires et les charges de contacts des particules sphériques de même diamètre.

The hypothesis does not apply when the stress ratio and the mean stress both vary simultaneously.

To test whether a consolidation curve fits the new hypothesis, the available results are plotted in such a way that if the new hypothesis applies over some stress range, that part of the graph will be straight. At low pressures, experimental (e.g. bedding) errors and the change in a K_0 consolidation test from isotropic stress to the full stress ratio $1/K_0$ will produce a transition curve. The locked-up stresses produced in compacted fills by heavy compaction will cause a long transition. In Figs. 2 to 6 the volumetric strain is plotted against a square root scale for the applied pressure*.

The general pattern of results suggests that the compressibility of many cohesionless soils and some fills with cohesion approaches the new hypothesis more closely than one would expect. No results have been found which showed that a sand did in fact obey the Hertzian law of contact consistently.

Unfortunately most published consolidation and K_0 consolidation curves lack detail near the origin. To see what order of strain was needed to reach the strain : (pressure)$^{1/2}$ relation from the isotropic state under which the sample was set up, some tests were performed on the 18-25 fraction of Leighton Buzzard sand (CHAPLIN, 1961). The results are shown in Fig. 6 and suggest that after an initial "elastic" section, a strain of about 0·025 per cent is sufficient to make the deviator stress come on to the linear relation. This is very

* Only those results from GOULD (1953) are included which showed a decreasing compressibility at higher pressures. The last 30 or 40 feet of fill at a dam due to their smaller lateral extent would not produce the full nominal increase of overburden at depth far away from the centre-line.

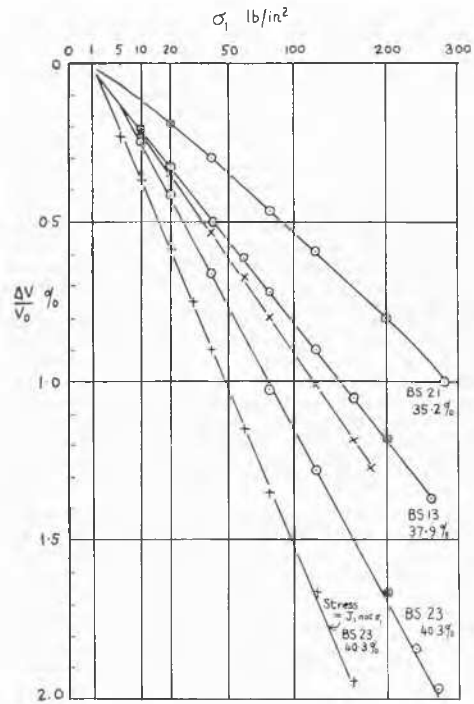

Fig. 2 Compressibility of Brasted Sand (Data : Frazer, 1957).
Compressibilité de sable de Brasted (Frazer, 1957).

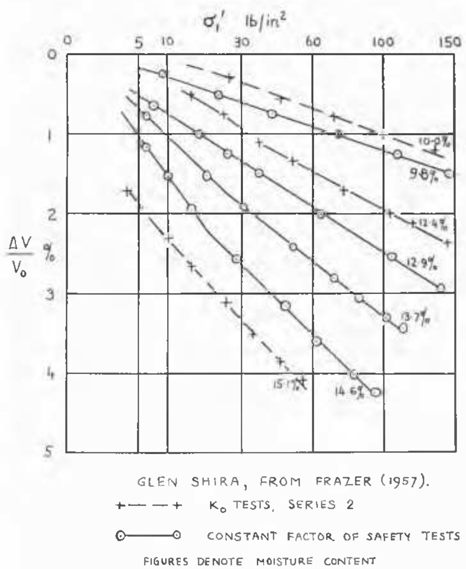

Fig. 4 Compressibility of Glen Shira Fill (Data : Frazer, 1957).
Compressibilité du remblai de Glen Shira (Frazer, 1957).

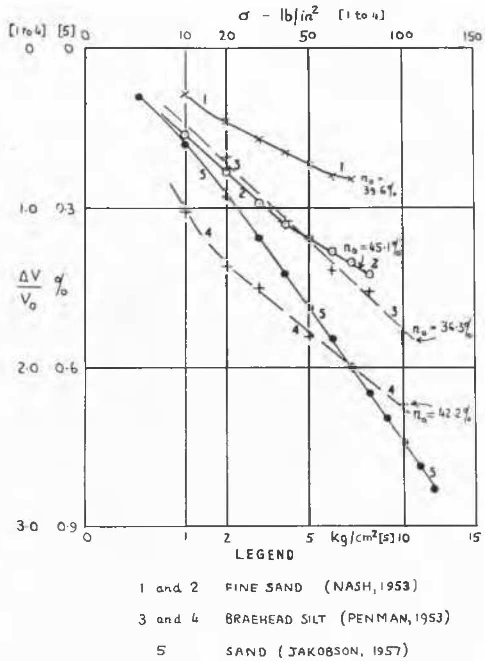

Fig. 3 Compressibilities of Silt and Sand.
Compressibilités des sols limoneux et sableux.

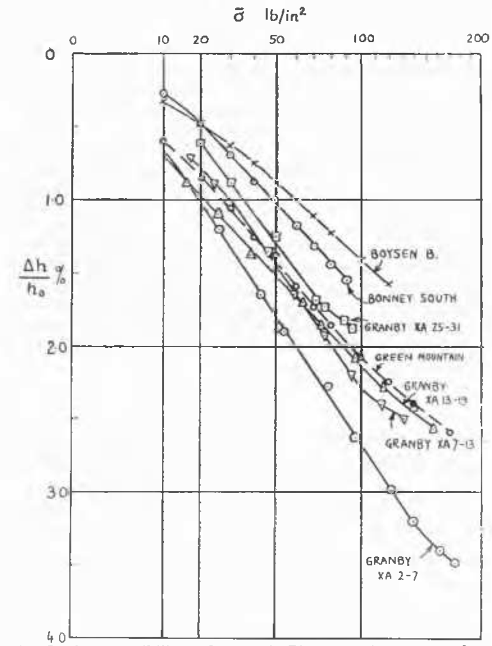

Fig. 5 Compressibility of Rolled Fill Materials (Data from Gould, 1953).
Compressibilites des remblais (Gould, 1953).

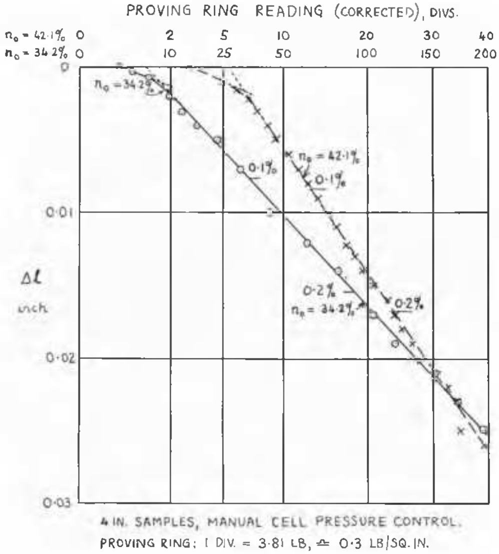

Fig. 6 K_0 Consolidation Tests on Leighton Buzzard 18/25 Sand at Relative Porosities of 0·7 and 0·0 (Chaplin; 1961).

Essais K_0 de consolidation sur sable de Leighton Buzzard de porosités 0,7 et 0,0 (Chaplin, 1960).

similar to the strain at which the tests of CHEN (1948) departed from the initial elasticity.

While these tests were being performed manually, it was observed that the proving ring reading increased remarkably steadily, despite the inevitable fluctuations of cell pressure control which raised it. Slight expansion of the sample if accidentally allowed to persist caused the proving ring reading to drop, as also did a too-rapid raising of the cell pressure. The low compressibility of the Leighton Buzzard sand at 34·2 per cent porosity made it apparent that smooth control of the *rate* of cell pressure rise is needed to make the best use of the Bishop lateral strain indicator on dense sands, as indeed it is for other soils.

(e) *Conclusion*—The remarkable straightness of the K_o consolidation graphs, though regarded as fortuitous at first, suggests what mechanics may be involved in K_o and isotropic consolidation. The physical basis for Bishop's statement at the BRUSSELS CONFERENCE (1958) concerning the fundamental nature of K_o (c.f. Poisson's ratio for metals) may now be examined further.

Part 2

An Experimental Study of the Settlement of Footings in Sand

(a) *General*—The tests on model foundations in 25/52 Ham River sand provided data on bearing capacities for comparison with the theoretical work of MEYERHOF (1948, 1950. 1951 and some later papers). As the settlements of those model foundations at ultimate bearing capacity were often very large, those settlements have been studied in detail (CHAPLIN, 1961). One point should be emphasised : the buried footings were suspended (e.g. from the proving ring) while the sand was deposited around them, so they represent cast in-situ foundations.

In most of the tests on dense sand, at the ultimate bearing capacity there was either a peak resistence or a clear transition to a sensibly linear load-settlement relationship. Tests in loose sand generally gave a linear relationship after a curved transition. Tests in compact sand were much more difficult because some of then gave an almost continuously curving load-settlement curve, sometimes finishing in an asymptote of constant load at a very large penetration.

There was some suggestion that as the initial depth of a footing was increased, an upper limit existed to the settlement at failure, even though the ultimate bearing capacity steadily increased. It was not possible to test very large depth/breadth ratios to verify the suggestion.

To study water table effects, a few tests were carried out in wet sand, some with a lowered static water table and others with an upward flow of water.

The initial slopes of the load-settlement curves for dense sand agreed fairly well with those calculated from the conventional triaxial tests. In compact and loose sand, settlements were greater than the theory.

In a few tests the progressive creep after each increment of load was measured, and was small in general. However we know that in some full-scale loading tests there is much more time lag in the response of the sand structure. K_o consolidation tests in a high pressure triaxial would be needed to elucidate this.

For full details, including a list of revised settlements at failure for the tests described in MEYERHOF'S 1951 thesis, see the thesis by CHAPLIN (1961). Tests in dry 25/52 Ham River sand are referred to unless otherwise stated.

(b) *Summary of results, surface footings*—The ultimate settlement s_u for a given width increased with length until an asymptote was apparently reached, see Fig. 7, at a length/width ratio of about 7·5 or less, which is slightly less then for asymptotic bearing capacity.

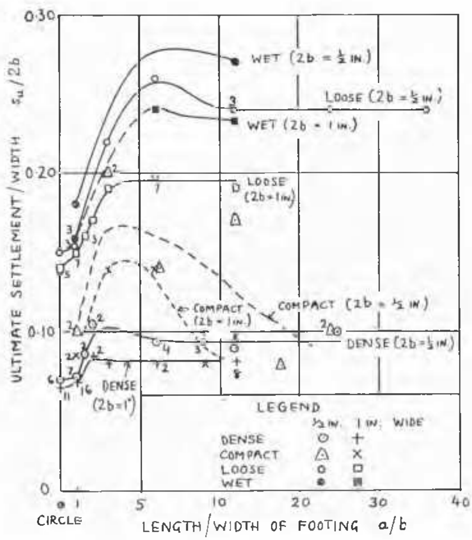

Fig. 7 Ultimate Settlements of Surface Footings, Ham River Sand.

Tassements limites des fondations superficielles, sable de la compagnie Ham River.

On dense sand a circular footing settled only slightly less than a square footing. For long strips, the ultimate settlement was generally less than 10 per cent of the width. The ultimate settlements on compact sand lay between those for the loose and dense states, and the settlements were very sensitive to small changes of relative porosity.

On loose sand the ultimate settlements of circular and square footings were very close.

In loose wet sand, which had a slightly lower porosity than the dry loose sand due to stirring, the ultimate settlements were 35 to 60 per cent greater than dry sand values interpolated for porosity, but never by more than the empirical ratio suggested by Terzaghi and Peck (1948) for *normal* loadings (i.e., not ultimate). There was a peak ultimate settlement (Fig. 8) *before* the full suction and bearing capacity were reached. With an upward hydraulic gradient in saturated sand, the ultimate settlements (Fig. 9) tended to zero as the piping gradient was approached, which was at about $i = 1{\cdot}03$.

(c) *Shallow and deep footings*—In dense sand the variation of maximum ultimate settlement with shape was not large see Figs. 10 and 11. As expected, there were larger ultimate settlements in compact sand and very large values in loose sand. The initial depth of placing to reach an apparent maximum ultimate settlement was reduced by increasing length of footing.

The reduction in ultimate settlement under an upwards hydraulic gradient (Fig. 9) is very marked, and would tend to make the associated loss of bearing capacity of lesser importance if this phenomenon also occurred in full-scale footings. A peak settlement again occurred with a static lowered water table, see Fig. 8, Fig. 12 summarises the variation of settlement with porosity from which the settlements can be calculated for dry sand at the same porosity as the

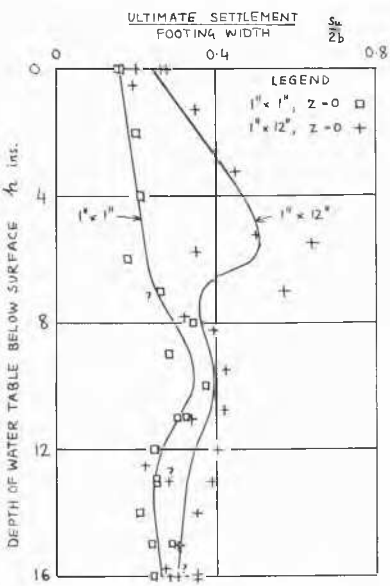

Fig. 8 Influence of Height of Water Table on Settlement of Surface Footings, Ham River Sand.

Influence du niveau de la nappe d'eau sur les tassements des semelles superficielles, sable de la Compagnie Ham River.

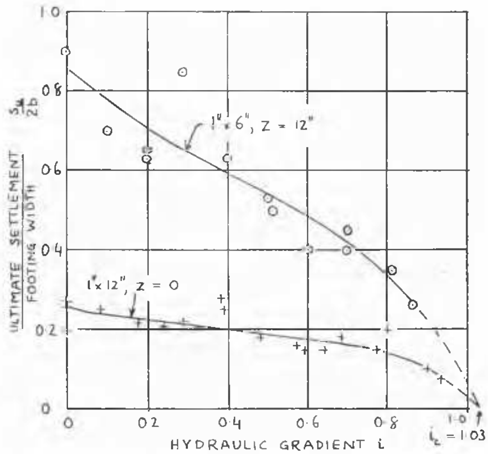

Fig. 9 Influence of Upward Hydraulic Gradient on Strip Footings, Ham River Sand.

Influence d'un gradient hydraulique ascendant sur des fondations superficielles, sable de la Compagnie Ham River.

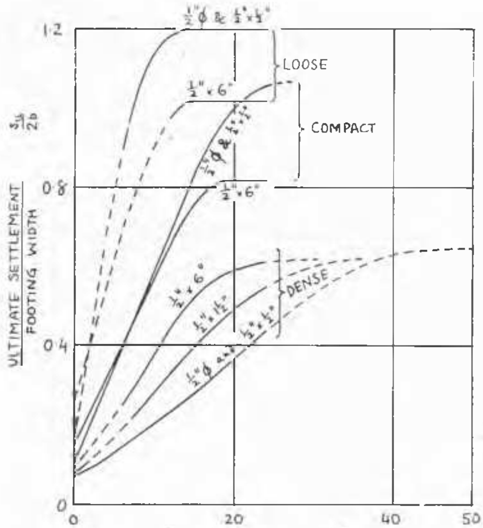

Fig. 10 Influence of Depth of Embedment of Footings in Ham River Sand.

Influence de la profondeur des fondations, sable de la Compagnie Ham River.

wet sand. Then, using Terzaghi and Peck's rule (1948) the observed and expected settlements are compared in Fig. 13.

(d) *Variation of settlement with footing size*—The load-settlement curves tended to be smoother and more regular as the size of surface footing increased, and the peak in the curves

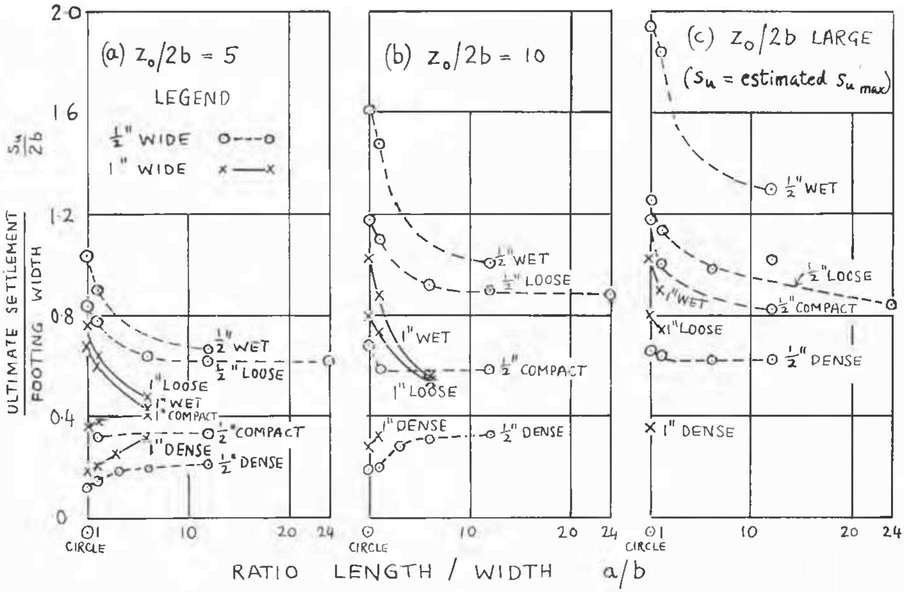

Fig. 11 Influence of Footing Shape on Ultimate Settlement of Shallow and Deep Footings, Ham River Sand.
Influence de la forme des fondations sur les tassements limites des semelles peu ou assez profondes.

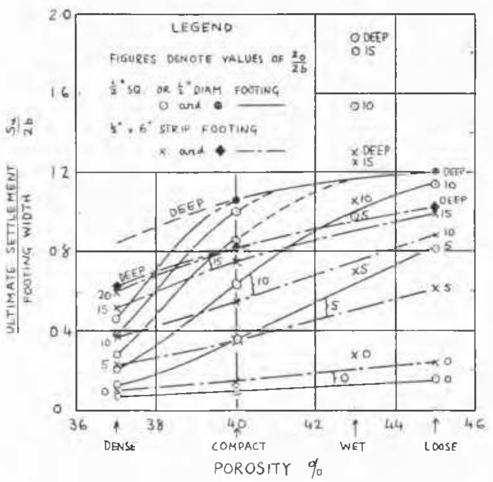

Fig. 12 Variation of Ultimate Settlement with Porosity in Dry and Wet Ham River Sand.
Effet de la porosité sur les tassements limites avec le sable sec ou humide.

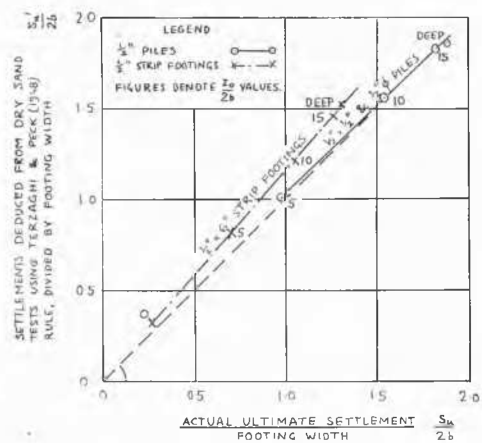

Fig. 13 Influence of Submergence on Ultimate Settlement : Application of Terzaghi and Peck's Rule.
Influence de la submersion sur les tassements limites : application de la règle Terzaghi-Peck.

to become less marked or absent. None of the footings were large enough for the load per unit area at a given settlement to be reduced. Probably the tests on the largest sizes of footing were slightly affected by the tank walls. Fig. 14 illustrates the close relationship of settlement to footing size.

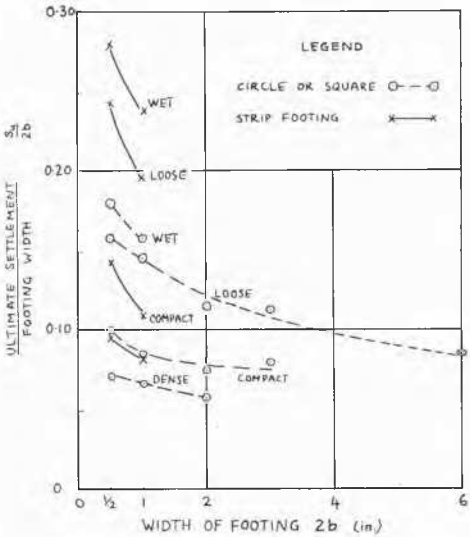

Fig. 14 Influence of Footing Size on Ultimate Settlement of Surface Footings on Ham River Sand.
Influence des dimensions des semelles superficielles sur les tassements limites, sable de la Compagnie Ham River.

(e) *Settlement rates*—Owing to lack of space, the detailed results are not included. A footing in dense sand took *longer* to reach a state of steady creep than a similar footing in a loose sand. The ratio of creep (per 10 : 1 cycle of time) to the immediate settlement within 10 seconds, varied from 10 to 0·1 for dense sand, and 0·1 to 0·01 for loose sand.

(f) *Conventional consolidation tests on sands*—Ham River sand and Thanet sand from Deptford Creek (MEYERHOF, 1953) were tested. Side friction caused a large loss of normal load for a thick sample. The long-continuing "creep" observed under each increment of load was perhaps partly due to the nature of the friction between the sand grains and the brass oedometer.

The Thanet sand pressure-void ratio curves were originally plotted at the time of testing against the square root of the loading, and they gave linear or slightly curving relations between void ratio and (pressure) $^{1/2}$. Unfortunately the theoretical implications of those relations were not then realised. The Ham River sand curves were less nearly straight, due perhaps to the larger grains.

(g) *Surface footing tests on other sands*— Two series of footings were more recently tested : equal width and equal area. The results are shown in Figs. 15 and 16, and are clearly different from the pattern of behaviour of the Han River sand, Fig. 7. These results are a valuable reminder that conclusions from one single sand must not be applied blindly to other sands (and sizes of footing).

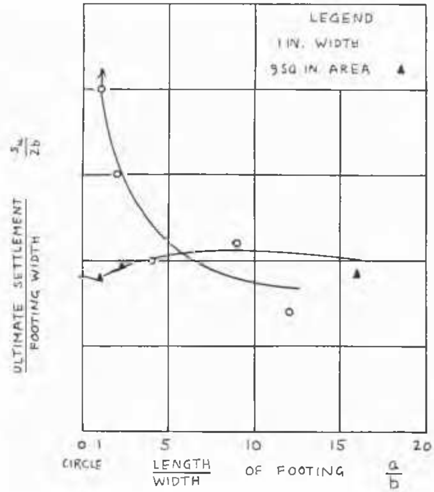

Fig. 15 Ultimate Settlement of Surface Footings, Redhill Sand.
Tassements limites des semelles superficielles, sable fin de Redhill.

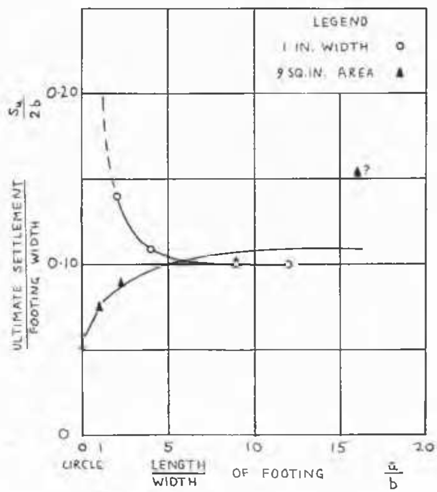

Fig. 16 Ultimate Settlement of Surface Footings on a Well-Graded Sand.
Tassements limites des semelles superficielles, sable avec grand coefficient d'uniformité.

(h) *Conclusions*—The test results cannot yet be interpreted fully, due to the very great number of variables involved. The influence of particle shape and surface roundness on the fundamental stress-strain properties of sand urgently need detailed study.

Acknowledgments

The tests on footings in Ham River sand were performed while the author was in the Soil Mechanics Division of the Building Research Station. The rest were carried out in the Foundations Laboratory of the Department of Civil Engineering, University of Birmingham. The encouragement of Dr. L.F. Cooling, Professor G.G. Meyerhof and Professor J. Kolbuszewski is gratefully remembered.

Appendix : Physical Properties of Sands

The average physical properties of the sands concerned in the Paper are given below. The roundness and sphericity were estimated visually with a microscope and camera lucida (Wadell, 1935; Rittenhouse, 1943), and the limiting porosities by Kolbuszewski's methods (1948 a).

Sand	*Roundness*	*Sphericity*	*Maximum Porosity* %	*Minimum Porosity* %
Ham River 25/52	0·3	0·8	45	29
Leighton Buzzard 18/25	0·4	0·8	42	31
Redhill H.	0·2	0·8	50	37

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