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A Method of Designing Pavements for Road and Airports

Contribution au progrès des méthodes de calcul des chaussées de routes et de pistes d'envol

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Summary

The authors have attempted to improve methods of designing pavements for roads and airport runways so that they can be readily analysed, at the same time keeping them in line with empirical methods, such as the CBR.

The problem was therefore studied of the basis of the Theory of Elasticity as applied to two-layer systems but assuming hypotheses for simplicity. The authors endeavoured to establish relationships between the modulus of strength and the CBR, in order to be able to apply the same design principles to both rigid and flexible pavements.

A method was thereby outlined for an approximate practical approach to pavement design problems. The basis of this simplified method was given in a paper presented to the Tenth International Road Congress [1] in Istanbul, and the results of an experimental research on the relationships between the CBR and the modulus of strength were presented at the Fourth International Congress of Soil Mechanics [2].

The authors review some improvements that they have made on Burmister's suggestions for applying the Theory of Elasticity to a two-layer system [3]. They extend their method to multilayer pavements designed on the CBR basis [5], giving the results that they have obtained from scale model and experimental road tests.

Sommaire

Des tentatives ont été entreprises au Laboratório Nacional de Engenharia Civil dans le but de perfectionner les méthodes de dimensionnement des revêtements de routes et d'aérodromes, en les rendant plus rationnelles, sans toutefois abandonner l'expérience étendue obtenue dans tous les pays par l'application des méthodes empiriques notamment la méthode CBR.

Dans ce but, on a tout d'abord procédé à une analyse du problème en partant de la Théorie de l'Elasticité appliquée aux systèmes à deux couches mais avec des hypothèses simplificatrices du type de celles admises dans la Résistance des Matériaux. On a cherché aussi à établir un rapport entre le module de résistance et le CBR de façon à rendre possible le calcul, sur les mêmes bases, de tous les revêtements, aussi bien rigides que flexibles.

On a obtenu ainsi, dans ses grandes lignes, une méthode assez pratique pour rendre possible une première analyse, bien qu'approchée, des problèmes du dimensionnement des revêtements. Les premières bases de cette méthode simplifiée ont été présentées au X^e Congrès International de la Route [1], à Istanbul et les résultats de l'étude expérimentale du rapport entre le module de résistance et le CBR ont été exposés au IV^e Congrès International de la Mécanique des Sols [2] à Londres.

Dans la présente communication, on résume les principes de la méthode, on introduit quelques perfectionnements fondés sur les travaux de Burmister sur l'application de la Théorie de l'Elasticité à un système à deux couches [3], on établit son application à la détermination de la déformabilité de revêtements dimensionnés par la méthode CBR [5], enfin on présente les premiers résultats d'essais sur modèle et d'essais sur des routes expérimentales.

1. Foundations of the simplified analytical method [5]

(a) *Modulus of strength of a material :*

The modulus of strength (R), a characteristic parameter of a material is obtained from a load-bearing test, and is defined as the ratio

$$R = \frac{\sigma}{\delta/d} = \frac{\sigma d}{\delta} \tag{1}$$

(σ) being the mean pressure exerted by a plate with a diameter (d) on the material and (δ) the resulting settlement.

The ratio $\varepsilon = \delta/d$ expressed in percentage is defined as *unit settlement.*

For elastic homogeneous isotropic materials, if the load test is carried out with a *flexible plate*, i.e. if a cushion filled with fluid is interposed between the loaded plate and the material, the contact pressure will be practically uniform and equal to the mean pressure, but the settlement of the material will be variable, decreasing from a maximum at the center of the plate to a minimum at its periphery.

The maximum settlement (δ_0) at the center of the plate calculated according to Boussinesq's theory is given by the expression

$$\delta_0 = \frac{\sigma d(1 - \mu^2)}{E}, \tag{2}$$

E and μ being respectively, the modulus of elasticity and the Poisson's ratio of the material.

Substituting this value in eq. (1), the following expression is obtained for the modulus of strength *under a flexible plate* or better *under a uniform pressure*, which shall be represented by $\overline{R}$,

$$\overline{R} = \frac{E}{1 - \mu^2} \tag{3}$$

This value $\overline{R}$ is a constant, whatever the diameter of the load plate.

Still for elastic homogeneous isotropic materials but performing the test on a *rigid plate*, the contact pressure is variable, ranging from a minimum value at the centre to a maximum at the periphery, but the settlement of the material remains constant at the loaded surface.

The settlement (δ) thus obtained for a given pressure is less than the maximum settlement (δ_0) of a flexible plate, hence the corresponding modulus of strength *under a uniform settlement* will exceed the value obtained under a uniform pressure.

It so happens, however, that while ($\bar{R}$) is, at least on theory, independent of (d), R depends on the diameter, increasing when it diminishes.

(b) *Apparent modulus of strength of a single-layer pavement :*

Let us consider an elastic system made up of a single-layer pavement with a thickness (h) and a modulus of strength (R_p) lying on a semi-indefinite foundation with a modulus of strength (R_f). In a first approximation it is assumed that a load (Q) uniformly distributed on a circular surface of diameter (d) is uniformly transmitted to the foundation in the form of a cone with an angle *(angle of distribution)* (α). Consequently the diameter (d_f) of the loaded area in the foundation will be

$$d_f = d + kh \tag{4}$$

k being the *coefficient of distribution* which is related with the angle of distribution by the expression

$$k = 2 \tan \alpha \tag{5}$$

For simplicity it is assumed that this coefficient of distribution depends exclusively on the ratio R_p/R_f of the modulus of strength of the materials of the pavement to those of the foundation and is independent of the thickness of the pavement (Fig. 1).

The values of k, as obtained by replacing the curves of Burmister's diagram (3) by straight lines, are presented in Table 1 and in Fig. 2.

Within the hypotheses admitted, the pressure σ_f at the foundation, assumed uniform, will be given by the expression

$$\sigma_f = \sigma\left(\frac{d}{d_f}\right)^2 \tag{6}$$

In order to simplify the problem it is further assumed that the settlement (δ) at the pavement surface is exclusively due to the settlement of the foundation, i.e. in the total settlement the part due to the thickness decrease of the pavement is neglected.

According to this simplified scheme, the pavement behaves as if the foundation were subjected to a uniform pressure (σ_f) due to a load plate with a diameter (d_f). In these conditions the settlement will be given by

$$\delta = \frac{\sigma_f d_f}{R_f} \tag{7}$$

The apparent modulus of strength (R) of a pavement, i.e. the modulus of strength which a homogeneous semi-indefinite material should possess to undergo the same settlement (δ) as the actual pavement under the same loading conditions, will be given by eq. (1). The values of (δ) supplied by (1) and (7) being equal, the following expression is obtained for R, taking eq. (6) into account

$$R = \frac{d_f}{d} R_f \tag{8}$$

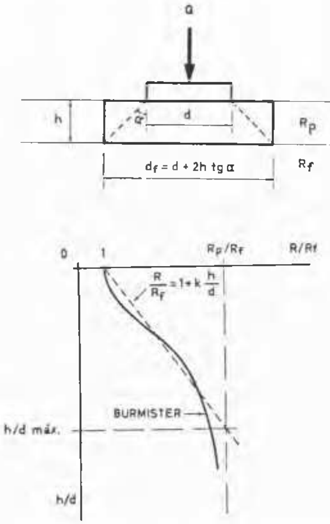

Fig. 1 Simplified sketch of the behaviour of a single-layer pavement (a) and corresponding simplified curve of R/R_f versus h/d (b).

Schéma simplifié du comportement d'un revêtement à une seule couche (a) et simplification correspondante de la loi de variation de R/R_f en fonction de h/d (b).

Table 1

Values of $K = 2 \tan \alpha$ and of α obtained from the straight lines which replace the curves in Burmister's diagram

R_p/R_f	k	α	R_p/R_f	k	α	R_p/R_f	k	α	R_p/R_f	k	α
1	0·0	0° 00′	10	2·0	45° 00′	100	6·8	73° 40′	1 000	17·7	83° 30′
2	0·4	11° 20′	20	3·1	57° 10′	200	9·3	77° 50′	2 000	23·7	85° 10′
3	0·6	16° 40′	30	3·8	62° 20′	300	10·9	79° 40′	3 000	27·2	85° 50′
4	0·9	24° 10′	40	4·4	65° 20′	400	12·3	80° 50′	4 000	30·0	86° 10′
5	1·2	31° 00′	50	4·9	67° 50′	500	13·6	81° 40′	5 000	32·6	86° 30′
6	1·4	35° 00′	60	5·5	70° 00′	600	14·5	82° 10′	6 000	35·8	86° 50′
7	1·6	38° 40′	70	5·8	71° 00′	700	15·5	82° 40′	7 000	38·0	87° 00′
8	1·8	42° 00′	80	6·3	72° 20′	800	16·3	83° 00′	8 000	40·0	87° 10′
9	1·9	43° 30′	90	6·6	73° 10′	900	17·0	83° 20′	9 000	42·0	87° 20′
									10 000	44·0	87° 30′

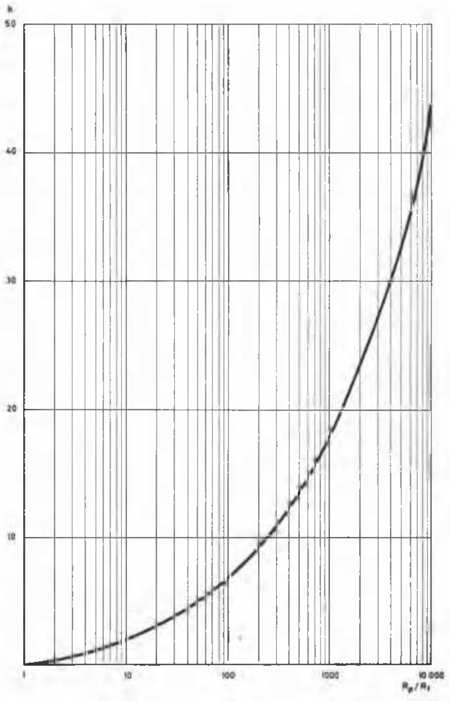

Fig. 2 Coefficient of distribution k as a function of R_p/R_f.
Coefficient de distribution k en fonction de R_p/R_f.

(c) *Apparent modulus of strength of a multi-layer pavement :*

Let us now consider an elastic system consisting of several layers with thickness h_1, h_2, h_3, ..., h_n and moduli of strength R_1, R_2, R_3, ..., R_n, carried by a foundation with a modulus of strength R_f.

It is assumed likewise that a load Q uniformly distributed on a circular area of diameter (d) is transmitted to the different layers of the system through distribution angles α_1, α_2, ..., α_n in the form of truncated cones with bases d_1, d_2, ..., $d_n = d_f$ lying on the surfaces of separation of the different layers.

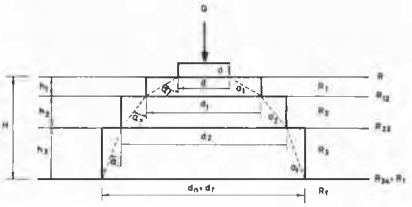

Fig. 3 Simplified sketch of the behaviour of a three-layer pavement.

Schéma simplifié du comportement d'un revêtement à trois couches.

Consequently the diameters d_1, d_2, ... and the diameter (d) are connected by the general expression

$$d_i = d + k_1h_1 + k_2h_2 + \ldots + k_ih_i \tag{9}$$

where

$$k_i = 2 \tan \alpha_i, \tag{10}$$

are the coefficients of distribution of the loads.

In order to simplify the problem it is further assumed that a given layer (i) behaves as if belonging to a single-layer system in which consequently the coefficient of distribution depends exclusively on the ratio of the corresponding modulus of strength (R_i) to the apparent modulus of strength on the layer immmediately below ($R_{i/i+1}$).

The values of (k_i) and (α_i) are given in Table 1 in function of the ratio $\frac{R_p}{R_f}$ which, for this purpose, represents the ratio $R_i/R_{i/i+1}$:

After obtaining the diameter d_f of the loaded circle in the foundation by means of eq. (9), it becomes possible to determine, by means of eq. (6), the pressure σ_f at the foundation and, by means of eq. (10), the apparent modulus of strength of the pavement and consequently, through eq. (1), its corresponding settlement.

The system of (n) layers of a pavement with a thickness $H = h_1 + h_2 + \ldots + h_n$ can be considered as a single layer with a *total distribution coefficient* (K) such that

$$d_f = d + KH \tag{11}$$

where

$$K = \frac{k_1h_1 + k_2h_2 + \ldots + k_nh_n}{H} \tag{12}$$

For an n-layer pavement, R will be obtained by solving $3n$ equations. In the case of, e.g. three layers, these equations will be written as follows :

$$\begin{aligned} R &= R_{12}\, d = R_{23}\, d_2 = R_f d_f \\ K_1 &= f(R_1/R_{12}) \\ d_1 &= d + K_1h_1 \\ K_2 &= f(R_2/R_{23}) \\ d_2 &= d_1 + K_2h_2 \\ K_3 &= f(R_3/R_f) \\ d_f &= d_2 + K_3h_3 \end{aligned} \tag{13}$$

Notice that the solution of this system is much simplified by using a trial and error method. An instance of application of this method to the design of a three-layer pavement is presented below.

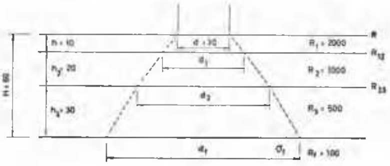

Fig. 4 Flexible three-layer pavement, the design of which is presented in the paper.

Revêtement flexible à trois couches dont le calcul est présenté dans le texte, à titre d'exemple.

Instance of application. Design of a flexible three-layer pavement (Fig. 4)
Apparent modulus of strength for $d = 30$ cm

Computation stages and corresponding formulas	*Computation*		
	1st trial	*2nd trial*	*3rd trial*
1st : $R_{12} = \frac{d_2}{d_1} R_{23}$	200 (assumed)	$\frac{80}{50} \times 150 = 240$	$\frac{78}{48} \times 145 = 236$
2nd : R_1/R_{12}	$\frac{2\,000}{200} = 10$	$\frac{2\,000}{240} = 8{\cdot}3$	$\frac{2\,000}{236} = 8{\cdot}5$
3rd : k_1 (table)	2	1·8	1·9
4th : $d_1 = d + k_1 h_1$	$30 + 2 \times 10 = 50$	$30 + 1{\cdot}8 \times 10 = 48$	—
5th : $R_{23} = \frac{d_f}{d_2} R_4$	150 (assumed)	$\frac{116}{80} \times 100 = 145$	$\frac{114}{78} \times 100 = 146$
6th : R_2/R_{23}	$\frac{1\,000}{150} = 6{\cdot}7$	$\frac{1\,000}{145} = 6{\cdot}9$	$\frac{1\,000}{146} = 6{\cdot}8$
7th : k_2 (table)	1·5	1·5	—
8th : $d_2 = d_1 + k_2 h_2$	$50 + 1{\cdot}5 \times 20 = 80$	$48 + 1{\cdot}5 \times 20 = 78$	—
9th : R_3/R_f	$\frac{500}{100} = 5$	—	—
10th : k_3	1·2	—	—
11th : $d_f = d_2 + k_3 h_3$	$80 + 1{\cdot}2 \times 30 = 116$	$78 + 1{\cdot}2 \times 30 = 114$	—
12th : $R = \frac{d_f}{d} R_f$	—	—	$\frac{114}{30} \times 100 = 380$

Result $R = 380$ kg per sq. cm.

Effect of repeated loading :

The settlement (δ_n) due to n repeated loadings of a pavement will be given by the well-known logarithmic law

$$\delta_n = \delta(1 + a_f \log n), \qquad (13)$$

in which the *settlement coefficient* (a_f) is assumed proportional to the pressure (σ_f) in the foundation

$$a_f = b_f \sigma_f$$

(b_f) being a *unit settlement coefficient.*

2. Application of the method for determining the strength of pavements designed by the current method [5]

Graphs of apparent moduli of strength of road pavements designed by means of diagrams of the California State Highway Department, assuming the pavement to consist of a single macadam layer with a modulus of strength $R_p = 2\,000$ kg per sq. cm are presented in Fig. 5. The CBR of the foundation soil was assumed to be 2, 5, 10, 30 and 50.

The CBR method omitting tire inflation pressure, a very important parameter in this analytical method, pressures of 2, 4, 6, 8, 10 and 12 kg per sq. cm were considered, it being easy to enter any other value by interpolation.

The dotted curves in Fig. 5 represent the apparent moduli of strength of airfield pavements as designed by the diagrams of the U.S. Corps of Engineers, for inflation pressures of 7 and 14 kg per sq. cm and CBR values of 3, 5, 10 and 30.

The apparent moduli of strength of rigid pavements designed with the help of the diagrams of the U.S. Corps of Engineers were determined likewise.

The results obtained, which are presented in Fig. 6, considerably exceed the values yielded by the diagrams, as the modulus of strength of concrete is much higher than that of macadam.

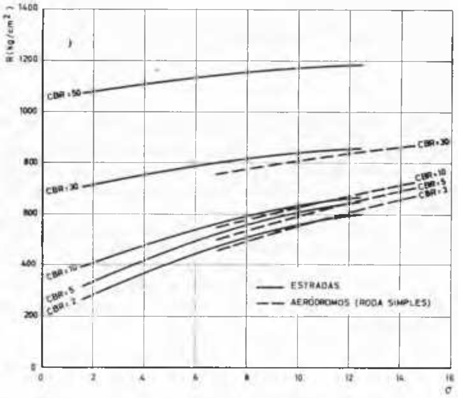

Fig. 5 Apparent moduli of strength (R) of flexible road pavements (full lines) and airfield pavements (dotted lines) designed by the CBR method versus tyre inflation pressure (σ).

Module apparent de résistance (R), en fonction de la pression de gonflement des pneux (σ), de revêtements flexibles de routes (courbes à trait plein) et d'aérodromes (courbes à trait interrompu) dimensionnés par la méthode CBR.

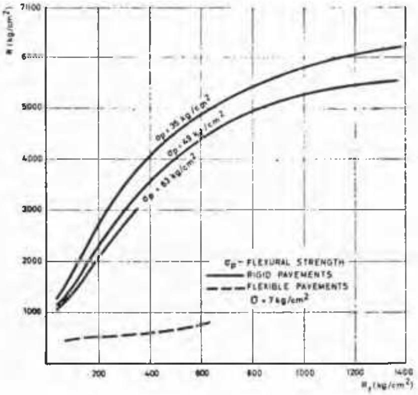

Fig. 6 Moduli of strength (R) of rigid pavements designed by means of the diagrams of the U.S. Corps of Engineers as a function of the modulus of strength of the foundation.

Modules de résistance (R) de revêtements rigides dimensionnés au moyen des diagrammes du U. S. Corps of Engineers, en fonction du module de résistance de la fondation.

3. Model study

The set up employed in model loading tests is shown in Fig. 7. By these means and starting from the theoretical foundations of model tests developed by the authors, they seek to improve and adjust the analytical method so that it shall be applicable to multi-layer pavements and to vehicles with several wheels.

The model is made up of foam rubber with an $R_f = 0{\cdot}56$ kg per sq. cm representing the foundation and current rubber, with an $R_p = 70$ kg per sq. cm, 0·5 to 1·0 cm thick, representing the pavement. The tests were carried out with three diameters, 7·5, 8·8 and 10.0 cm, and with both rigid and flexible plates. The tests with the latter were indirectly carried out by interposing a cushion of foam rubber between the rigid plate and the model.

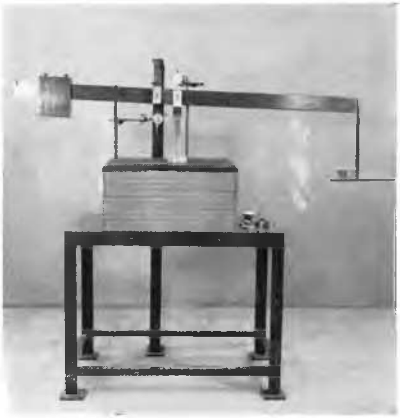

Fig. 7 Setup for model loading tests. The moael is, made of rubber, and represents an elastic single-layer pavement.

Dispositif employé dans les essais de charge sur modèle. Le modèle étudié, en caoutchouc reposant sur du caoutchouc mousse, représente un revêtement élastique à une seule couche.

The results presented in Table 2 were obtained on flexible plates, by means of which R was determined as the mean value of the unit settlements determined for $\varepsilon = 1$ per cent and $\varepsilon = 5$ per cent. The values obtained for $k = 7{\cdot}8$, taken from Table 1 for $R/R_f = 70/0{\cdot}56 = 125$, are also presented in Table 2.

The results obtained with the rigid plate are higher than those obtained with the flexible plate (about 40 per cent for $h = 0{\cdot}5$ cm and 20 per cent for $h = 1{\cdot}00$ cm) and the computed values differ but little from those obtained on flexible plates (more than 10 per cent for $h = 0{\cdot}5$ cm and less than 15 per cent for $h = 1{\cdot}0$ cm).

Table 2

Comparison of the apparent moduli of strength as computed and as determined in model tests

h (cm)	h (cm)	Computed (kg/cm²)	*Obtained in model tests*	
			Flexible plate (kg/cm²)	*Rigid plate* (kg/cm²)
7·5	0·5	0·86	0·78	1·18
	1·0	1·14	—	1·64
8·8	0·5	0·81	—	1·12
	1·0	1·06	1·28	1·54
10·0	0·5	0·78	0·75	1·06
	1·0	0·99	1·16	1·42

4. Experimental road studies

Full scale studies were undertaken on experimental roads at the Lisbon airport and the National Road No. 247-8, in Cascais near Sintra, with a view to determining the apparent modulus of strength of pavements from the moduli of strength of the foundation soil and the macadam.

The first results obtained were :

Lisbon airport :

	kg per sq. cm
Modulus of strength of the foundation	3 300
Modulus of strength of the macadam	5 300 ($h_2 = 0{\cdot}40$ m) + 6 400 ($h_1 = 0{\cdot}40$ m)
Apparent modulus of strength :	
Observed	4 000
Computed	4 600

Cascais roads :

Modulus of strength of the foundation	1 720
Modulus of strength of the macadam	5 300 ($h = 0{\cdot}30$ m)
Apparent modulus of strength:	
Observed	3 360
Computed	2 400

5. Conclusions

Although still in an early stage, it seems possible to draw the following conclusions may be drawn from this research.

(a) The simplified analytical method makes it possible to design a rigid or flexible pavement, with any number of layers, from the mechanical characteristics (modulus of strength) of the constituent materials, the deformability of the whole being fixed by means of the apparent modulus of strength.

(b) Owing to the lack of data on the characteristics of the materials and the inadequate experience of the application of the method which make it difficult to estimate the correct value of the apparent modulus of elasticity of the pavement, this should always be checked by conventional methods. A diagram which facilitates this check is given by the authors.

(c) The first results obtained in model tests show that the values of the apparent modulus of strength as computed and as obtained in load tests on flexible plates differ by about 10 to 15 per cent. It also shows that load tests on rigid plates yield values 20 to 40 per cent higher.

(d) The results obtained for the experimental pavement of Lisbon airport gave values differing by 13 per cent from the computed modulus of strength. The results obtained for National Road No. 247-8 showed much greater differences (40 per cent) which are partially explained by the fact that the modulus of strength of the macadam was determined immediately after its conclusion, that is, when it was still rather moist.

It is to be noted besides that the computations were made for R_p/R_f below 10, that is, in a not very sensitive zone of the graph on Fig. 2. The values of R_f were determined immediately after compaction of the filling and in summer. Usually and for designing pavements, these values would have to be determined in worse conditions of humidity and the values obtained for R_p/R_f would be above 10.

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