The Ultimate Bearing Capacity of Wedge-shaped Foundations

La force portante des fondations en coins

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Summary

The previous theory of the bearing capacity of foundations is extended to wedge-shaped bases and cones. The analysis is compared with the results of tests on cones and model piles of different roughnesses and with various shapes of tips in clays and sands.

Sommaire

La théorie antérieure de la force portante des fondations est étendue aux bases en coins et en cônes. L'analyse est comparée avec les résultats d'essais sur cônes et modèles réduits de pieux de rugosités différentes et avec des pointes de formes diverses dans les argiles et les sables.

Introduction

Piles frequently have pointed rather than flat tips, and cone penetrometers are used in the field and laboratory. The bearing capacity theory previously published by the Author (1951, 1953, 1955) can readily be extended to cover such loading conditions. The present paper gives an outline of the methods and the results of some tests on cones and model piles in clays and sands.

Bearing Capacity of Wedges

When a foundation with a wedge-shaped base carries a central load, the zones of plastic flow in the soil at failure are similar to those of an inclined strip foundation for which a solution of the ultimate bearing capacity was derived previously (MEYERHOF, 1953). Thus, for a perfectly smooth wedge with a semi-angle α (Fig. 1) the region above the failure surface on each side of the foundation centre line is assumed to be divided into a plane shear zone ACD, a radial shear zone ADE and a mixed shear zone AEFG (shallow wedge) or a plane shear zone AEF (deep wedge). As the roughness of the wedge increases, the angle ψ at A in zone ACD decreases as under an inclined load on a horizontal base (MEYERHOF, 1953). For a perfectly rough wedge (Fig. 1) a central elastic zone ACD forms a false base on a blunt wedge when the bearing capacity is identical to that of a horizontal base (MEYERHOF, 1955), while for a sharp wedge this elastic zone coalesces with the wedge.

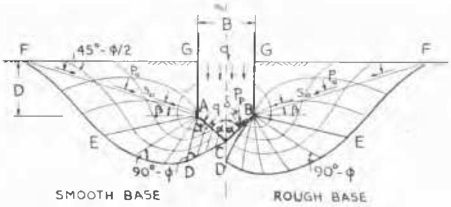
The stresses in the zones of plastic equilibrium can be found as shown for a horizontal foundation (MEYERHOF, 1951, 1953) by replacing the weight of the soil wedge AFG by the equivalent stresses p_o and s_o, normal and tangential, respectively, to the plane AF inclined at an angle β to the horizontal. The bearing capacity can be represented by (TERZAGHI, 1943)

$$q = cN_c + p_oN_q + \frac{\gamma B}{2} N\gamma \qquad (1)$$

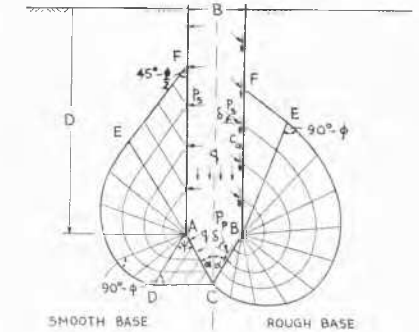
where c = cohesion, γ = unit weight of soil,
B = width of foundation, and N_c, N_q and $N\gamma$ = bearing capacity factors depending on β, angle of internal friction Φ and depth/width ratio D/B of foundation. For shallow foundations ($D/B \leqslant 1$) the stress $p_o = \gamma D$, where D = base depth of wedge, while for deep foundations ($D/B \geqslant 4$ to 10, depending on Φ)

$$p_o = K_b\gamma D \qquad (2)$$

where K_b = earth pressure coefficient on shaft near base, which is about 0·5 for sands and 1·0 for clays (MEYERHOF 1951, 1959).

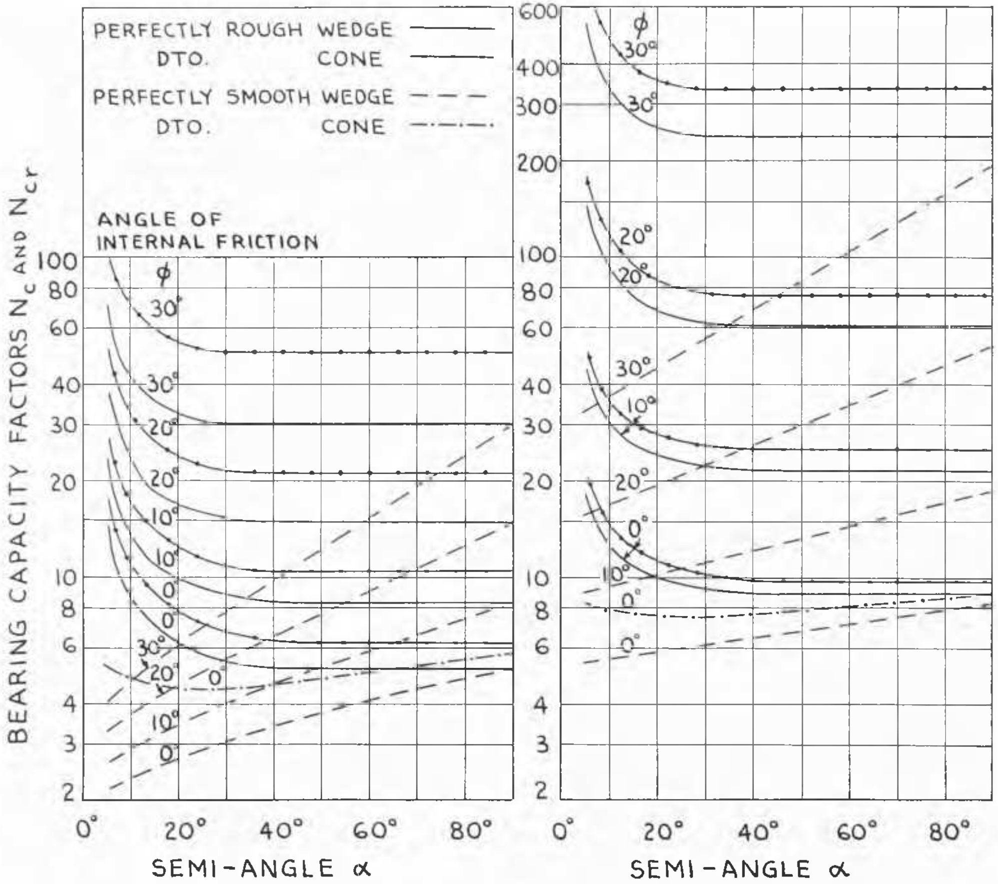


(a) SHALLOW BLUNT WEDGE OR CONE



(b) DEEP SHARP WEDGE OR CONE

Fig. 1 Plastic Zones Near Wedge-Shaped Foundations.
Zones plastiques près des fondations en coins.



(a) SHALLOW DEPTH

(b) GREAT DEPTH

Fig. 2 Bearing Capacity Factors N_c and N_{cr}.
Facteurs de force portante N_c et N_{cr}.

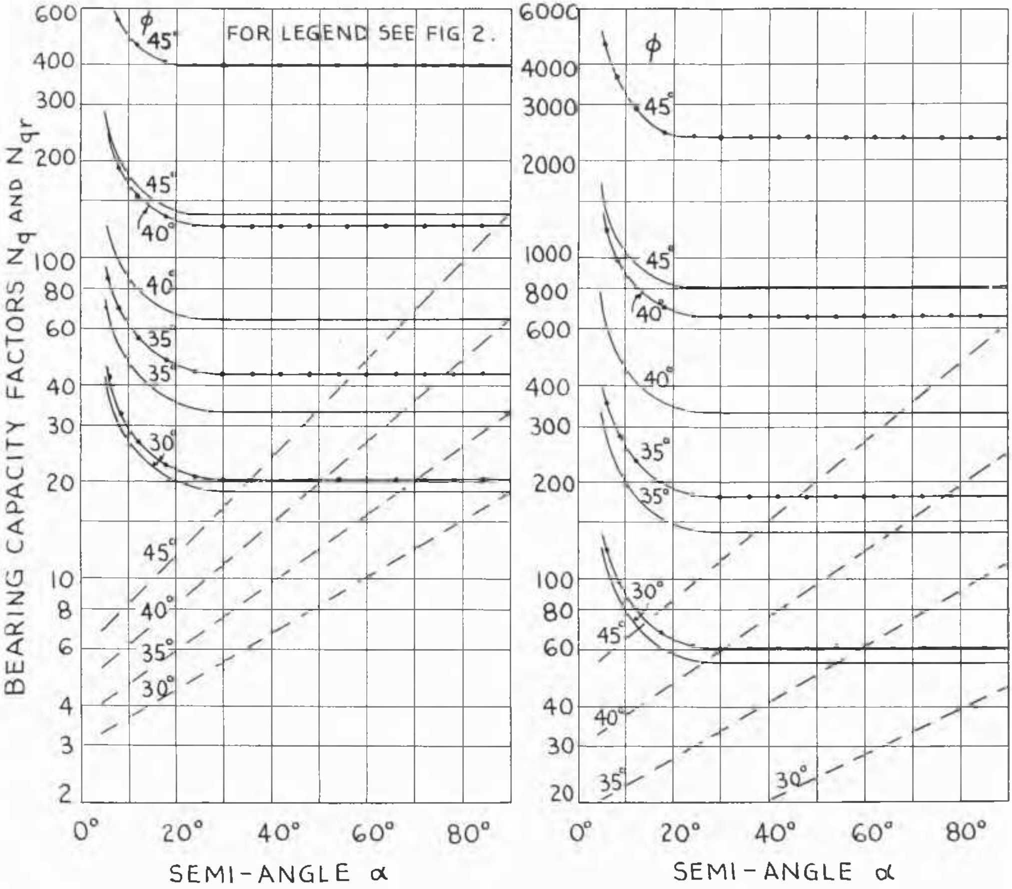
The bearing capacity factors are given in Figs. 2 to 4 for the limiting conditions of perfectly smooth and perfectly rough wedges at shallow and great depths. The factors for smooth wedges decrease rapidly with smaller semi-angles α, but for $\alpha < 30°$, approximately, the factor $N\gamma$ increases again. For rough wedges the factors are sensibly unaffected by the wedge angle (false base) except for about $\alpha < 30°$ when the factors increase rapidly with smaller angles. The factors for smooth wedges are much smaller than those for rough wedges. Bearing capacity factors for intermediate degrees of roughness can be found by linear interpolation between the above limits with good approximation, and such factors decrease with smaller α to a minimum and then increase again.

The above expressions give only the base or wedge resistance to which must be added any skin friction ($c_a + p_s \sin \delta$, see Fig. 1) on the shaft to obtain the total bearing capacity of the foundation.

Bearing Capacity of Cones

At the ultimate bearing capacity of a cone plastic flow of the soil induces circumferential stresses, which raise the bearing capacity above that for a corresponding wedge. The previous solution for the bearing capacity of circular foundations in purely cohesive soils (Meyerhof, 1951) has been extended in the Appendix to derive corresponding bearing capacity factors N_{cr} for perfectly smooth and perfectly rough cones, which are shown in Fig. 2. The factors for rough cones vary with α in a similar way to those of wedges and the shape factor (ratio of cone/wedge bearing capacity) is sensibly independent of α. The factors N_{cr} for smooth cones do not vary appreciably with α; they are less than those for rough cones and of the same order as for rough blunt wedges ($\alpha > 30°$). Bearing capacity factors for cones of intermediate roughnesses can be interpolated linearly.

For cohesive soils with internal friction the bearing capacity



(a) SHALLOW DEPTH

(b) GREAT DEPTH

Fig. 3 Bearing Capacity Factors N_q and N_{qr}.
Facteurs de force portante N_q et N_{qr}.

of cones can at present only be obtained from empirical shape factors in conjunction with eq. (1) to give the cone resistance

$$q_r = cN_{cr} + p_o N_{qr} + \frac{\gamma B}{2} N\gamma_r \tag{3}$$

On the assumption that the shape factors are the same as observed for circular foundations with horizontal bases (MEYERHOF, 1951, 1955), the bearing capacity factors for perfectly rough cones are given in Figs. 2 to 4. While the factors N_{cr} and N_{qr} for cones are greater than those for wedges, as would be expected, the semi-empirical factors N_{γ_r} are smaller, although an approximate theory for circular footings (BEREZANTZEV, 1952) gives the opposite result. This difference appears to be due to the effect of the intermediate principal stress, which raises the actual bearing capacity of wedges relative to that of cones in frictional soils. Thus, for circular surface footings on sands the empirical shape factors are less than unity compared with theoretical values exceeding 2; this would correspond to an increase in Φ under strip foundations of some 14 per cent ($30° < \Phi < 45°$), which is in reasonable agreement with the amount of about 10 per cent found by comparing some plane strain and triaxial compression test results (BISHOP, 1957).

Experiments with Cones and Piles

Some loading tests were made, first at the Building Research Station and more recently at the Nova Scotia Technical College, using either brass (semi-rough) or sanded (rough) cones and model piles of $\frac{1}{2}$ and 1 in. dia. with tips of various angles, which were pushed into soft remoulded clays ($c = 2$ to 3 lb./in^2) and medium sands of various densities ($\Phi = 35°$ to $45°$). The experimental procedure was similar to that described previously (MEYERHOF, 1948, 1951).

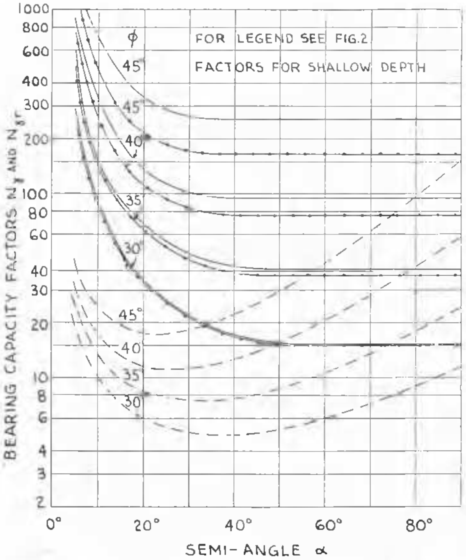


Fig. 4 Bearing Capacity Factors N_γ and $N_{\gamma r}$.
Facteurs de force portante N_γ et $N_{\gamma r}$.

The test results for clays (Fig. 5a) show that the cone resistance and point resistance of brass piles agree well with theoretical estimates based on perfectly rough tips; ancillary pure torsion tests on the cones gave an adhesion of about 0·8 *c*, which is likely to be increased by vertical load. The theory for perfectly smooth cones can be compared with the results of shallow indentation tests using lubricated steel cones in copper and aluminium (Dugdale, 1954); the experimental cone resistance is somewhat less than predicted unless an allowance is made for the raised lip around the indentation (Fig. 5a). Similar indentation tests with 60° cones in cohesive-frictional soils (Evans, 1950) also support the theoretical relationships and indicate a skin friction of about 20 to 80 per cent of the shearing strength (Fig. 5b).

Exploratory model tests with rough piles in compact sand indicated that the point resistance increases little with smaller cone angles (Habib, 1953); this is supported by the present test results, which show that the observed point resistance of rough piles is somewhat greater than estimated (Fig. 6). The measured point resistance of brass piles in sand is in fair agreement with estimates based on a skin friction of about 0·5 Φ, which compares well with the results of direct shearing tests under the same conditions.

Although large-scale tests would be useful as a check, the proposed methods of analysis are probably sufficiently accurate for practical purposes. For steel (semi-rough) piles and penetrometers the point resistance decreases, while for concrete (rough) piles the point resistance increases, as the cone angle of the tip decreases (sharper points). Since the ultimate bearing capacity of piles in cohesionless soils is largely due to point resistance, the shape of the tip may have a considerable influence on the bearing capacity and penetration resistance in such soils and should be taken into account in estimates. In cohesive soils the bearing capacity of piles is mainly due to skin friction and the shape of the tip is less important. The interpretation of laboratory cone tests on clays is, however, difficult due to the unknown amount of adhesion and lip of the material; thus ignoring consolidation and time effects, the resistance of a 60° cone may be only one-half of that of a perfectly rough cone which would be preferable in practice.

Conclusions

The previous bearing capacity theory of foundations with horizontal bases has been extended to wedge-shaped bases and cones. The theory, which indicates that the point resistance of piles with smooth tips decreases and with rough tips increases as the cone angle decreases, is supported by the results of loading tests on cones and model piles in clays and sands.

Acknowledgement

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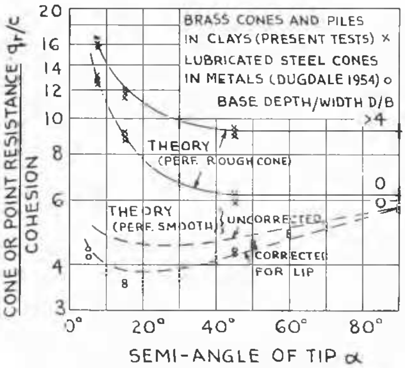
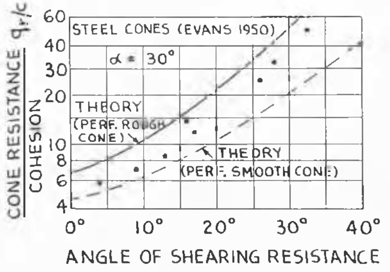


(a) CONE RESISTANCE AND POINT RESISTANCE OF PILES IN CLAYS



(b) CONE RESISTANCE IN COHESIVE SOILS

Fig. 5 Cone and Point Resistance of Piles in Cohesive Soils.
Résistance de cône et de pointe des pieux dans les sols cohérents.

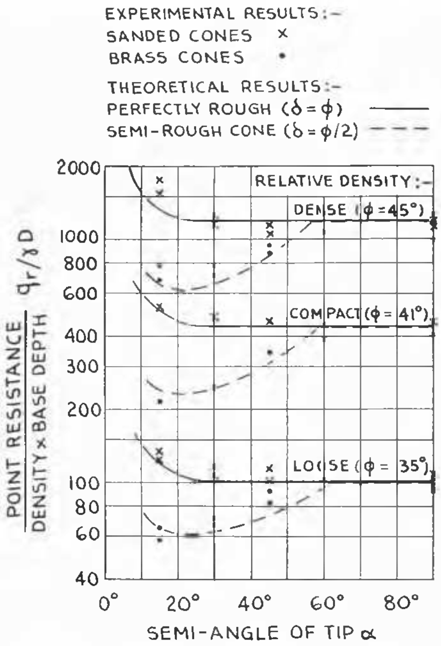


Fig. 6 Point Resistance of Piles in Sands.
Résistance de pointe des pieux dans les sables.

APPENDIX

Bearing Capacity of Cones in Purely Cohesive Material

On the assumption that the plastic zones on radial planes of cones are identical to those on transverse sections of wedges (Fig. 1) and that the circumferential stresses are equal to the minor principal stresses, it was shown (MEYERHOF, 1951) that at failure the vertical contact pressure on the base q_x at any radius $r = x$ from the foundation axis with cylindrical coordinates (r, z) is

$$q_x = q + c\left(\log_e \frac{x'}{x} - \int_{C'}^{F'} \frac{d_z}{r}\right) \quad \ldots\ (4)$$

$$= q + \Delta q \quad \ldots\ (4a)$$

where q = bearing capacity of similar wedge (eq. 1), Δq = contact pressure due to circumferential stresses at failure, x and x' = radial coordinates of C' at beginning and F' at end, respectively, of the slip line (parallel to failure surface $CDEF$) governing the contact pressure q_x.

The bearing capacity factor N_{cr} in eq. (3) is then given by

$$N_{cr} = N_c + \frac{8}{cB^2} \cdot \int_0^{B/2} \Delta q x dx \quad (5)$$

which integration must be carried out numerically with Δq given by the last term of eq. (4). The results of this analysis show that the bearing capacity increases almost linearly with depth (or β), and the factors N_{cr} are given in Fig. 2 for the limits of surface cones (β = 0) and deep cones (β = 90°).

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