3B/23

# Groups of Deep Foundations : A Theoretical and Experimental Investigation

## Groupes de fondations profondes : recherche théorique et expérimentale

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### Summary

The present methods of calculating the bearing capacity of foundations, assume they act as individuals. In fact this is rarely the case, and the problem of interference between strip foundations in sand is discussed.

A theoretical solution to the problem of a pair of infinitely long strips in sand, at any depth and spacing has been developed. Experimental work has been carried out in the laboratory on small scale models, using pairs of rough and smooth strips at various spacings and depths, and the results are compared with the theoretical values.

It is concluded that the theoretical approach will enable the loads at failure on a pair of strip foundations to be estimated to within 15 per cent to 20 per cent.

### Sommaire

On trouvera exposés dans ce rapport, les résultats de recherches expérimentales sur le problème du pouvoir portant des groupes de fondation sur semelles, en surface et en profondeur dans un sable fin.

L'on a mis en avant une théorie qui attribue les phénomènes observés à une interférence entre les surfaces de rupture, et les résultats des calculs sont donnés, et comparés avec les essais sur modèles réduits.

Les auteurs ont conclu qu'il est possible de calculer le pouvoir portant, et de limiter l'erreur à environ 15 pour cent ou 20 pour cent des valeurs expérimentales.

### Introduction

In spite of the fact that foundations seldom occur individually, bearing capacity calculations are usually made on the assumption that the foundation is isolated. When placed close together interference between the stressed zones results. This presents considerable difficulty in design. In the case of clay soils several methods have been suggested for the calculation of the ultimate bearing capacity of such foundations, all of which consist effectively of reducing the allowable shaft area per foundation as the group size and spacing change. Such reduction formulae are arbitrary in origin and take few of the variables present into account.

The problem of foundation groups in sand has received much less attention and only a few experimental results are available. (Press (1933), Kezdi (1957) (1960), Fleming (1958), Stuart Hanna and Naylor (1960)). Some theoretical suggestions for surface footings only on sand have been put forward by Stuart. The authors have been concerned mainly with the problem of groups of piles in sand and have used as a basis for comparison the behaviour of a single pile. Attempts to develop a theoretical solution to this problem (Fleming) were not successful, and in order to limit the number of variables and simplify the theoretical approach, the case of a pair of parallel surface strips has been extended to cover the case of a pair of deep strips.

### Theory

Direct observation of the failure mechanisms involved when a pair of foundations are placed close together show that a modification of the plastic failure zones results. The extent of the free side mechanism increases and a distortion of the base elastic wedge occurs with spacing change resulting in a curtailment of the inside mechanism. The roughness of both the foundation shaft and base controls the failure pattern. The mechanisms assumed for the surface foundation by Stuart are shown in Fig. 1a and 1b for a pair of parallel rough and smooth strips.

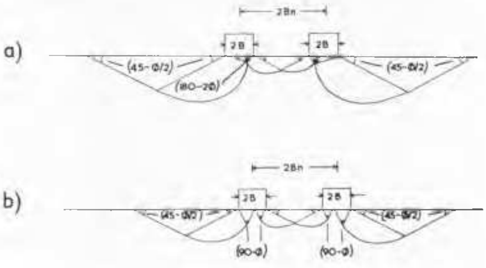

Fig. 1 (a) Failure mechanism — rough foundation (Stuart). Mécanisme de rupture — fondation rugueuse.
(b) Failure mechanism — smooth foundation (Stuart). Mécanisme de rupture — fondation lisse.

The extension of the surface mechanism to the deep case appears on Figs. 2a and 2b. The results of photographs (Figs. 3 and 4) confirm this qualitatively for the case of a rough foundation.

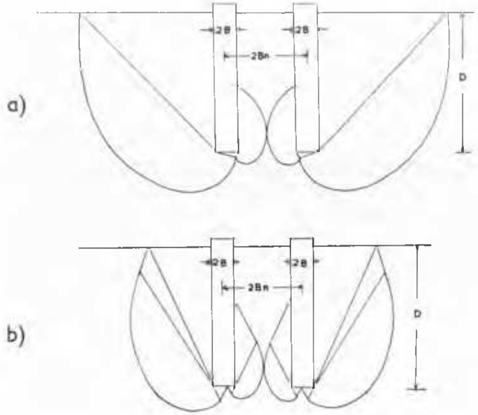

Fig. 2 (a) Failure mechanism — rough foundation.
Mécanisme de rupture — fondation rugueuse.
(b) Failure mechanism — smooth foundation.
Mécanisme du rupture — fondation lisse.

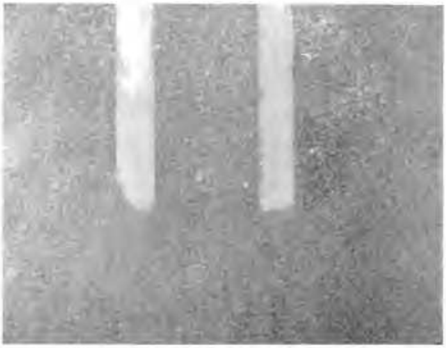

Fig. 3 Pair of rough strips. (Time exposure showing the soil movement and the limiting slip lines) D/2B = 6.
Deux fondations rugueuses. (Pose photographique montrant le mouvement du sable et la ligne de glissement) D/2B = 6.

### Notes on the calculation of the bearing capacity of deep parallel strips

Trial calculations show that the roughness of the foundation shaft and the values assumed for the coefficient of earth pressure between and outside the foundations affect the loads considerably. In the first place the mechanism assumed is symmetrical, whilst asymmetrical patterns have been observed where failure is predominantly to the one side. (See Fig. 4).

The smooth foundation case offers itself to solution readily since the effects of soil arching and hence changes in shaft pressure between the strips do not need to be taken into account.

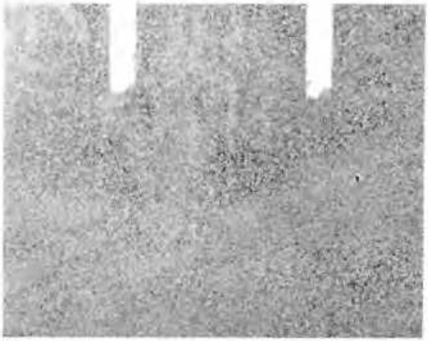

Fig. 4 Pair of rough strips. (Time exposure showing the soil movement and the limiting slip lines) D/2B = 5.
Deux fondations rugueuses. (Pose photographique montrant le mouvement du sable et la ligne de glissement) D/2B = 5.

In both the rough and smooth case the soil reaction on the base of the foundation becomes eccentric, and in the case of the rough based foundation it becomes inclined. This is caused by the changes in the effective depth/width ratio of the free and curtailed sides of the foundation and the distortion of the elastic base wedge.

### Method of Calculation

Only approximate methods are available at present for the calculation of a single foundation load where the self weight and surcharge terms are calculated separately (e.g. Terzaghi and Meyerhof). For deep foundations such methods become very laborious (especially in the case of a pair of foundations) due to the large number of trials required to arrive at a minimum value of the bearing capacity coefficient. The nature of the Authors' work suggested that an approximate solution would suffice to show whether the trend of the proposed theory was correct. Accordingly we assumed that the free and curtailed sides of the foundation behaved as if made up of separate foundations of effective widths of $B_I$ and $B_{II}$ (see Fig. 5). The soil reactions were calculated from the coeffi-

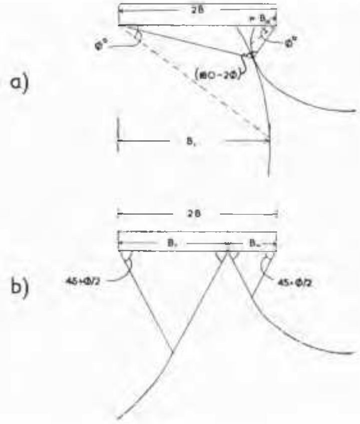

Fig. 5 The approximate mechanism used for the estimation of bearing capacity.
Mécanisme approximatif utilisé pour l'appréciation de la force portante.

cients already available for the deep foundation case (MEYERHOF (1950), (1951). By using these the approximate ultimate loads may be calculated once the geometry of the foundation is fixed. The appropriate values of the coefficients $N_{\gamma q}$ are designated by $N_{I\gamma q}$ and $N_{II\gamma q}$ according to which side of the foundation they refer.

Using the definition of efficiency as given in our previous articles, namely :

$$\text{Efficiency } \xi = \frac{\text{Load on pair of footings}}{\text{twice the load on a single footing}}$$

we arrive at the following general expressions.

$$\xi = \frac{\gamma B_I{}^2 N_{I\gamma q} + \gamma B_{II}{}^2 N_{II\gamma q}}{2\gamma B^2 N_{\gamma q}}$$

where :

$N_{I\gamma q}$ corresponds to a foundation of depth/width ratio $D/2B_I$
$N_{II\gamma q}$ — — $D/2B_{II}$
$N_{\gamma q}$ — — $D/2B$

For foundations on the surface the expression becomes

$$\xi = \frac{B_I{}^2 + B_{II}{}^2}{2B^2}$$

since the effects of surcharge are zero.

Using the expression for the surface case good agreement is shown (Figs. 6 and 7) with the values given by Stuart, who used a method involving the calculation of minimum values. The agreement is especially good for values of $\Phi = 35°$.

Since the mechanisms and their solutions only apply to two neighbouring strips and are approximate, no straightforward extension to the case of more than two strips has been possible at present. An interesting feature of the calculations is that they were completed before any of the experimental work was done, and the trends were not what we expected, but in fact they predicted the observed experimental results in the smooth sided case.

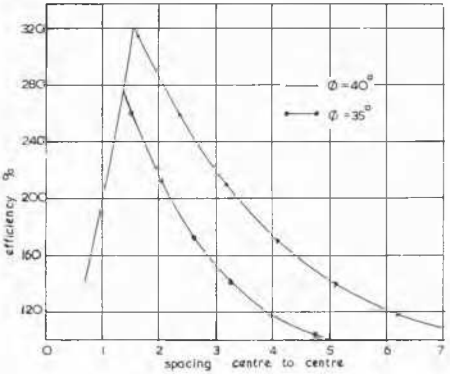

Fig. 6 Spacing — Efficiency graphs for two foundations in sand. ($\delta = \Phi$).
Graphiques distance-efficacité pour deux fondations dans le sable. ($\delta = \Phi$).

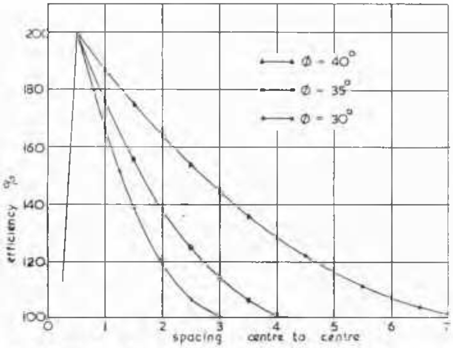

Fig. 7 Spacing — Efficiency graphs for two foundations in sand. ($\delta = 0$).
Graphiques distance-efficacité pour deux fondations dans le sable ($\delta = 0$).

**Experimental Laboratory Investigation**

The model testing apparatus is shown in Fig. 8. It consists of a steel box 40″ × 30″ × 25″ deep filled with a fine dry sand ($\gamma$ max. = 106 lb./ft.$^3$; $\gamma$ min = 86 lb./ft.$^3$) to give an average bulk density of 94·95 lb./ft.$^3$ Load was applied hydraulically through a proving ring via a guide pin to the model foundations. The foundations were of $\frac{1}{2}$″ × 9″ long section of wood and polished silver steel, fixed rigidly at the top end and spaced accurately with a variable sized block 3 $\frac{1}{2}$″ deep which was bolted through the pair of foundations. The foundations were pushed into the sand slowly to the required depth and there they were statically loaded and a 0·001″ dial gauge used to record the settlements. The test procedure was repeated a number of times and the curves obtained could be repeated to within 5 per cent or less in all cases.

Fig. 8 The apparatus.
L'appareil.

During loading surface heave took place in all cases and at close spacings the soil was partly carried down in between the foundations indicating blocking action. The extent of the heave and the blocking depended on the foundation type and the spacing.

A comparison of the efficiencies at a depth of $\frac{1}{4}$" (*D*/2*B* ratio of 0·5) between the two types of footing is shown in Figs. 9 and 10. Encouraging agreement is obtained with the theoretical results when corrections are applied to the experimental results to allow for the depth of penetration and the effective shape of the foundation. (Theory refers to infinitely long strips whereas the experimental values refer to footings having an isolated foundation length/width ratio of 18.) Where the footings are close together and where blocking occurs, the effective *L*/*B* and *D*/*B* ratios will be altered considerably. As depth becomes greater and hence more important, the efficiency drops off at the closer spacing and reaches a maximum value in the region of 2 to 3 widths spacing for the polished steel models. These results are compared with the theoretical values in Figs. 11, 12 and 13 for *D*/2*B* ratios of 8, 16 and 20.

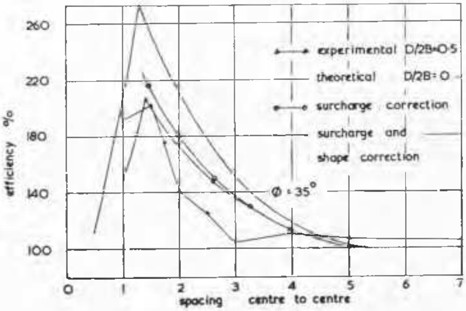

Fig. 9 Spacing-efficiency graphs for two foundations in sand compared with theory. (Rough foundation) D/2B = 0·5.

Graphiques distance-efficacité pour deux fondations dans le sable : comparaison avec la théorie. (Fondation rugueuse) D/2B = 0.5.

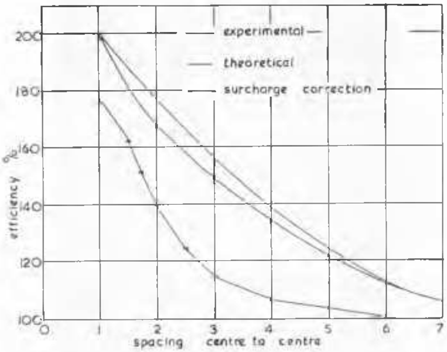

Fig. 10 Spacing-efficiency graphs for two foundations in sand, compared with theory. (Smooth foundation) D/2B = 0·5.

Graphiques distance-efficacité pour deux fondations dans le sable : comparaison avec la théorie. (Fondation lisse) D/2B = 0.5.

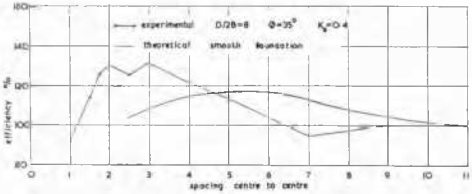

Fig. 11 Spacing-efficiency graphs for two foundations in sand, compared with theory (Smooth foundation D/2B = 8.

Graphiques distance-efficacité pour deux fondations dans le sable : comparaison avec la théorie. (Fondation lisse) D/2B = 8.

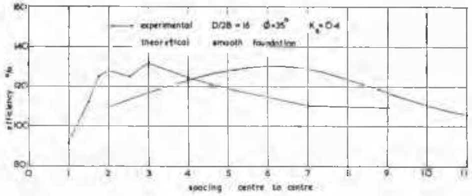

Fig. 12 Spacing-efficiency graphs for two foundations in sand, compared with theory. (Smooth foundation) D/2B = 16.

Graphiques distance-efficacité pour deux fondations dans le sable : comparaison avec la théorie. (Fondation lisse) D/2B = 16.

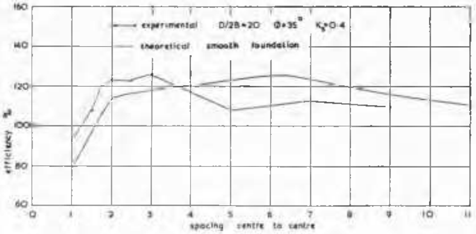

Fig. 13 Spacing-efficiency graphs for two foundations in sand, compared with theory. (Smooth foundations) D/2B = 20.

Graphiques distance-efficacité pour deux fondations dans le sable : comparaison avec la théorie. (Fondation lisse) D/2B = 20.

Similar experimental results were obtained using the rough wooden strips, but the maximum efficiency occured at a wider spacing than when using the polished steel strips. In both cases the critical spacing was found to be a function of the depth.

## General Discussion

The experimental results show efficiency to be a function of spacing, depth and shaft roughness, the critical spacing depending on the depth and roughness which suggests that the effects of soil arching between the strips becomes more important as depth increases. The load $Q_F$ taken by shaft friction per unit length is given by the expression

$$Q_F = \int_{Z=0}^{Z=D} \tan \delta K_s \gamma z dz$$

If $K_s\gamma$ and $\delta$ are all assumed constant between the footings the theoretical results do not agree with the experimental observations.

If however, they are assumed to vary (as in fact they are likely to) then the variation may be calculated and expressed as a function of foundation depth and spacing, by using the experimental values. Assuming that $\gamma$ and $\delta$ remain constant at their original values the changes in $K_s$ are shown in Fig. 14. It can be seen that the values lie between the active and passive values. The higher values occur at close spacings at greatest depth.

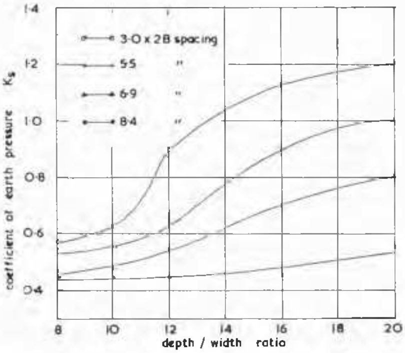

Fig. 14 Variation of the earth pressure coefficient $K_s$ between the foundations, with depth.

Variation du coefficient de pression du sol $K_s$ entre les fondations, en profondeur.

The authors wish to point out that this course has been taken purely from convenience and in fact the graph reflects the changes in soil properties which occur due to compression and arching stresses. It is open to objections but we feel that in the absence of full data about these changes in soil properties* it is most convenient to choose $K_s$ as the variable and to use values of $\gamma$ and $\delta$ which correspond to the soil's undisturbed state.

In general fair agreement has been found between the experimental and theoretical results for the surface case and for the smooth sided foundations. Differences occur which can partly be explained by imperfect smoothness of the polished steel strips ($\delta = 10°$), but mostly by imperfections in the theory which apart from the main approximations do not take into account in a detailed fashion the effects of shape factor in plan and elevation which occur as the foundations are moved in relation to the soil surface and to each other. With all these defects the theory can be used to predict the load at failure with an error in most cases less than 20 per cent and more usually to within 15 per cent.

## Settlements

As regards the settlement pattern of the pairs of foundations difficulties in the interpretation of the results were found due to insufficient readings being taken during the tests. The settlement ratios swing about a set of values which are as follows for factors of safety between 1 ½ and 3.

| Spacing | 1 | 2 | 4 | 9 |
|---|---|---|---|---|
| Settlement Ratio | 2 | 2·5 | 1·5 | 1·0 |

These values follow the same general pattern as previously reported by us in 1960 for groups of model piles where the group size was increased keeping the total number of piles constant.

## Conclusions

An approximate theoretical solution to the problem of the bearing capacity of two neighbouring strip foundations at any depth and spacing has been shown to give reasonable agreement with experiment.

The work has shown the need for further investigation into the changes in soil properties which occur near driven or pushed foundations and this is being carried out.

The settlement behaviour observed in experiments has confirmed the previous results reported by the authors.

## Acknowledgement

The work was carried out in the Soil Mechanics Labs. of the Civil Engineering Department of the Queen's University of Belfast and the authors wish to thank Professor A. H. Naylor for his interest and criticism.

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* Some field observations have been made in the changes occuring round isolated piles by Plantema and Nolet (1957) and Meyerhof (1959) and (1960).