Stresses and Deflections in Homogeneous Soil Masses

Contraintes et déformations dans des masses de sol homogène

by W. J. Turnbull, Chief, Soils Division
A. A. Maxwell, Chief, Flexible Pavement Branch, Soils Division
and
R. G. Ahlvin, Chief, Special Projects Section, Soils Division, U.S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi, U.S.A.

Abstract

Typical results are presented from a study of the stresses and deflections induced in two soil test sections by surface loads. Comparisons are made between theoretically determined and measured values.

Sommaire

Dans ce rapport sont présentés les résultats caractéristiques obtenus à la suite d'une étude des contraintes développées et des flèches résultant dans deux sections d'essais de sol de l'application de charges en surface. Les valeurs obtenues par le calcul sont comparées aux résultats expérimentaux.

Introduction

For a number of years a research program has been under way at the U.S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi, that has as its purpose the advancement of knowledge of the distribution of stresses, strains, and deflections in airfield pavements. Under this program, testing has been completed on two homogeneous test sections. These two studies, which are briefly documented here, were much more extensive than this presentation indicates, and there were many tributary and subsidiary facets to the investigations that cannot be treated in this paper. Complete reports on them are contained in references 4 and 5 of the bibliography.

Description of Tests

The studies included the measurement of stresses and deflections induced by a uniform circular load in each of the two test sections. The first was constructed of a clayey-silt material placed at about its optimum moisture content. This material has a liquid limit of 36 and a plasticity index of 13. In place in the test section, it had a CBR of about 15, density near 105 lb/ft^3, and moisture content of about 18 per cent. The other test section was constructed of a fairly uniform air-dried sand. This material is nonplastic; more than 95 per cent passes the No. 10 sieve and is retained on the No. 40 sieve. In place it had a CBR of about 7·5, density near 108·5 lb./ft^3, relative density of about 83 per cent, void ratio of about 0·53, and moisture content less than 0·3 per cent.

In both test sections, pressure cells and deflection gages were installed at various lateral positions 5 ft below the surface. Loads were applied along two lines down the center of each test section such that stress and deflection readings could be taken for each foot of offset from the load axis between 0 and 9 ft. This was accomplished by proper spacing of instruments either side of the load lines. The test sections were cut down in successive 1-ft layers so that all measurements could be repeated for each foot of depth from 1 to 5 ft. By this means, stress and deflection patterns were established in the test sections within limits of 9 ft horizontally and 5 ft vertically.

Loads were applied using water-filled, flexible-face loading plates of the type shown in Fig. 1. A truss mounted on railroad carts and loaded to 300,000 lb. gross provided the reaction

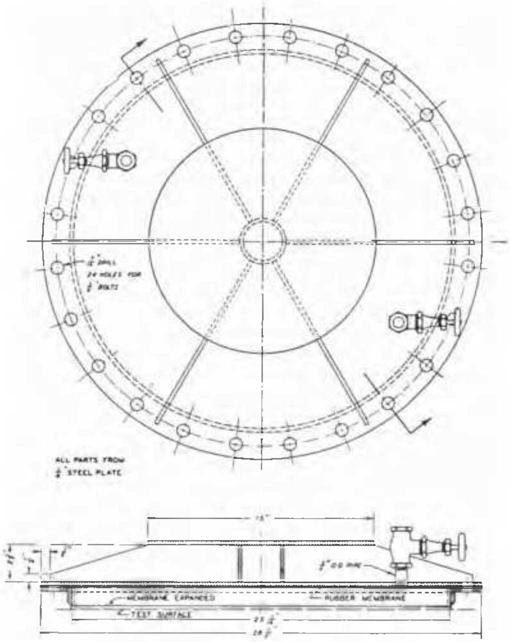

Fig. 1 Flexible-Faced Pressure Plate.
Plaque de charge à face flexible.

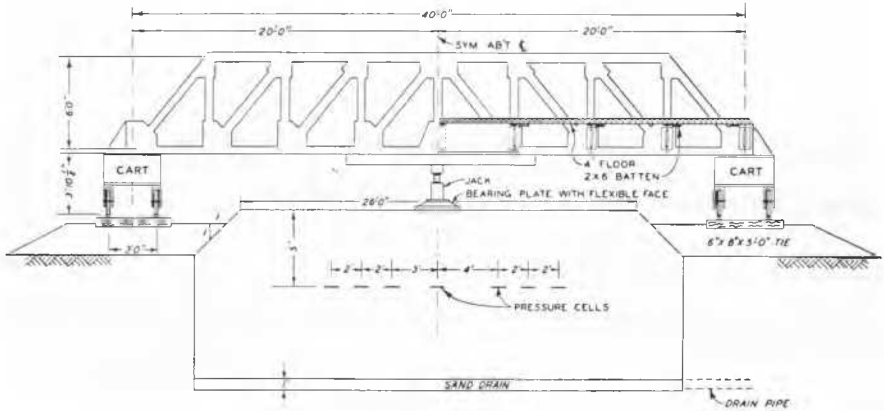

Fig. 2 Loading Truss.
Charpente de mise en charge.

for applying test loads. Fig. 2 shows this truss in place over the clayey-silt test section. Parallel pairs of railroad tracks on each side of the test section permitted moving the truss to allow passage of equipment during construction as well as to provide reactions at various positions.

Vertical, horizontal, and diagonal (45°) stresses, σ_z, σ_x, σ_y, σ_u, and σ_v, and vertical deflections, ω, were measured at each foot of depth and offset beneath both single and dual loads. In the first (clayey-silt) test section, single, 1 000-sq-in., circular uniform loads and dual, 500-sq.-in., circular uniform loads at 3, 4·5, 6, and 7·5 ft center-to-center spacings were used. In the second (sand) test section, single, 1 000-, 500-, and 250-sq.-in., circular uniform loads were used as well as dual, 1 000-sq.-in., circular loads at 4·5-ft spacing; dual, 500-sq.-in., circular loads at 3-and 6-ft spacings; and dual, 250-sq.-in., circular loads at 2.5-ft spacing.

In the first test section, loading intensities of 15, 30, 45, and 60 psi were used, while in the second, intensities of 15, 30, and 60 psi were adopted. For convenience of presentation of all data in common plots, the stress measurements were all reduced to percentage of contact pressure (loading intensity). The deflections were reduced to a ratio of the deflection to a loading intensity of 1-psi surface load in the clayey-silt test, and to a ratio of deflections for a 100-psi surface load in the sand test. The only reason for the latter is that it gives a more convenient number to handle.

Theoretical stresses and deflections were developed for comparison with measurements. For this purpose a semi-infinite elastic mass was assumed. The load was taken to be circular and uniform. Newmark charts [2]* were used to produce theoretical results for the first (clayey-silt) test section, while those for the second (sand) test section were computed directly. A 0·5 value of Poisson's ratio was satisfactory for theoretical data for the clayey-silt test section, but results developed for a 0·3 value were more suitable for the sand test section. Computation of deflections required the selection of several values for the modulus of elasticity, E_m, in order to bracket test data. For the clayey-silt test section the values used were 5,000, 10,000, and 25,000 psi. For the sand test section they were 20,000, 40,000, and 60,000 psi.

The pressure cells used for measuring stresses were the Waterways Experiment Station type incorporating a fluid-filled space or pocket to transmit pressure from the face plate to an inner diaphragm. The resulting bulge of this diaphragm is measured by electrical-resistance type strain gages. Cells in the first (clayey-silt) test section were 12 in. in diameter and 1 in. thick, while those in the second (sand) section were of an improved design and were 6 in. in diameter and 1 in. thick. Further details of design are beyond the scope of this paper but may be found in the complete reports of the two test sections [4,5]. The stresses measured, σ_z, σ_x, σ_y, σ_u, and σ_v, give a complete definition of stress at a point since the two 45° diagonal stresses can be resolved into the shear stress, τ_{zx}, or its equal, τ_{xz}, and the remaining shear stresses are zero. Fig. 3 shows the stress directions.

The deflection gages used incorporate Selsyn motor units for remote actuation. They measured between reference flanges at the gage level and reference rods driven deep in the subsoil. Here again, details are not within the scope of this paper but may be found in the complete reports [4,5].

Stress Measurements

Clayey silt—Comparisons of measured and computed stresses are shown for selected typical cases from the clayey-silt test section in Figs. 4 and 5 [6].* The computed stresses are for a Poisson's ratio of 0·5 but the plots include results for concentration factors (N) other than for $N = 3$ which represents the Boussinesq or elastic case. Explanation of the concentration factors can be found in some of the work of O. K. Fröhlich [1].

Fig. 4 shows the vertical stress at 1-ft depth for all loadings and spacings used. Fig. 5 shows the vertical stress for 2- and 4-ft depths, and two horizontal stresses and the vertical-horizontal shear stress for 1-ft depth, all for single loading. Both plots show the stress in per cent of surface load intensity versus offset for a given depth. All stresses were treated at five depths for all loadings but, again, limited space prevents inclusion of these data.

These plots indicate a rather remarkable agreement between theoretical computed stresses, assuming elastic action of the mass, and the stresses measured within the test section.

* Bracketed numbers refer to Bibliography.

* Some of the results from the clayey-silt test section were reported in reference 6.

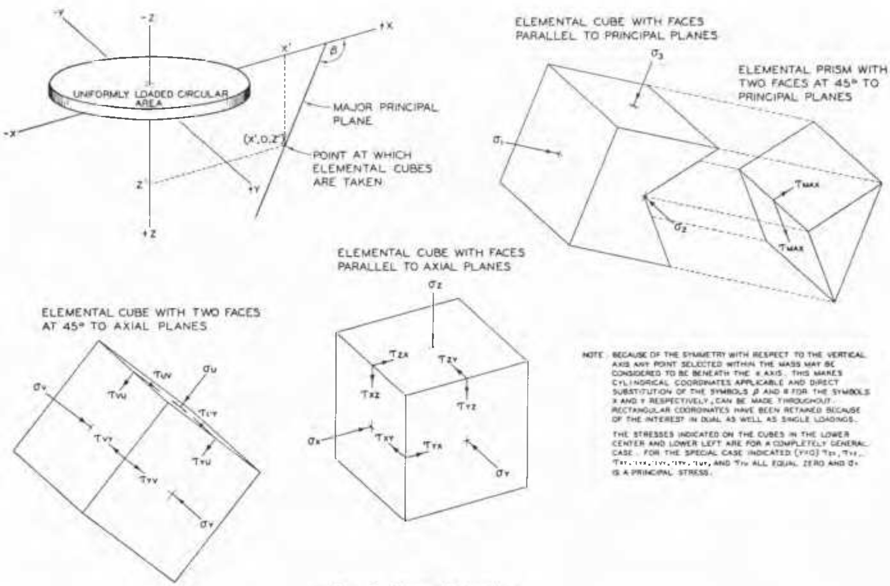

Fig. 3 Stress Directions.
Directions des contraintes.

The computed curves for concentration factors other than 3 did not, in general, show better agreement with measurements than those of the $N = 3$ curves.

Sand—Comparisons of measured and computed stresses are shown for selected typical cases from the sand test section in Figs. 6 and 7. Computed curves are given for Poisson's ratio of 0·3 and only the elastic ($N = 3$) case is considered. Fig. 6 shows the vertical stress at 1-ft depth for all loadings and spacings used. Fig. 7 shows the vertical stress for 2-, 4-, and 5-ft depths, and two horizontal stresses and the vertical-horizontal shear stress for 1-ft depth, all for single loading. Both plots show the stress in per cent of surface load intensity versus offset for given depths. The study included comparisons for all loadings for 1-, 2-, 3-, 4-, and 5-ft depths for all the stresses shown in Fig. 7 and for the major and minor principal stresses as well. Space limitations prevent their presentation here.

These plots also show a generally good agreement between computed theoretical stresses and those measured within the test section. In Fig. 6, the stress measurements beneath the loaded areas plot slightly above the theoretical curves. It was established in later work on the program (beyond the scope of this paper) that these overlarge measurements are probably due to pressure cell overregistration. For a detailed treatment of pressure cell theory, see reference 3.

Deflection Measurements

Clayey silt—Comparisons of measured and computed deflections are shown for the clayey-silt test section in Fig. 8. These are plots of the vertical deflection for a 1-ft depth. The figure shows deflection versus offset for all loadings and spacings. Theoretical curves are shown for two values of the modulus of elasticity, E_m (25,000 psi and 10,000 psi), and in one plot, a theoretical curve for the 5,000-psi modulus value is included for comparison.

Sand—Comparisons of measured and computed deflections are shown for the sand test section in Fig. 9. These are also plots of the vertical deflection for a 1-ft depth. And again, each graph plots deflection versus offset for all loadings and spacings. Theoretical curves are shown for two values of the modulus of elasticity (40,000 psi and 20,000 psi).

Measured versus computed deflections—Comparison of the theoretical curves with the plotted data points for either the clayey-silt (Fig. 8) or the sand (Fig. 9) test section shows only general correlation. Beneath the load, at shallow depths where deflections are large, the theoretical curves for the lesser values of the modulus of elasticity apply, while measured deflections out from beneath the load are smaller than any of the theoretical computed deflections. This implies for shallow depths either a nonagreement with the theory used or a variation of the modulus of elasticity with stressing of the soil. In both test sections, agreement with theoretical results is quite good at the greater depths, and in both cases, this agreement is with the theoretical curve based on the larger modulus of elasticity used for computations (25,000 psi for clayey silt, 40,000 psi for sand). Space limitations prevent inclusion of these comparisons.

As was to be expected, deflections in the sand test section for equivalent conditions were, in general, less than half as great as those in the clayey-silt test section. It is perhaps notable that the two soil materials used in these tests show such similar deflection behavior patterns.

Stress-Strain Relations

Measurement of the three co-ordinate stresses, σ_x, σ_y, and σ_z, and the vertical deflection, ω, for common points with respect to the loadings applied permit the development of stress-strain relations for the test sections. Vertical strains were derived from plots of the vertical deflection versus

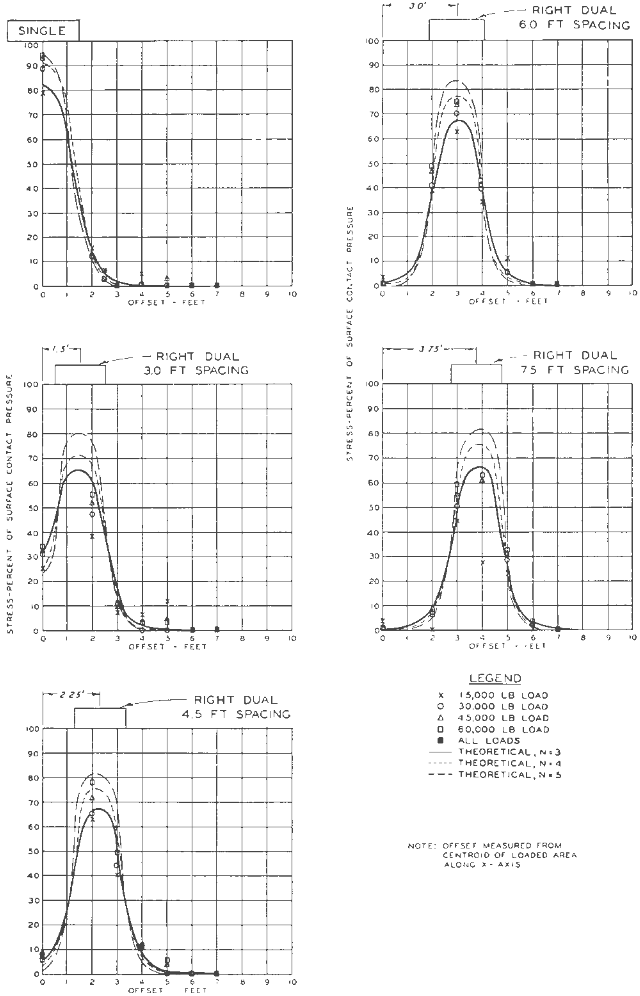

Fig. 4 Stress vs Offset Distance, σ_z at 1-ft Depth, Clayey-Silt Test Section.
Contrainte en fonction de la distance horizontale σ_z à 1 pied de profondeur. Essais sur limon argileux.

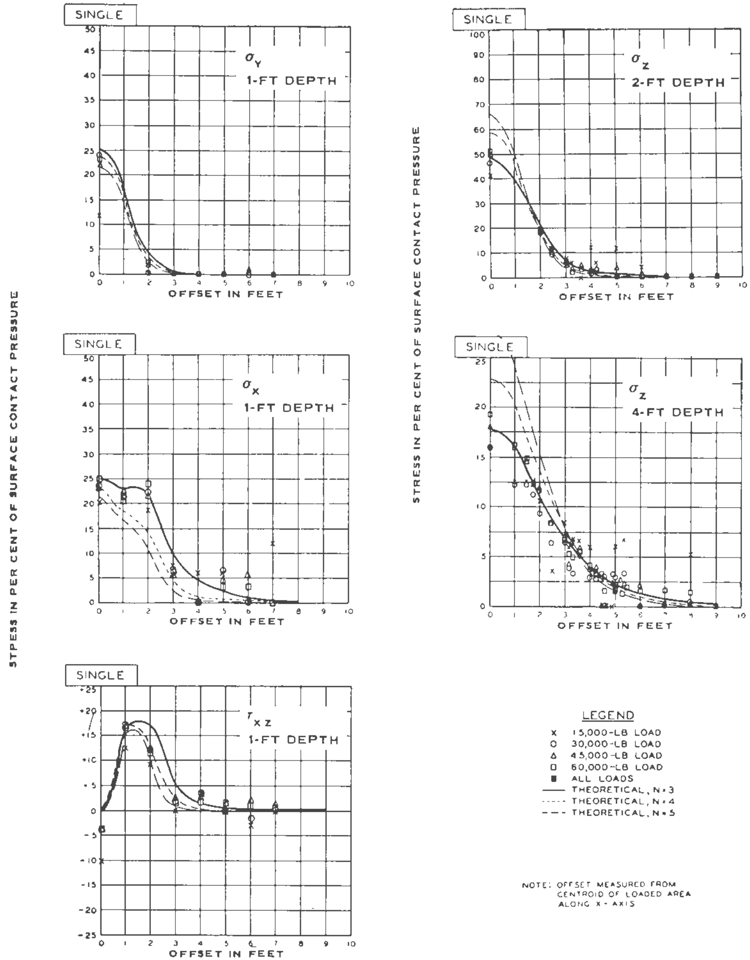

Fig. 5 Various Stresses vs Offset Distance Single Plate Load, Clayey-Silt Test Section.
Contraintes diverses en fonction de la distance horizontale. Charge sur plaque simple. Essais sur limon silt-argile.

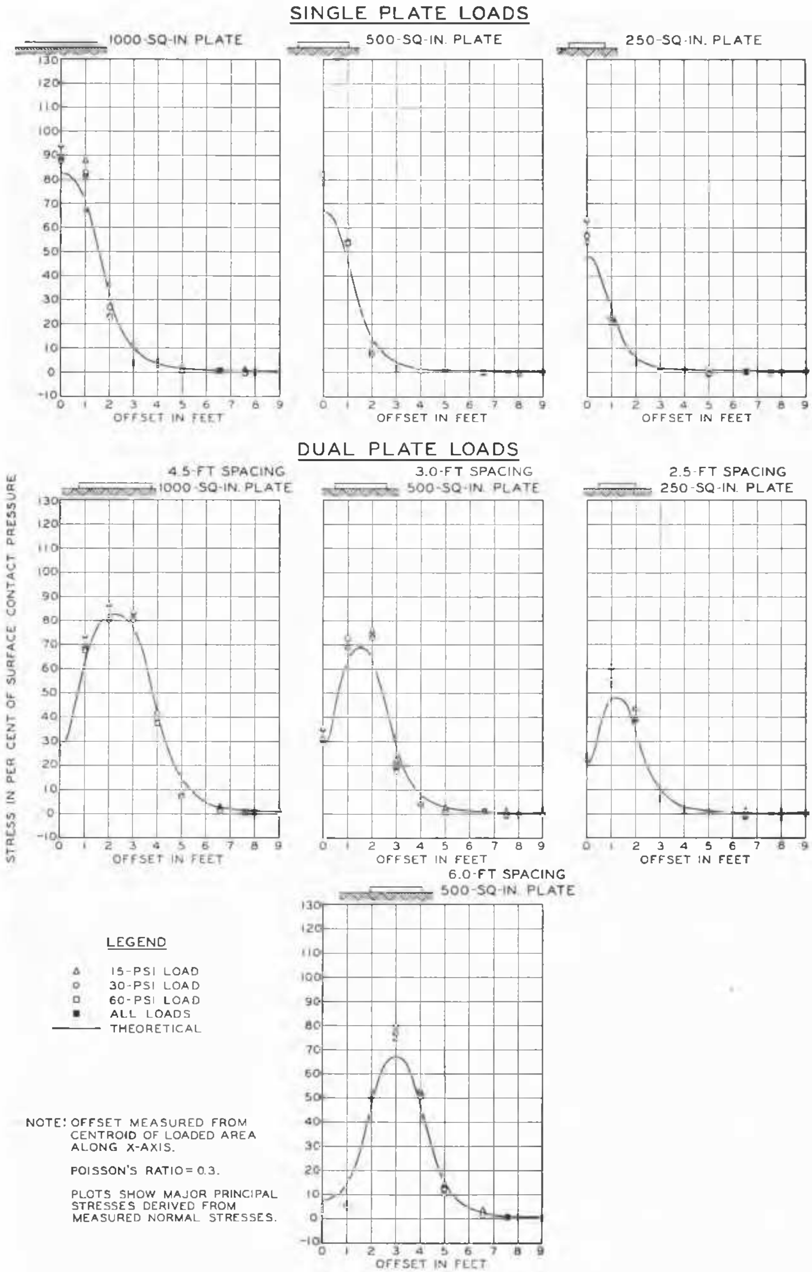

Fig. 6 Stress vs Offset Distance, σ_2 at 1-ft Depth, Sand Test Section.
Contrainte en fonction de la distance horizontale, à 1 pied de profondeur. Essais sur sable.

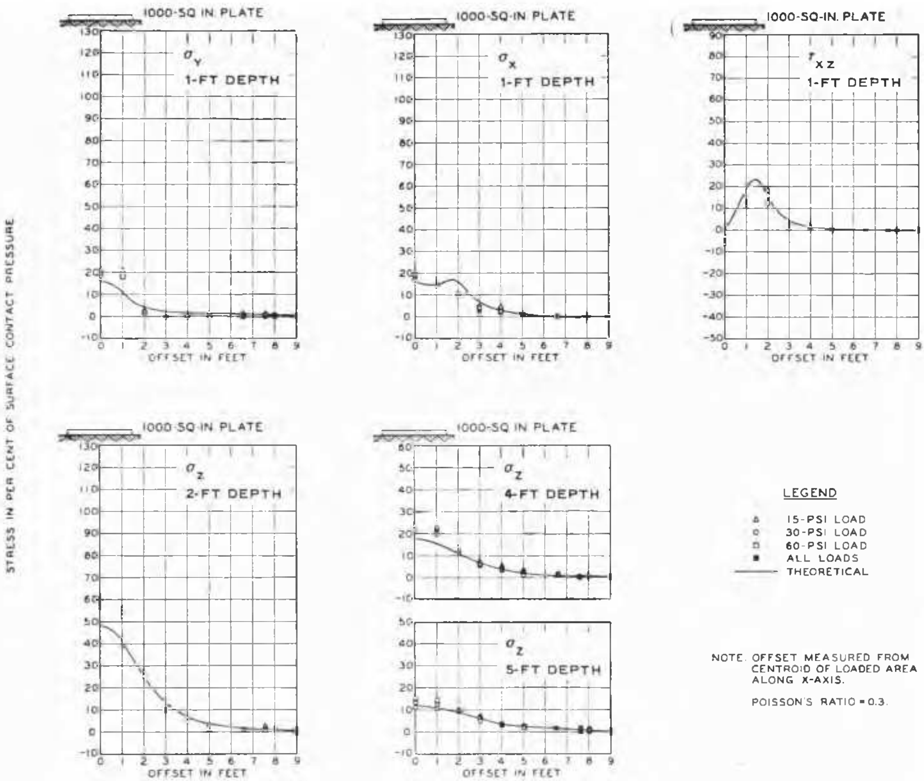

Fig. 7 Various Stresses vs Offset Distance, Single Plate Load, Sand Test Section.
Contraintes diverses en fonction de la distance hozizontale. Charge sur plaque simple. Essais sur sable.

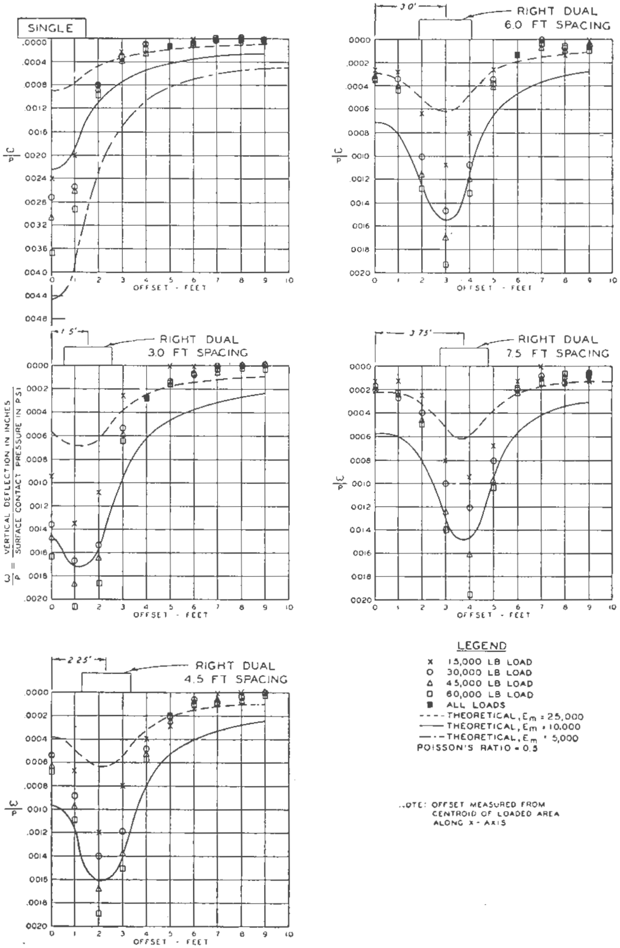

Fig. 8 Deflection vs Offset Distance, ω at 1-ft Depth, Clayey-silt Test Section.
Flèche en fonction de la distance horizontale. ω à 1 pied de profondeur. Essais sur limon argileux.

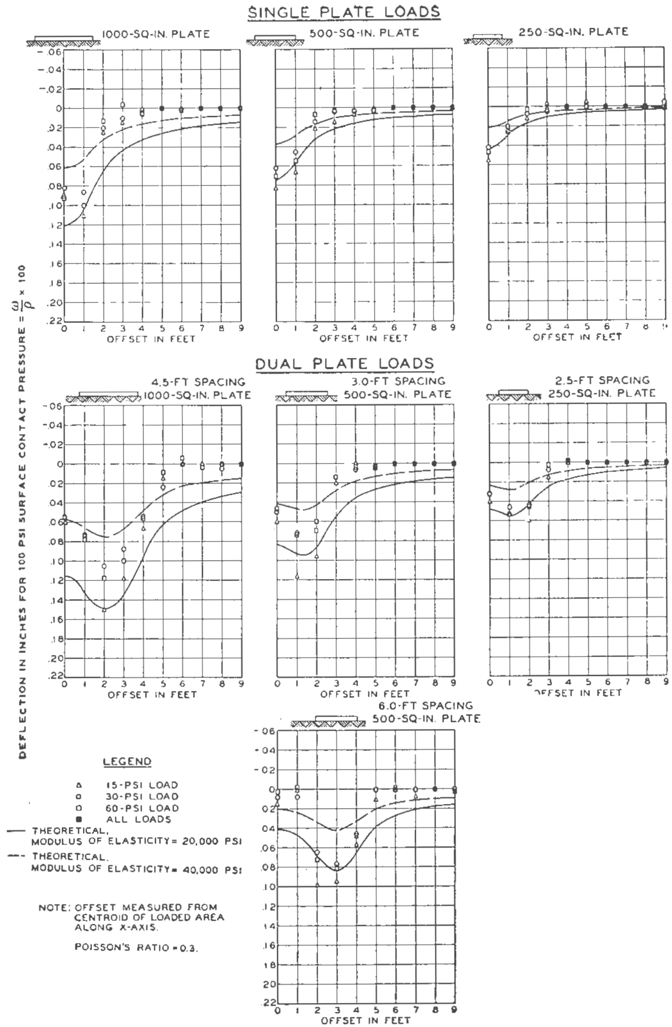

Fig. 9 Deflection vs Offset Distance, ω_z at 1-ft Depth, Sand Test Section.
Flèche en fonction de la distance horizontale. ω_z à 1 pied de profondeur. Essais sur sable.

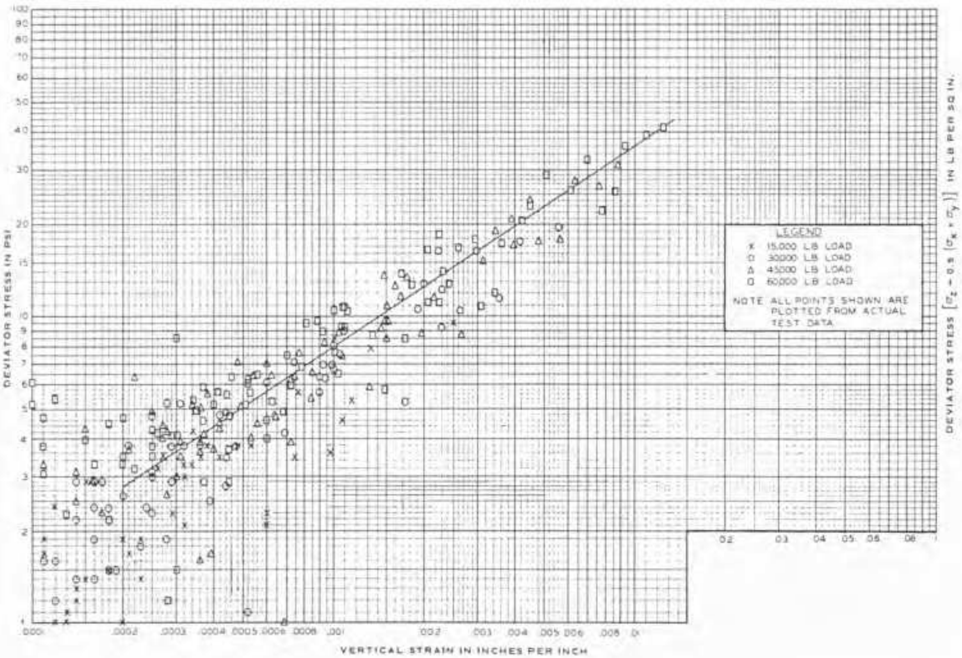

Fig. 10 Logarithmic Plot, Stress-Strain Data, Clayey-Silt Test Section.
Représentation logarithmique. Données contrainte-déformation. Essais sur limon argileux.

depth merely by taking the slope of tangents to the deflection-depth curve. These strains are related to the stresses by the familiar equation :

$$\varepsilon_z = \frac{1}{E_m}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

where

ε_z = vertical strain
E_m = modulus of elasticity
σ_z = vertical stress
σ_x and σ_y = mutually perpendicular horizontal stresses
ν = Poisson's ratio

Thus, the nature of the modulus of elasticity or the relation between stress and strain can be examined by plotting ε_z versus $\sigma_z - \nu(\sigma_x + \sigma_y)$. This has been done for each test section. In the case of the clayey-silt test section, ν was taken as 0·5. Fig. 10 is a logarithmic plot of all the pertinent test data collected. A curve giving the best visual average fit has been drawn through the plotted points. In the case of the sand test section, a 0·3 value of ν was used. This value was found from many facets of the over-all analysis of the sand test section data to better represent the action of the material than the more commonly used 0·5 value. Fig. 11 shows all pertinent data on an arithmetical stress-strain plot. Here again, the best visual average curve has been drawn through the points.

It is interesting that the stress-strain curve for the sand test section is nearly a straight-line relation on an arithmetical scale, while that for the clayey-silt test is roughly linear beyond a vertical strain of about 0·001 in./in. The indicated modulus for the sand is about three times that for the clayey silt.

Conclusions

Some of the primary conclusions derived from this study are :

(*a*) Stresses measured in the test sections are, in general, in good agreement with those predicted by the theory of elasticity.

(*b*) Deflections measured in the test sections show a distribution somewhat different than that predicted by the theory of elasticity at shallow depths, but show relatively good agreement at depths of 3, 4, and 5 ft.

(*c*) The stress-strain curves determined from stress and deflection measurements were nearly linear for the sand test section, and were roughly linear for the clayey-silt test section beyond vertical strains of about 0·001 in./in.

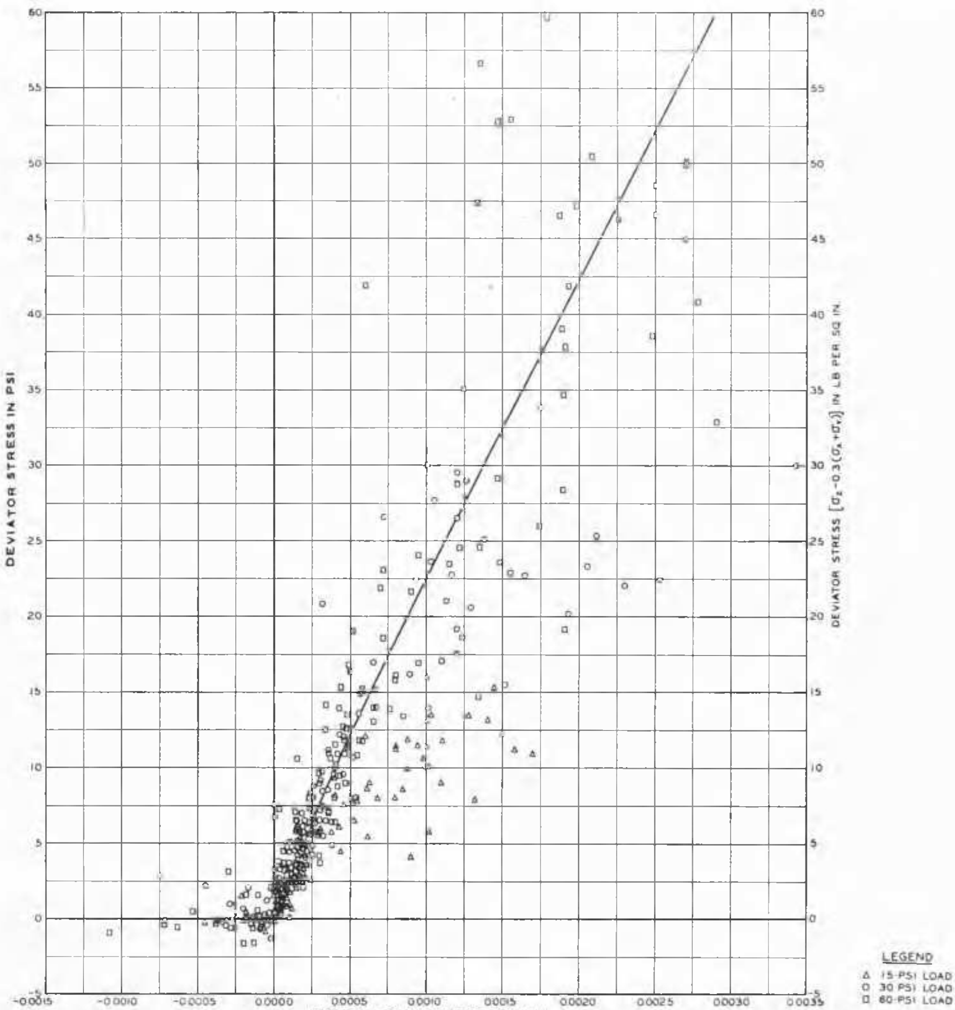

Fig. 11 Arithmetical Plot, Stress-Strain Data, Sand Test Section.
Représentation arithmétique. Données contrainte-déformation. Essais sur sable.

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