Foundation Analysis of Offshore Pile Supported Structures

Analyse des fondations des constructions sur pieux en mer

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Summary

Where lateral loads on pile-supported structures are significant, the critical factor for determining the size of the piles is frequently the portion of the stress resulting from bending moment. While the analysis of the vertical load capacity of a pile can proceed by conventional methods, the analysis for lateral load is more difficult, requiring the solution of a fourth-order differential equation. Complicating the rational solution of the problem is that static equilibrium must be maintained and compatibility must be achieved between the behavior of the superstructure, the foundation piling and the supporting soil.

Two rational methods are presented for analyzing piles under lateral loads, one a hand solution and the other requiring a digital computer. By iterative procedures, each of the methods achieves compatibility between an inelastic soil and an elastic pile which is elastically restrained by the superstructure. The soil stiffness constants are adjusted for each trial in accordance with predicted force-deformation relations for the soil.

Example problems are solved by each of the methods and the results are compared. Charts and tables are included and computation procedures are shown for the hand solution. The digital computer solution is rigorous and more adaptable but the hand solution is indicated to be satisfactory for many problems.

Sommaire

Quand les charges latérales qui pèsent sur des structures supportées par des pieux sont importantes, le facteur critique pour déterminer les dimensions de ces pieux est fréquemment la composante de la contrainte résultant du moment de flexion. Alors que l'analyse de la capacité de charge verticale d'un pieu peut être effectué par les méthodes ordinaires, l'analyse de la capacité de charge latérale est plus complexe et exige pour sa solution une équation différentielle du quatrième ordre. L'équilibre statique, qui doit être maintenu, et la condition de compatibilité entre le comportement de la structure, les pieux de fondation et le sol qui les supporte, qui doit être satisfaite complique encore la solution rationnelle du problème.

L'auteur propose deux méthodes pour analyser le comportement des pieux soumis à des charges latérales, l'une ne comportant que des calculs pouvant se faire à la main, l'autre exigeant l'emploi d'un calculateur électronique. L'une et l'autre permettent par tâtonnements d'arriver à la compatibilité entre un sol non élastique et une pile élastique soumise à l'action de la superstructure. Les constantes de rigidité du sol sont choisies pour chaque essai en accord avec les relations force-déformation prévues pour le sol.

A titre d'exemple l'auteur a résolu des problèmes par chacune de ces deux méthodes et en a comparé les résultats. Des courbes et des tables sont incluses et les méthodes de calcul sont indiquées pour la solution manuelle. La solution par calculateur est rigoureuse et plus adaptable, mais la solution manuelle est déjà satisfaisante pour beaucoup de problèmes.

Introduction

Offshore structures have been erected in many parts of the world for the production of oil and for many other purposes. Although a wide variety of structural forms and concepts has been employed most of the structures are supported by piles.

The design of offshore structures involves consideration of unusually large ratios of lateral to vertical load, particularly in areas subject to severe storms. While the analysis of the foundation for vertical load capacity follows conventional procedures, the lateral-load analysis poses a more complex problem. Since combined flexural and axial stresses are used to determine required pile sizes and since the flexural stresses are usually the major factor, bending moments in the piles must be reliably predicted. This requires that the interaction between the structure and the foundation elements be rationally analyzed.

The successful application of a rational method of lateral-load analysis depends upon the availability of detailed information concerning soil properties. It is especially important to have accurate soils information very near the ground surface, at depths less than 10 to 20 pile diameters. Offshore soil borings, usually made from a floating vessel, are expensive but are indispensable to the analysis and ultimate safety of any offshore structure.

In order to solve the problem of a laterally loaded pile, it is necessary to predict the lateral soil resistance along the pile as a function of deflection of the pile. This problem has been discussed by TERZAGHI (1955) and MCCLELLAND and FOCHT (1958), and has been the subject of extensive research (MATLOCK and RIPPERGER, 1956 and 1958). A potentially useful concept has been presented by SKEMPTON (1951).

Rational Methods of Analysis—For rational solutions of structure-soil interaction problems it is necessary that conditions of both static equilibrium and compatibility of deformations be achieved simultaneously for all parts of the system. In the case of laterally loaded pile-supported structures, it is usually possible to treat the structure and the piles as linearly elastic components, but in general the mechanical characteristics of the soil are very non-linear. Solutions may be obtained by repeated elastic-theory computations, with soil stiffness values adjusted after each iteration.

An Example of Lateral-Load Analysis—As an example of the type of soil-pile-structure interaction problems which can be solved, a typical offshore structure is shown in Fig. 1.

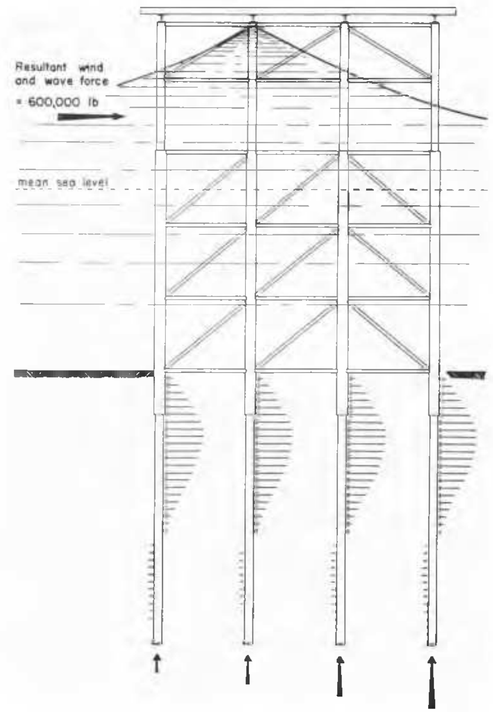


Fig. 1 Lateral forces applied to an offshore structure.
Forces latérales agissant sur une structure en mer.

The specific problem considered is that of solving for the bending moments in the portion of the structural system which lies beneath the soil surface. In erecting such a structure, a prefabricated welded-pipe framework or "jacket" is set in place on the ocean bottom and pipe piles are driven through the vertical members of the jacket.

The primary solution will consist of finding the set of elastic deflections of the pile (including the short jacket-leg extension) which simultaneously will satisfy (1) non-linear resistance-deformation relations which are predicted for the soil, (2) the elastic bending properties of the piles, and (3) the angular stiffness of the upper structure at the pile-to-structure connection.

The elastic elements of the problem are described in Fig. 2. The annular space between the pile and the jacket column is assumed to be grouted so that the two members will bend as a composite section. This is frequently but not always done in actual practice.

The elastic angular restraint provided by the portion of the structure above the soil many be analyzed by determining the moment required to produce a unit value of rotation at the connection. This value, and the imposed lateral load, constitute the boundary conditions for this particular problem. For the example, the elastic angular restraint $M_t/S_t = 6{\cdot}176 \times 10^9$ in.-lb/radian and the lateral load P_t = 150,000 lb per pile.

The force-deformation characteristics of the soil are described by a set of predicted "*p-y*" curves, such as are shown for the example problem in Fig. 3. Such curves may be developed from soil test data by methods previously mentioned. Since solutions for the interaction problem rely on repeated applications of elastic theory, a secant modulus of soil reaction E_s is required which is defined as

$$E_s = \frac{-p}{y} \qquad (1)$$

This modulus is essentially only a computation device which varies with both depth and pile deflection. It is not a unique soil property.

The differential equation for a beam, from conventional theory, is

$$EI\,\frac{d^4y}{dx^4} = p \qquad \ldots\ldots \quad (2)$$

Combining Equations 1 and 2, the general differential equation for the laterally loaded pile is

$$\frac{d^4y}{dx^4} + \frac{E_s}{EI}\,y = 0 \qquad \ldots\ldots \quad (3)$$

Two methods of solution will be described for the example problem, by which the correct set of E_s values are found and deflections and bending moments are computed.

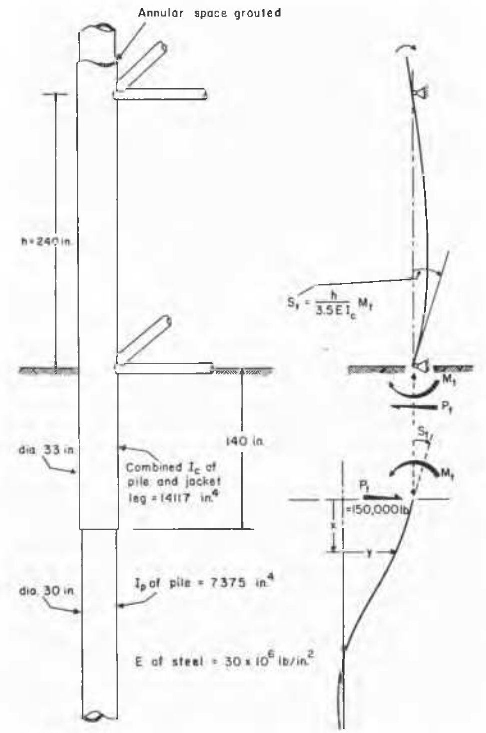


Fig. 2 The superstructure and the pile, considered as elastic elements of the problem.
La superstructure et la pile, considérées comme les éléments élastiques du problème.

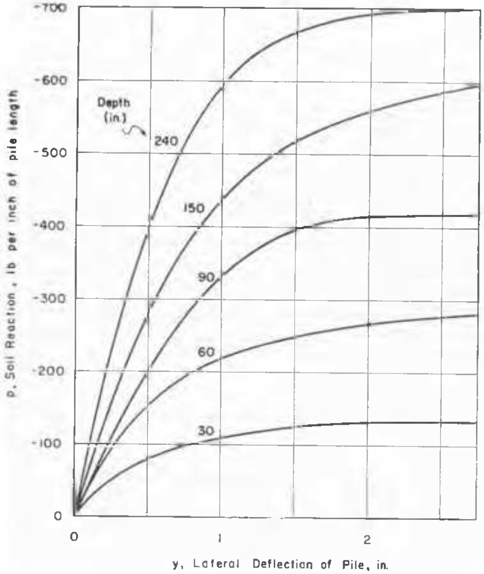


Fig. 3 Typical resistance-deflection curves predicted for the soil at various depths.

Courbe typique de variation de résistance prévue pour le sol à diverses profondeurs.

Hand Solution — Non-dimensional solutions may be developed for any fixed form of variation of E_s with depth (MATLOCK and REESE, 1960). One set of such solutions, for E_s proportional to depth x, or $E_s = kx$, is available (REESE and MATLOCK, 1956). In this set of solutions, the deflection y of the pile, at any depth x, is

$$y = A_y \frac{P_t T^3}{EI} + B_y \frac{M_t T^2}{EI} \qquad (4)$$

where EI is the flexural rigidity of the pile, where T is the relative stiffness factor, defined by

$$T^5 = \frac{EI}{k} \qquad (5)$$

and where P_t and M_t are as shown in Fig. 2. A_y and B_y are non-dimensional coefficients for deflection due to shear and deflection due to moment, respectively. They are functions of only a depth coefficient Z which is equal to x/T. Coefficients for the case of very long piles, and appropriate equations and sign conventions, are given in Table 1.

It is convenient to define an additional set of non-dimensional deflection coefficients by rearranging Equation 4 as follows.

$$y = C_y \frac{P_t T^3}{EI} \qquad \ldots . \ (6)$$

where, at any depth coefficient Z,

$$C_y = A_y + \frac{M_t}{P_t T} B_y$$

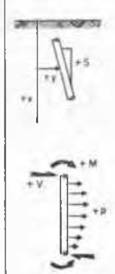
Table 1

Coefficients and Equations for Long Piles, $E_s = kx$

Z	A_y	A_S	A_M	A_V	A_p
0.0	2.435	-1.623	0.000	1.000	0.000
0.1	2.273	-1.618	0.100	0.989	-0.227
0.2	2.112	-1.603	0.198	0.956	-0.422
0.3	1.952	-1.578	0.291	0.906	-0.586
0.4	1.796	-1.545	0.379	0.840	-0.718
0.5	1.644	-1.503	0.459	0.764	-0.822
0.6	1.496	-1.454	0.532	0.677	-0.897
0.7	1.353	-1.397	0.595	0.585	-0.947
0.8	1.216	-1.335	0.649	0.489	-0.973
0.9	1.086	-1.268	0.693	0.392	-0.977
1.0	0.962	-1.197	0.727	0.295	-0.962
1.2	0.738	-1.047	0.767	0.109	-0.885
1.4	0.544	-0.893	0.772	-0.056	-0.761
1.6	0.381	-0.741	0.746	-0.193	-0.609
1.8	0.247	-0.596	0.696	-0.298	-0.445
2.0	0.142	-0.464	0.628	-0.371	-0.283
3.0	-0.075	-0.040	0.225	-0.349	0.226
4.0	-0.050	0.052	0.000	-0.106	0.201
5.0	-0.009	0.025	-0.033	0.013	0.046

Z	B_y	B_S	B_M	B_V	B_p
0.0	1.623	-1.750	1.000	0.000	0.000
0.1	1.453	-1.650	1.000	-0.007	-0.145
0.2	1.293	-1.550	0.999	-0.028	-0.259
0.3	1.143	-1.450	0.994	-0.058	-0.343
0.4	1.003	-1.351	0.987	-0.095	-0.401
0.5	0.873	-1.253	0.976	-0.137	-0.436
0.6	0.752	-1.156	0.960	-0.181	-0.451
0.7	J.642	-1.061	0.939	-0.226	-0.449
0.8	0.540	-0.968	0.914	-0.270	-0.432
0.9	0.448	-0.878	0.885	-0.312	-0.403
1.0	0.364	-0.792	0.852	-0.350	-0.364
1.2	0.223	-0.629	0.775	-0.414	-0.268
1.4	0.112	-0.482	0.688	-0.456	-0.157
1.6	0.029	-0.354	0.594	-0.477	-0.047
1.8	-0.030	-0.245	0.498	-0.476	0.054
2.0	-0.070	-0.155	0.404	-0.456	0.140
3.0	-0.089	0.057	0.059	-0.213	0.268
4.0	-0.028	0.049	-0.042	0.017	0.112
5.0	0.000	0.011	-0.026	0.029	-0.002

Term	Equation	Sign Convention
Depth	$x = ZT$	
Deflection	$y = A_y \frac{P_t T^3}{EI} + B_y \frac{M_t T^2}{EI}$	
Slope	$S = A_s \frac{P_t T^2}{EI} + B_s \frac{M_t T}{EI}$	
Moment	$M = A_M P_t T + B_M M_t$	
Shear	$V = A_V P_t + B_V \frac{M_t}{T}$	
Soil Reaction	$p = A_p \frac{P_t}{T} + B_p \frac{M_t}{T^2}$	



Depending on the angular restraint provided by the structure, values of $M_t/P_t T$ will range from zero for the pinned-end case to -0·93 for the case where the structure prevents any rotation of the pile head. Values of C_y are given by the curves in Fig. 4.

To begin the solution of the example problem it is necessary to assume, temporarily at least, that the form of soil modulus variation $E_s = kx$ will be a satisfactory approximation of the actual final E_s variation. Also, available non-dimensional solutions are limited to a pile of constant bending stiffness. For the example hand solution the pile stiffness will be assumed equal to that of the combined pile and jacket leg. The effect of this assumption will be considered subsequently.

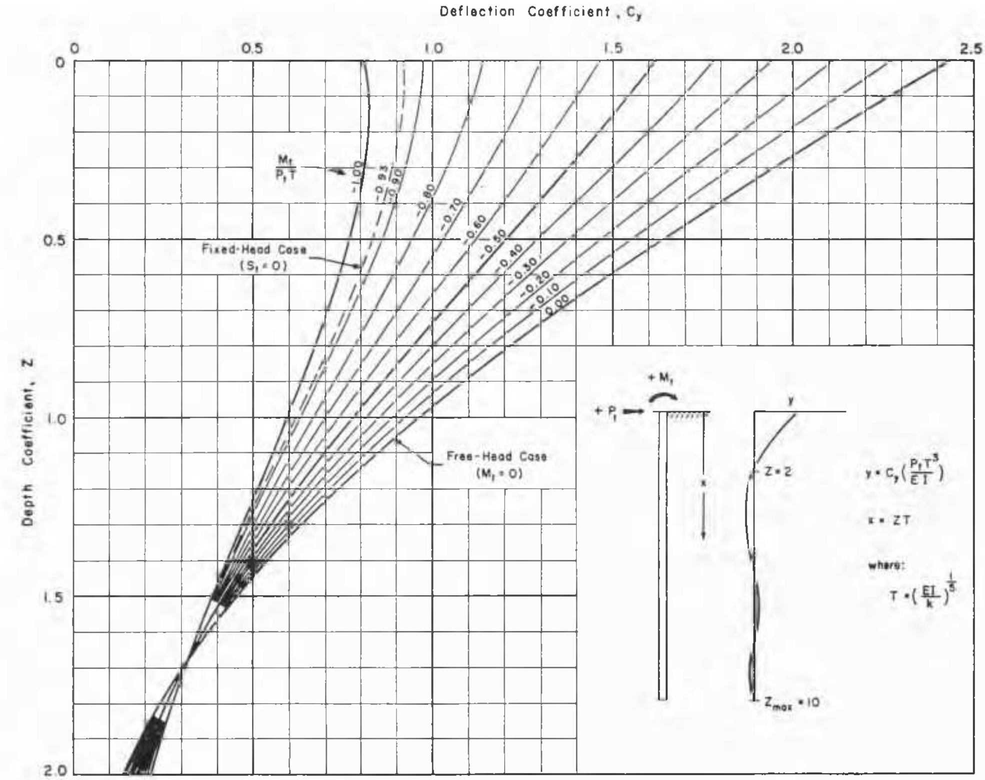


Fig. 4 Non-dimensional coefficients for lateral deflection of a pile, assuming soil modulus proportional to depth, or $E_s = kx$.

Coefficients sans dimensions pour déplacement latéral d'une pile avec comme hypothèse le module du sol proportionnel à la profondeur, ou $E_s = kx$.

The slope at the top of the pile is

$$S_t = A_{s_t} \frac{P_t T^2}{EI_c} + B_{s_t} \frac{M_t T}{EI_c} \quad \ldots\ldots (7)$$

where the subscript $_t$ indicates values at $Z = 0$. The relation between M_t and S_t from Fig. 2 is

$$S_t = \frac{h}{3{\cdot}5\ EI_c} M_t \quad (8)$$

Combining Equations 7 and 8, and rearranging,

$$\frac{M_t}{P_t T} = \frac{A_{s_t} T}{\frac{h}{3{\cdot}5} - B_{s_t} T} = \frac{-\ 1{\cdot}623\ T}{\frac{240}{3{\cdot}5} + 1{\cdot}750\ T} = \frac{T}{42{\cdot}25 + 1{\cdot}078\ T} \quad \ldots\ldots (9)$$

Since the relative stiffness factor T depends on the coefficient of soil modulus variation k and this quantity in turn depends on non-linear soil resistance characteristics, the solution must proceed by a process of repeated trial and adjustment of values of T (or k) until the deflection and resistance patterns of the pile are made to agree as closely as possible with the resistance-deflection (p-y) relations previously estimated for the soil and shown in Fig. 3. Even though the final set of secant soil moduli (E_s — p/y) may not vary in a perfectly linear fashion with depth, proper fitting of $E_s = kx$ will usually produce satisfactory solutions. (See MATLOCK and REESE 1960.)

For the first trial, T will be assumed equal to 200 in. From Equation 9, the corresponding value of M_t/P_tT is -0·776. For this value of M_t/P_tT, values of C_y are interpolated from Fig. 4 and are given in Table 2 at depths corresponding to the positions of the several p-y curves of Fig. 3. Values of deflection y are then computed at each depth. By reference to Fig. 3, values of soil resistance p are obtained, and soil modulus values E_s are computed. This is similar to the method of MCCLELLAND and FOCHT (1958).

Table 2

Sample Computations for First Trial

Depth	Depth Coefficient	Deflection Coefficient	Deflection	Soil Resistance	Soil Modulus
x	Z	C_y	y	p	E_s
	$= \frac{x}{T}$	from Fig. 4	$= C_y \frac{P_t T^3}{EI_c}$	from Fig. 3	$= \frac{-p}{y}$
in.	--	--	in.	lb/in.	lb/in.^{2}
30	0.15	1.13	3.20	-132	41
60	0.30	1.06	3.00	-285	95
90	0.45	0.99	2.81	-420	149
150	0.75	0.82	2.32	-578	249
240	1.20	0.57	1.62	-675	416

Values of soil modulus from the first trial are plotted versus depth as shown in Fig. 5. A straight line through the origin is fitted to the points, with more weight being given to points at depths less than $x = 0{\cdot}5T$ than at greater depths. For this straight line, the coefficient of soil modulus variation resulting from the first trial is computed as

$$k = \frac{E_s}{x} = 1{\cdot}6 \text{ lb/in.}^3 \qquad (10)$$

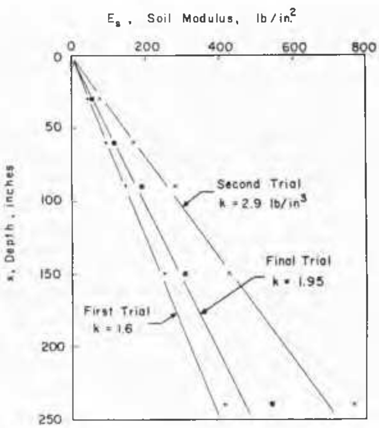


Fig. 5 Trial plots of soil modulus values. The first trial corresponds to computations in Table 2.

Courbes correspondant à diverses valeurs du module du sol. La première correspond au calcul figurant Table 2.

The corresponding value of the relative stiffness factor is

$$T_{(\text{obtained})} = \sqrt[5]{\frac{EI_c}{k}} = 194 \text{ in.} \qquad \ldots. \quad (11)$$

If the value of $T_{(\text{obtained})}$ were equal to the value of $T_{(\text{tried})}$, the trial and error process would have been completed. To facilitate additional estimating and to reach closure with a minimum of trials, a plot of T-values is used, as shown in Fig. 6. Two trials will usually allow interpolation for the final value of T. A final set of computations for E_s values is then made as a check.

Computation of values of bending moment along the pile (and of deflection, slope, shear and soil reaction, if desired) are made by application of the equations and non-dimensional coefficients given in Table 1.

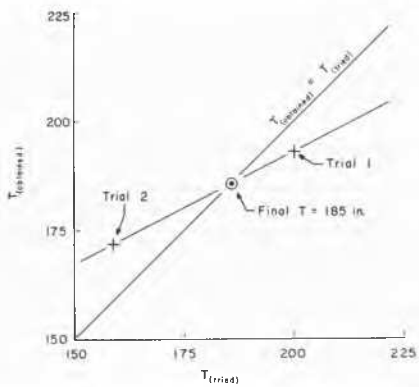


Fig. 6 Interpolation for final value of relative stiffness factor T.

Interpolation pour la valeur finale du facteur de rigidité relative T.

The moment curve from the hand solution of the example problem is given as a solid curve in Fig. 7. Subsequent comparisons with more rigorous computer solutions will demonstrate the effects of the simplifying assumptions used in the above solution. These assumptions are (1) that $E_s = kx$ is a satisfactory approximation of the real variation of E_s values, and (2) that the use of a constant value for the flexural stiffness of the pile does not introduce excessive error.

Computer Solutions—Where unusual variations in soil resistance are encountered and where it is desirable to consider properly any changes in flexural stiffness of the pile, the use of a digital computer is a practical necessity.

The difference-equation method offers a convenient means of solving the problem of the laterally loaded pile (GLESER, 1953) (REESE and GINZBARG, 1960). A program has been developed for the IBM 650 computer, which provides the following principal features.

(1) Step-changes in flexural stiffness may be introduced at any depths.

(2) Length of the pile may be varied as desired.

(3) The soil *p-y* data may be introduced in several ways, including a simple numerical tabulation to define a set of individual *p-y* curves of any form and of any variation with depth.

(4) Various combinations of boundary conditions may be introduced, including lateral load P_t and either (a) moment M_t, (b) slope S_t, or (c) moment divided by slope, M_t/S_t. (The last form is used with the example problem.)

(5) By successive elastic-theory difference-equation computations, based on repeated reference to the soil *p-y* data, the computer will determine, independently at each increment along the pile, the value of soil modulus which represents the proper compatibility and equilibrium conditions for the soil, the pile, and the superstructure.

Comparison of Solutions—The results of three computer solutions are shown by the dashed curves in Fig. 7 and may be compared with the hand solution previously described.

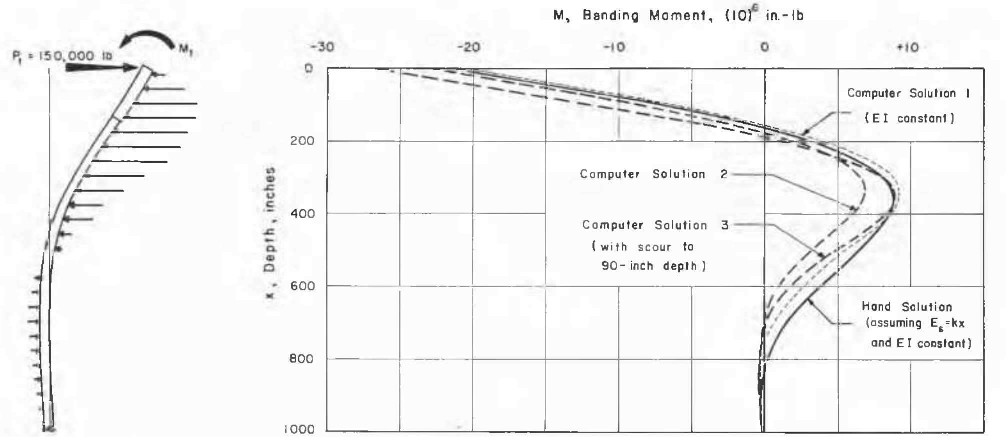


Fig. 7 Comparison of moment curves for Hand Solution (assuming constant flexural stiffness EI_c of pile, and soil modulus $E_s = kx$) with (a) Computer Solution 1 (same as Hand Solution except E_s varies as required), (b) Computer Solution 2 (the correct solution), and (c) Computer Solution 3 (a correct solution with soil resistance eliminated by scour to a depth of 90 inches).

Fig. 7 Comparaison des courbes des moments pour la solution manuelle (supposant une rigidité à la flexion du pieu constante EI_c et le module du sol $E_s = kx$) avec, (a) solution du calculateur (identique à la solution manuelle à l'exception des conditions de variation de E_s), (b) solution 2 au calculateur (solution correcte), et (c) solution 3 au calculateur (solution correcte où la résistance du sol est éliminée par le creusement jusqu'à une profondeur de 90 inches).

To test the validity of the assumption that $E_s = kx$, Computer Solution 1 was performed on exactly the same basis as the hand solution except that the computer was allowed to seek the proper values of soil moduli independently at each depth. The flexural stiffness of the pile was held constant. While good agreement with the previous hand solution is obtained at points of maximum negative and maximum positive moment, some divergence is noted at greater depths.

The actual variation of flexural stiffness at the end of the jacket-leg extension was added to the conditions of the problem for Computer Solution 2. The differences between the results of Computer Solutions 1 and 2 represent the net change due to the step-change in flexural stiffness of the pile.

Scour of the ocean bottom around offshore structures is a matter of considerable importance to the safety of the structures. For Computer Solution 3 the imput conditions of Computer Solution 2 were modified by eliminating completely the soil resistance to a depth of about eight feet. This modification would be difficult to introduce into a hand solution but is easily done with the computer by simple changes in the input data which describe the soil characteristics. An appreciable increase in both maximum negative and maximum positive bending moments may be noted.

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