Pile Loading Tests in Stiff Clays

Essais de chargement de pieux dans des argiles consistantes

by Richard J. Woodward, Senior Partner, Woodward-Clyde-Sherard & Associates, Oakland, California
Raymond Lundgren, Partner, Woodward-Clyde-Sherard & Associates, Oakland, California
and
Joseph D. Boitano, Jr., Director, Materials Testing Laboratory, District Public Works Office, U. S. Department of the Navy, San Bruno, California.

Summary

The results of load tests on pipe piles and drilled piers constructed in stiff clay soils are presented and analyzed. Conclusions are drawn concerning the relationship between soil strength and maximum skin friction for pipe piles driven in stiff clay. Comparisons are also made between the performance of pipe piles and drilled piers, the actual supporting capacity of the piles and values computed from soil test data, and the developed and computed values of skin friction in sands.

Sommaire

Nous présentons et analysons dans ce rapport, les résultats d'essais de chargement de pieux creux et pieux forés dans une argile de consistance moyenne. Nous en avons tiré des conclusions sur la relation entre la résistance au cisaillement du sol et le frottement du terrain sur la surface du pieu.

Nous avons fait une comparaison entre la charge portante réelle et les valeurs calculées d'après les résultats des essais de sol et entre le frottement superficiel réel et calculé pour des sables.

Introduction

The development of airport facilities at Lemoore, California, included the design and construction of a series of hangars for which even small differential settlements would have undesirable effects. Thus, although the site was underlain by sand and relatively stiff clays, it was necessary to transfer the column loads by means of piles to relatively incompressible clays located at depths of 45 to 60 feet below the ground surface. Because of the large numbers of piles involved, a comprehensive series of load tests on pipe piles was conducted which enabled values for the maximum skin friction developed in the stiff clay to be determined. Additional tests on a series of drilled piers provided similar information for this type of foundation.

In a previous survey of the adhesion of piles driven in clay soils, Tomlinson [1] concluded that the available data concerning the performance of pipe piles was too meager to permit even approximate recommendations to be offered. It was hoped, therefore, that the information obtained from the tests might help to remedy this deficiency and at the same time enable comparisons to be drawn between the performances of pipe piles and drilled piers, the actual supporting capacity of the piles and values computed from soil test data, and the developed and computed values of skin friction in sands.

Soil Conditions at Site

The soil conditions at the site consist primarily of silty and sandy clays, with occasional sand lenses. However, a layer of medium dense silty sand, 12 to 30 ft. in thickness is located about 8 ft. below the ground surface in some areas, and a layer of dense silty sand, about 10 ft. thick, appears to underly the entire site at depths varying from 42 to 85 ft.

Details of the soil conditions at various locations are shown in Figs. 1, 2 and 3. The unconfined compression strength (q_u) of the clays varied throughout the site from about 1,500 lb. per sq. ft. to 7,400 lb. per sq. ft. and the average penetration resistance of the sand (N) in the standard penetration test from about 18 to 45.

Pile Tests

The pipe piles on which tests were conducted had an internal diameter of 12 in. and an external diameter of 12 3/4 in. The bottom of each pile was sealed by a one-half inch plate, fillet welded to the outside of the pipe. The piles were driven with a single acting Raymond No. 1 hammer (weight of ram = 5 000 lbs., height of fall = 3 ft.); the resistance to penetration during driving is shown in Figs. 1, 2 and 3. When each pile had been driven to the required depth (45 or 60 ft. below the ground surface) the inside was inspected to ensure that the bottom plate was intact, and the pile was filled with concrete.

For compression tests, loads were applied by jacking against a loaded sandbox located centrally above the pile. For tension tests loads were applied by jacking against a heavy cross member welded to the pile, using a timber grillage resting on the ground on each side of the pile as a reaction.

Loads were applied in a series of increments, each increment being maintained until the rate of settlement of the pile had reduced to 0·012 in. per hour or less. Deflections were measured by dial gauges relative to a stiffened beam supported on the ground at a distance of 8 ft. from the center of the pile.

A total of 14 individual pipe piles (numbered I to XIV), together with a 3-pile group were driven, but not all the piles were loaded to failure. The results of all tests carried to failure together with pertinent information concerning the soil conditions and the penetration resistance during driving are shown in Figs. 1, 2 and 3.

Drilled Piers

The drilled piers were constructed by excavating a shaft to the desired depth (30 to 46 ft.) using a bucket rig, casing

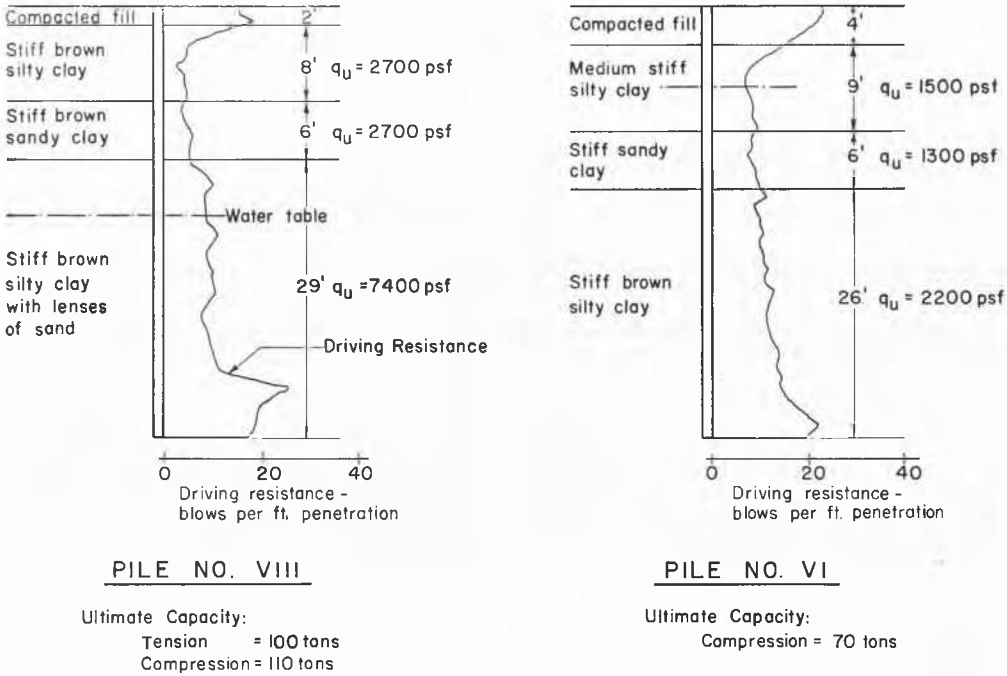

Fig. 1 Test data for piles driven in clay.
Résultats des essais relatifs à des pieux battus dans de l'argile.

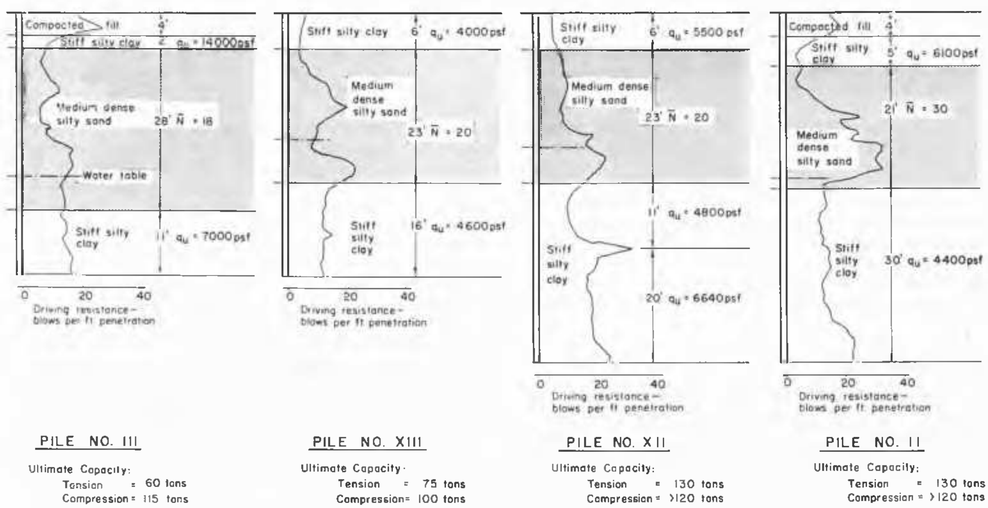

Fig. 2 Test data for piles driven in sand and clay.
Résultats des essais relatifs à des pieux battus dans du sable et de l'argile.

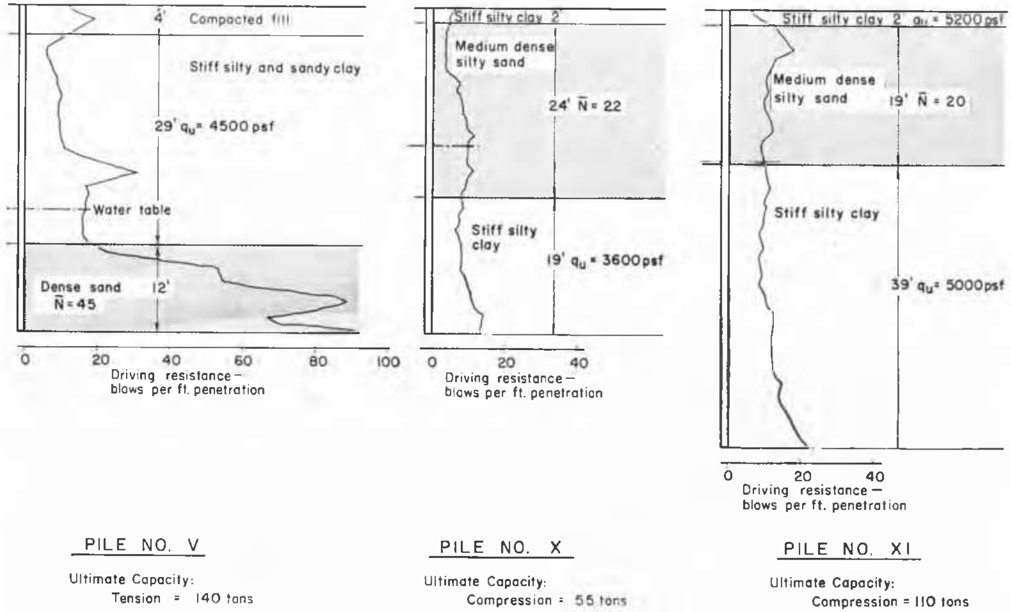

Fig. 3 Test data for piles driven in sand and clay.
Résultats des essais relatifs à des pieux battus dans du sable et de l'argile.

being provided only when the sides of the holes tended to cave in. When casing had to be used it was found that both the drilling time and concreting time increased considerably, as the installing and removal of the casing was difficult.

False bottoms were placed at the bottoms of the majority of the pier excavations prior to placing concrete. These consisted of 15-in. diameter cylinders of No. 10 gauge sheet metal, 8 in. in depth, with an 18-in. diameter plate on the top side. Concrete was placed to fill the shafts immediately on completion of the excavation.

Compression tests on the piers were conducted in the same manner as for the piles. However, in cases where false bottoms were used, only skin friction on the sides of the shaft contributed to the supporting capacity of the pier. Although a total of 10 piers (numbered I to X) were constructed, not all tests were carried to failure. The supporting capacities of pier shafts and the soil conditions at four test locations permitting direct comparison of the performance of piers and piles are presented in Fig. 6.

Analysis of Pile Load Test Results

(a) *Piles Driven Essentially in Stiff Clay*—Test data for two piles, numbers VI and VIII, driven essentially in clay, are presented in Fig. 1. For pile VI, the ultimate capacity of the pile tip was computed using the bearing capacity formula, $q_{ult} = 9s$ lb. per. sq. ft. where s is the shear strength of the soil at the tip of the pile (and is equal to one half the unconfined compression strength, q_u). Subtracting the tip capacity from the total compression capacity of the pile, the support provided by the shaft is determined, and dividing this by the embedded area of the pile leads to an average value of the skin friction at failure of 950 lb. per sq. ft. In computing this value it was assumed, since the deformations of a pile during driving cause a lateral deformation of the soil, that in stiff clay there would be a loss of contact between the pile and soil in the upper four feet of the pile. The same assumption is made throughout the following analyses.

Reference to Fig. 1 shows that the value for the maximum skin friction is less than the average shear strength of the soil surrounding the pile. However, the maximum skin friction can conveniently be expressed by the relationship :

Maximum skin friction = Reduction coefficient (K) $\times$ shear strength of clay,

and the appropriate value of the reduction coefficient K can readily be determined; for this pile, driven in clay having an average shear strength of 1,000 lb. per sq. ft., $K = 0{\cdot}95$.

Applying a similar approach to pile VIII and utilizing the approximate ratio of the reduction coefficients for s = 1,350 and s = 3,700 lb. per sq. ft. indicated by Tomlinson leads to the values of K indicated in Table 1.

(b) *Piles Driven in Similar Deposits of Sand and Clay*—Four piles, for which the tension capacities were determined, were driven in essentially similar conditions of sand and clay (Fig. 2). Since these piles had different lengths, a direct comparison of the shaft support for any two of the piles can be used to establish values for the maximum skin friction provided by the sand and the appropriate reduction coefficients for the stiff clays. For example, piles XII and XIII may be compared as follows :

Let K_1 = reduction coefficient for s = 2,350 lb. per sq. ft.
K_2 = reduction coefficient for s = 3,300 lb. per sq. ft.
S = total maximum skin friction in sand.

Then for pile No. XII, for which the diameter is 1·06 ft. and the measured value of the shaft support is 130 tons :

$$3{\cdot}14 \times 1{\cdot}06 \times 13 \times 1{\cdot}2 \times K_1 + 3{\cdot}14 \times 1{\cdot}06 \times 20 \times 1{\cdot}65 \times K_2 + S = 130 \text{ tons}$$

TABLE 1

Reduction Coefficients for Piles in Clay

Pile No.	Average Shear Strength of Clay Along Shaft (lb./ sq. ft)	Ultimate Capacity of Pile (tons)		Computed Tip Capacity (tons)	Reduction Coefficient
		Tension	Compression		
VI	41 ft. @ 1,000	-	70	5	s = 1,000 lb.sq.ft. K = 0.95
VIII	12 ft. @ 1,350 29 ft. @ 3,700	100	110	-	s = 1,350 lb.sq.ft. K = 0.73 s = 3,700 lb.sq.ft. K = 0.43

TABLE 2

Analysis of Load Test Data for Piles Driven in Similar Deposits of Sand and Clay

Pile No.	Depth Sand (ft.)	Penetration Resistance of sand (blows/ft.)	Unit Skin Friction of Sand (tons sq. ft.)		Depth Clay (ft.)	Reduction Coefficient for Adhesion of Clay
XII	23	20	0.38	$\frac{N}{53}$	33	s = 2,350 lb.sq.ft. K = 0.67 s = 3,300 lb.sq.ft. K = 0.59
XIII	23	20	0.38	$\frac{N}{53}$	18	s = 2,350 lb.sq.ft. K = 0.67
II	21	30	0.64	$\frac{N}{47}$	35	s = 2,200 lb.sq.ft. K = 0.73 s = 3,050 lb.sq.ft. K = 0.51
III	28	18	0.33	$\frac{N}{55}$	13	s = 3,500 lb.sq.ft. K = 0.54

TABLE 3

Reduction Coefficients for Piles Driven in Sand and Clay

Pile No.	Ultimate Capacity (tons)		Computed Tip Capacity (tons)	Computed Shaft Support From Sand (tons)	Reduction Coefficient
	Tension	Compression			
V	140	-		42 to 84	s = 2,250 lb.sq.ft. K = 0.59 to 0.92
X	-	55	7	35	s = 1,800 lb.sq.ft. K = 0.23
XI	-	110	20	25.5	s = 2,500 lb.sq.ft. K = 0.39

Similarly, for pile No. XIII, which has a measured value of shaft support of 75 tons :

$$3{\cdot}14 \times 1{\cdot}06 \times 18 \times 1{\cdot}15 \times K_1 + S = 75 \text{ tons}$$

If, in addition, the relationship between reduction coefficient K and shear strength s is considered to be similar in form to that shown by Tomlinson [1] for concrete piles, then approximately $\frac{K_1}{K_2} = 1{\cdot}13$

Reduction of these equations leads to the following values :

$$S = 29 \text{ tons}; \ K_1 = 0{\cdot}67; \ K_2 = 0{\cdot}59$$

Analysis of the unit skin friction in the sand based on $S = 29$ tons leads to a value of 750 lb. per sq. ft. If this is expressed as a proportion of the standard penetration resistance as suggested by Meyerhof [2] the skin friction in the sand ($N = 20$) is thus equal to $\frac{N}{53}$ tons per sq. ft.

Similar analyses with minor modifications for variations in conditions for the other piles shown in Figs. 2 leads to the values of reduction coefficients and skin friction values for sand shown in Table 2.

It will be noted that the skin friction in the sand, indicated by these analyses, is in excellent agreement with the value of $\frac{N}{50}$ suggested by Meyerhof. In view of this agreement it is considered reasonable to adopt this value for analysis of other piles at this site for which load test data are available.

(c) *Other Piles Loaded to Failure*—The load test data and soil conditions for three other piles loaded to failure are shown in Fig. 3. By computing the shaft support contributed by the sand as discussed above and determining the tip capacity for piles in clay as previously described, values of the skin friction developed in the clays and the corresponding reduction coefficients can readily be determined. Values determined in this way are presented in Table 3.

(d) *Summary of Reduction Coefficients*—The reduction coefficients listed in Tables 1 to 3 are plotted in relation to the shear strength of the clay in Fig. 4. As noted by Tomlinson, the reduction coefficient tends to decrease as the shear strength of the soil increases. However, the average values determined at Lemoore are appreciably higher than the average values for concrete piles and for all types of pile suggested by Tomlinson.

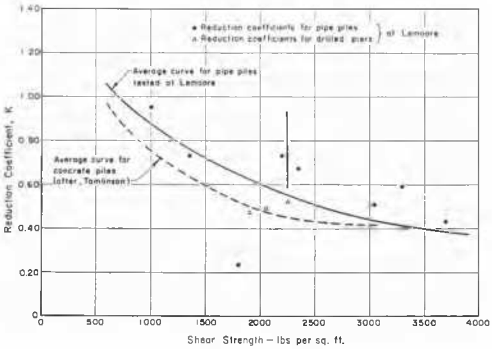

Fig. 4 Relationships between reduction coefficient and shear strength of clay.

Relation entre le rapport du frottement du pieu sur le sol à la résistance au cisaillement de celui-ci.

In order to throw further light on the reduction coefficients applicable to pipe piles, a summary of all available values for this type of pile is presented in Fig. 5. In addition to values determined by the tests at Lemoore, the figure includes values determined from data reported by R. B. PECK [3] for tests conducted in the Cuyaoga River valley and by TOMLINSON [1] for a test at the Isle of Grain. The plotted values for tests in the Cuyahoga River valley were determined in essentially the same manner as that described above.

It will be seen that there is a considerable variation in values, the results for the Cuyahoga Valley tests being consistently lower than those at the other two sites. Possibly this variation is influenced to some extent by variations in the interval between pile driving and testing but no definite conclusion regarding the effect of this factor can be drawn from the information available.

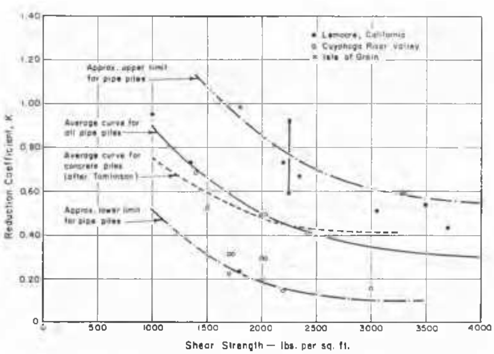

Fig. 5 Relationships between reduction coefficient and shear strength of clay for pipe piles.

Relations entre le coefficient de réduction et la résistance au cisaillement de l'argile pour des « pieux tuyaux ».

An average curve for all tests on pipe piles is shown in Fig. 5 which might be used for estimation of supporting capacities; it will be noted that this curve is quite similar to that suggested by Tomlinson for concrete piles. However, upper and lower limits are also indicated on the figure to emphasize the fact that substantial variations from these average values may be applicable to different sites. Further information of this type for other sites is extremely desirable.

(f) *Prediction of Pile Performance*—The average curves shown in Fig. 5 may be useful in predicting the supporting capacity of friction piles. To illustrate the potential accuracy of the results, computations of the supporting capacity in tension and compression have been made for the 14 piles test loaded at Lemoore. Computations were made as follows :

(1) the tip resistance and skin friction for piles in sand were determined by the methods suggested by MEYERHOF [2] based on the results of standard penetration tests,

(2) the ultimate bearing capacity of pile tips in clay was considered to be equal to 9 times the shear strength of the clay,

(3) the maximum skin friction in clay was assumed to be equal to the shear strength of the soil multiplied by the appropriate reduction coefficient determined from the average curve in Fig. 5.

A comparison of the pile supporting capacities computed in this way with those measured in the tests is shown in Table 4. For the fourteen tests analyzed, the maximum discrepancies in computed values were about $\pm$ 40 per cent — a degree of accuracy compatible with that achieved in other aspects of soil mechanics.

Analysis of Pier Load Test Results

(a) *Direct Comparison of the Supporting Capacities of Piles and Piers*—The test program conducted for this project provided four locations at which it was possible to make a direct comparison of the shaft support developed on driven pipe piles and on drilled and cast-in-place concrete piers. The soil conditions and the lengths and supporting capacities of the piles and piers at these locations are presented in Fig. 6. It should be noted that the shaft support for the piers was determined by compression tests on piers constructed with false bottoms. However, at three of the four locations the shaft support for the piles was measured by tension tests; at the fourth location the supporting capacity of the pile in compression was determined and the computed tip capacity subtracted from this test result to determine the shaft support.

In general, the piles and piers also had different lengths and diameters. However, the shaft capacity for a pile can be reduced to the value corresponding to the pier length, where necessary, by deducting from the measured capacity the skin friction developed in the additional length of piles, using the reduction factor or skin friction in sand computed previously for the pile in question. The shaft support for a pier of smaller diameter was determined by reducing the measured capacity in proportion to the ratio of the desired and actual embedded areas.

By applying appropriate corrections in this way, the test results permit a direct comparison of the shaft support on piles and piers constructed at the same location. The corrected values for shaft support are presented in Table 5. It will be seen that in general the piers at this site develop somewhat less shaft support than the piles, though the difference is not great and is unlikely in any case to exceed about 25 per cent.

TABLE 4

SUMMARY OF PILE DATA

Pile No	Length (ft.)	Soil at Tip	Soils Along Shaft	Computed Supporting Capacity		Actual Supporting Capacity		Ratio: Computed Supporting Capacity / Actual Supporting Capacity	
				Tension	Compression	Tension	Compression	Tension	Compression
I	45	Stiff silty clay q_u = 3600 psf	4 ft. fill 10 ft. clay q_u = 3800 psf 15 ft. sand N = 32 16 ft. clay q_u = 4000 psf	81	88	--	>120	--	<0.73
II	60	Stiff silty clay q_u = 2600 psf	4 ft. fill 5 ft. clay q_u = 6100 psf 21 ft. sand N = 30 30 ft. clay q_u = 4400 psf	100	105	135	>120	0.74	<0.78
III	45	Stiff silty clay q_u = 7000 psf	4 ft. fill 2 ft. clay q_u = 14000 psf 28 ft. sand N = 18 11 ft. clay q_u = 7000 psf	60	74	60	115	1.00	0.65
IV	80	Stiff silty clay q_u = 6600 psf	4 ft. fill 8 ft. clay q_u = 6100 psf 18 ft. sand N = 26 50 ft. clay q_u = 5800 psf	130	143	--	>130	--	<1.10
V	45	Dense sand N = 45	4 ft. fill 27 ft. clay q_u = 4500 psf 14 ft. sand N = 45	85	243	140	>120	0.61	--
VI	45	Stiff silty clay q_u = 2200	4 ft. fill 41 ft. clay q_u = 2000 psf	61.5	65	--	70	--	0.93
VII	45	Dense sand N = 45	2 ft. fill 16 ft. clay q_u = 2800 psf 24 ft. clay q_u = 4400 psf 3 ft. sand N = 40	74	215	--	>120	--	--
VIII	45	Stiff sandy clay q_u = 7400	2 ft. fill 14 ft. clay q_u = 2700 psf 29 ft. clay q_u = 7400 psf	75	90	100	110	0.75	0.82
IX	60	Dense sand N = 48	4 ft. fill 14 ft. clay q_u = 2800 psf 26 ft. clay q_u = 5200 psf 16 ft. sand N = 50	120	282	>150	>120	<0.8	--
X	45	Stiff clay q_u = 3600 psf	2 ft. clay q_u = 5000 psf 24 ft. sand N = 22 19 ft. clay q_u = 3600 psf	67	74	--	55	--	1.34
XI	60	Stiff silty clay q_u = 10000 psf	2 ft. clay q_u = 5200 psf 19 ft. sand N = 20 39 ft. clay q_u = 5000 psf	90	110	--	110	--	1.00
XII	60	Stiff silty clay q_u = 6600 psf	6 ft. clay q_u = 5500 psf 23 ft. sand N = 20 11 ft. clay q_u = 4800 psf	88	101	130	>120	0.68	<0.78
XIII	45	Stiff silty clay q_u = 4600 psf	6 ft. clay q_u = 4000 psf 23 ft. sand N = 20 16 ft. clay q_u = 4600 psf	60.5	69.5	75	100	0.81	0.70
XIV	60	Dense sand N = 45	5 ft. clay q_u = 5800 psf 31 ft. sand N = 30 15 ft. clay q_u = 4000 psf 9 ft. sand N = 40	111	269	--	>120	--	--

(b) *Reduction Coefficients for Piers in Clay*—Three of the test piers provided with false bottoms were constructed entirely in stiff clays. Thus the average skin friction at failure could be directly determined and compared with the average shear strength of the soil. The pertinent data for these piers are presented in Table 6. In each case the maximum skin friction is substantially less than the shear strength, corresponding to reduction coefficients ranging from 0·49 to 0·52 for soil shear strengths of about 2,000 lb. per sq. ft. The computed reduction coefficients are plotted in Fig. 4. It will be seen that the reduction coefficients for the piers are slightly less than those determined for piles driven at this site.

(c) *Piers in Sand and Clay*—Four of the test piers constructed with false bottoms were embedded in sand and clay. By deducting the shaft support mobilized in the clay from the measured capacity, the shaft support provided by the sand could be evaluated. Details of the soil conditions, supporting capacities, and pertinent calculations are presented in Table 7. The shaft support provided by the clay was computed using a reduction factor of 0·5 (as determined above) when the shear strength of the clay was of the order of 2,000 lb. per sq. ft.; the appropriate reduction coefficient was considered to be in the range between 0·5 and a value slightly less than that determined for piles when the shear strength was considerably greater than 2,000 lb. per sq. ft.

It will be seen that the unit skin friction of the sand acting on the piers ranges from $\frac{N}{40}$ to $\frac{N}{110}$ lb. per sq. ft. As might be expected, these values are somewhat less than the skin friction values determined for piles.

Conclusions

Based on the results of load tests on pipe piles, tapered piles and drilled piers conducted at Lemoore, California, the following conclusions may be drawn with regard to the supporting capacities of these types of foundations at this site :

(1) The maximum skin friction developed for pipe piles driven in stiff clay is substantially less than the shear strength of the clay; reduction coefficients expressing the maximum skin friction as a proportion of the shear strength determined

TABLE 5

Direct Comparison of Shaft Support
For Pipe Piles and Drilled Piers
(where piles and piers tested at same location)

Length (ft.)	Diameter (ft.)	Shaft Support for Pile (tons)	Shaft Support for Pier (tons)	Ratio: $\frac{\text{Shaft Support for Pier}}{\text{Shaft Support for Pile}}$
35	1.06	80 to 110	69	0.63 to 0.86
40	1.06	70	69	0.98
45	1.06	70 to 102	72.5	0.71 to 1.03
35	1.06	41	55	1.34

TABLE 6

Analysis of Load Test Data for Piers in Clay

Pier No.	Embedded Length (ft.)	Ultimate Shaft Capacity (tons)	Average Skin Friction (tons sq.ft.)	Average Shear Strength of Clay (tons sq.ft.)	Reduction Coefficient
III	45	105	0.59	1.12	0.52
IV	46	110	0.47	0.95	0.49
VI	36	92	0.50	1.02	0.49

TABLE 7

Analysis of Load Test Data for Piers in Sand and Clay

Pier No.	Soils Along Shaft	Ultimate Shaft Capacity (tons)	Estimated Shaft Support from Clay (tons)	Shaft Support from Sand (tons)	Unit Skin Friction from Sand (tons sq.ft.)
I	4 ft. fill 10 ft. silty clay, q_u = 3,800 psf 15 ft. dense sand, N = 32 6 ft silty clay, q_u = 4,000 psf	120	62.4 to 81.4	38.6 to 57.6	$\frac{N}{40}$ to $\frac{N}{64}$
VII	2 ft. fill 24 ft. sand, N = 22 9 ft. silty clay q_u = 3,600 psf	90	25	67	$\frac{N}{42}$
IX	6 ft. silty clay, q_u = 5,500 psf 23 ft. silty sand, N = 20 11 ft. silty clay, q_u = 4,800 psf	105	55	50	$\frac{N}{48}$
--	5 ft. silty clay, q_u = 5,800 psf 27 ft. sand, N = 30	50	13.5	36.5	$\frac{N}{110}$

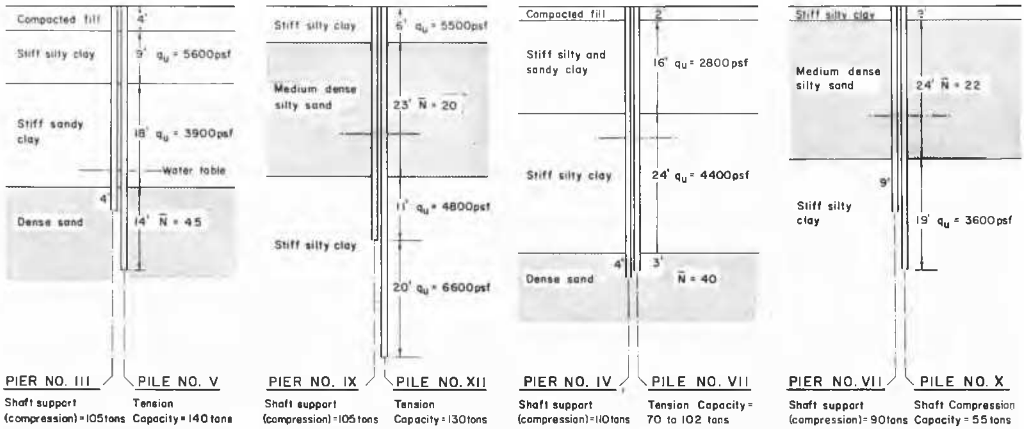

Fig. 6 Comparison of supporting capacities of piles and drilled piers.
Comparaison des capacités portantes des pieux battus et forés.

by the tests at Lemoore, presented in Fig. 4, are somewhat higher than values previously reported. Average values of reduction coefficients applicable for pipe piles, based on all available test data, are shown in Fig. 5.

(2) Analysis of load test data for four pipe piles shows that the unit skin friction of sand, at failure, varied from $\frac{N}{47}$ to $\frac{N}{55}$ tons per sq. ft. where N is the penetration resistance of the sand in blows per ft. measured in the standard penetration test.

(3) For fourteen test piles driven at this site, the use of the curve of average reduction coefficients (Fig. 5) and of the methods suggested by Meyerhof for determining the supporting capacity of piles driven in sands leads to computed values of supporting capacity which do not differ by more than 40 per cent from the values measured by load tests.

(4) The supporting capacity of drilled piers at this site is somewhat less than that of pipe piles of the same length and diameter; however, comparison of test results at four locations did not show a difference of more than 25 per cent.

(5) Reduction coefficients for drilled piers constructed in stiff clay were somewhat less than those applicable to pipe piles.

(6) Analysis of load test data for four piers shows that the unit skin friction developed by sand at failure varied from $\frac{N}{40}$ to $\frac{N}{110}$ tons per square ft.

Acknowledgment

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