Nonsteady-State Flow in Homogeneous Earth Dams after Rapid Drawdown

Ecoulement non permanent dans les barrages en terre après une baisse rapide du niveau du réservoir

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Summary

The nonsteady-state flow after rapid drawdown in homogeneous earth dams on impervious foundations was investigated. It was found that the seepage line became elliptical in shape after a short initial period. The equation describing the flow, which begins after this short period, was obtained using the method of continuous successions of steady states. The equation gives coordinates of the seepage line in function of time. An experimental investigation, which confirmed the validity of the obtained equations, was performed in a Hele-Shaw flume for dams with upstream slopes of 1/2·5, 1/3 and 1/5, for different ratios of head water to the dam width on the base, and for different types of drainage.

For some cases of dams with 1/2·5 and 1·3 uptream slope the equation describes the flow exactly ; for others it requires a constant correction factor, denoted c. Factor c was investigated in detail, and the value of it was obtained to use for dams with upstream slopes from 1/2·5 to 1/5, for different ratios of the headwater to the width of the dam. Dam shape characteristics were obtained for practical application of the equation and were represented in the form of graphs.

Sommaire

L'auteur étudie dans ce rapport l'écoulement non-permanent dans les barrages en terre sur bases imperméables après une baisse instantanée du niveau du réservoir. Il a été constaté qu'après une période initiale très courte, l'écoulement s'établit avec une surface elliptique. Une équation décrivant l'écoulement durant cette dernière période a été obtenu en appliquant la méthode de succession continue d'état d'écoulement permanent. L'équation donne les coordonnées de la surface de saturation en fonction du temps. Une recherche expérimentale a été conduite dans le chenal de Hele-Shaw avec des barrages ayant leur pente amont de 1/2·5, 1/3 et 1/5, des rapports de la profondeur amont initiale à la largeur de la base différents ainsi que des systèmes de drainage divers. Les résultats des essais ont démontré la validité de l'équation obtenue. Elle décrit l'écoulement d'une façon exacte dans des cas de barrage à pente de 1/2·5 et 1/3, pour d'autres elle demande l'introduction d'une constante c. Le facteur c a été étudié en détail, et les valeurs de c ont été obtenues pour l'emploi pratique de l'équation, en fonction de la pente amont et du rapport de la profondeur initiale amont à la base. D'autres facteurs entrant dans l'équation ont été obtenus et ont été mis sous forme de graphiques pour leur utilisation.

The interest in the problem of seepage flow in earth dams after drawdown of the reservoir is well-known. Its practical importance is primarily related to the analysis of stability of the upstream slope of dams. The purpose of this work is to establish the equation of the free surface of seepage flow in function of time after instantaneous drawdown of the reservoir to zero level. The problem treated concerns homogeneous dams of any geometrical shape with different drainage systems and without tailwater on impervious foundations·

The equation of the free surface of the gravity seepage flow on an impervious layer can be written, taking into account all assumptions and limitations attributed to it, according to Boussinesq :

$$\frac{\partial^2 h^2}{\partial^2 x} - 2\, n_e \frac{1}{k}\frac{\partial h}{\partial t} + \frac{2\varepsilon}{k} = 0 \qquad (1)$$

where n_e is the effective porosity, or the ratio of the part of the voids not filled by water to the volume of soil mass, k (LT^{-1}) is Darcy's coefficient of permeability, $\varepsilon(LT^{-1})$ is the rate of infiltration from the horizontal projection of the surface area and t is the time. Other notations are shown on Fig. 1. For steady state flow (1) is reduced to

$$\frac{d^2 h^2}{dx^2} + \frac{2\,\varepsilon}{k} = 0 \qquad (2)$$

and its integral is

$$h^2 = -\frac{\varepsilon}{k}x^2 + \text{const}_1\, x + \text{const}_2 \qquad \ldots\ldots (3)$$

For a rectangular earth massive, Fig. 1, with symmetrical flow conditions at the outlet where $x = a$ at $h = 0$, and $h = H$ at $x = 0$, (3) becomes

$$h^2 = -\frac{\varepsilon}{k}x^2 + \left(\frac{\varepsilon\, a}{k} - \frac{H^2}{a}\right)x + H^2 \qquad (4)$$

which is an equation of an ellipse.

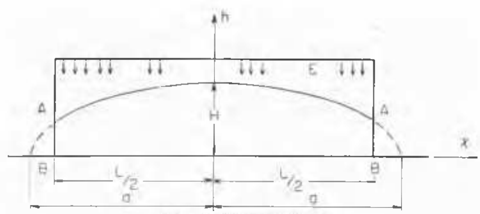

Fig. 1 Nomenclature
Notations

From Darcy's law

$$q = -kh\frac{dh}{dx} \quad \dots\dots (5)$$

and (4) we obtain

$$q = \left(x - \frac{a}{2}\right)\varepsilon + \frac{kH^2}{2a} \quad \dots\dots (6)$$

By definition of ε we have

$$q = \varepsilon x \quad (7)$$

Equating (6) and (7) we obtain

$$\varepsilon = k\frac{H^2}{a^2} \quad (8)$$

Substituting ε from (8) into (4) the equation of the free surface becomes

$$\frac{h^2}{H^2} + \frac{x^2}{a^2} = 1 \quad \dots\dots (9)$$

If the water levels in reservoirs are at A, the semi-axia a, of the ellipse may be determined. If there are point drains at B, $a = \frac{L}{2}$, but if there are outflow surfaces, AB, and a seepage flow into empty reservoirs, a cannot be determined in an elementary way. It is more difficult to determine a or AB in the case of sloping outflow surfaces.

Suppose levels of open water are at A. If they begin to draw down, a nonsteady flow will take place. In the process of drainage, the water comes out of pores similarly to infiltration when water is supplied from the surface. Since the nonsteady-state flow may be regarded as a continuous succession of steady states, (4) will represent the free surface at any time interval, with the condition that instead of infiltration ε, the water will come from pores above the free surface, i.e., from the volume ΔV per unit of width normal to the drawing, Fig. 2. In the vicinity of outflow, sections AB, (4) may not describe the seepage surface, but the influence of transient boundary conditions will be insignificant at some distance from them. In the case of sloping faces of the earth massive, i.e. earth dams, the local influence of faces will be smaller due to the resistance to the flow of the triangular areas below the intersection of the seepage line with the face.

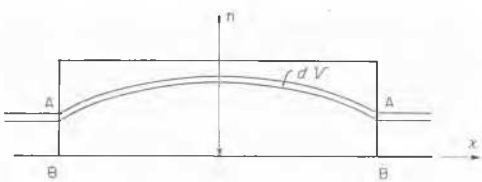

Fig. 2 Elementary volume supplying water.
Volume élémentaire fournissant l'eau.

The writer performed numerous experiments in a Hele-Shaw flume and discovered that the free surface after draw-down in dams of any practically encountered shape is actually elliptical and may be described by (9), Fig. 3, almost exactly, except in the very early stage, when the initial top point A_o of seepage line draws down quickly to the position A'_o. When this stage is terminated, the ellipse intersect the upstream slope and have the maximum point A at various positions. Investigations further prove that the points A are located approximately on a sloping straight line with cotangent m_2. The sloping line may be located from the heel by n_2L, and the distance a may be located by n_1L.

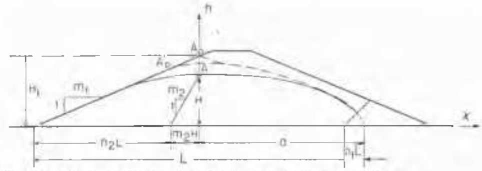

Fig. 3 Nomenclature for the transient flow equation.
Notations de l'équation de l'écoulement non permanent.

All factors, m and n, are functions of the shape of the dam. Since now the area under the seepage line can be expressed in terms of physical dimensions of the dam and the variable H, the differential equation of the method of continuous succession of steady states can be written.

The volume of water in the earth mass below the seepage line per unit of width of the dam normal to the drawing at the time t, may be expressed, with few per cent accuracy, by the area of a quarter of the ellipse located downstream of A and by a trapezoid upstream of it :

$$V = n_e\left[\frac{\pi}{4}aH + (L + n_1L - a)H - \frac{1}{2}m_1H_2\right] \quad \dots\dots (10)$$

The following notations will be used for shorter writing :

$$k_1 = \frac{\pi}{4}(1 + n_1) + \left(1 - \frac{\pi}{4}\right)n_2;\ k_2 = (1 + n_1) - n_2 \quad \dots\dots (11)$$

$$k_3 = H_1/L;\ k_4 = m_2;\ k_5 = \frac{m_1}{2} - \left(1 - \frac{\pi}{4}\right)m_2$$

In terms of k — factors (10) will be

$$V = n_e(k_1LH - k_5H^2) \quad \dots\dots (12)$$

Differentiating V by H (12) and dividing by dt, we obtain the discharge

$$q = \frac{dV}{dt} = -n_e(k_1L - 2k_5H)\frac{dH}{dt} \quad \dots\dots (13)$$

On the other hand, from Darcy's law, (7) and (8), we obtain in upstream direction $q = kH^2/a$ at $x = a$, and in both directions we obtain the total discharge

$$q = 2k\frac{H^2}{a} \quad \dots\dots (14)$$

a may be expressed by $a = k_2L - k_4H$ and q by

$$q = 2k\frac{H^2}{k_2L - k_4H} \quad \dots\dots (15)$$

Equating (13) and (15), we obtain

$$2k \frac{H^2}{k_2 L - k_4 H} = n_e (k_1 L - 2k_5 H)\frac{dH}{dt} \quad \ldots\ldots (16)$$

After rearrangement and the use of $L = H_1/k_3$

$$dt = -\frac{ne}{2k} L\left[C_1 \frac{H_1}{H^2} - C_2\frac{1}{H} + C_3\frac{1}{H_1}\right] dH \quad \ldots\ldots (17)$$

where

$$C_1 = \frac{k_1 k_2}{k_3};\ C_2 = k_1 k_4 + 2k_2 k_5 \quad \ldots\ldots (18)$$

$$C_3 = 2k_3 k_4 k_5$$

The integration results in the equation

$$t = \frac{ne}{2k} L\left[C_1\frac{H_1}{H} + C_2\log_e H - C_3\frac{H}{H_1} + \text{const}\right] \quad \ldots\ldots (19)$$

The constant of integration can be determined on the assumption that at the time $t = 0$, $H = H_1$, Fig. 3. (19) will finally become

$$t = \frac{cn_e}{2k} L\left[C_1\left(\frac{H_1}{H} - 1\right) + C_2\log_e\frac{H}{H_1} + C_3\left(1 - \frac{H}{H_1}\right)\right] \quad \ldots\ldots (20)$$

in which a factor c is introduced to correct, if necessary, possible inaccuracies involved as a consequence of assumptions made.

Experimental verification of the equation (20) was performed for dams with upstream slopes of 1:2·5, 1:3 and 1:5, and with different types of drainage and various ratios of H_1/L, Fig. 3. L was always the distance from the heel to the edge of the drainage facilities. The correction factor c was fond as shown in Table 1.

Table 1

Upstream slope $1/m_1$	*Ratio* H_1/L	c
1/2·5	0·14-0·25	0·90-1·10
1/3	0·12-0·20	1·00-1·30
1/5	0·11-0·17	1·20-1·40

Taking into account the upstream slopes, 1/2·5 to 1:3, most frequently encountered in practice, and the average ratios, H_1/L, between two extremes given in Table 1, it is seen that the time computed by (20) does not require more than 20 per cent correction. For practical application of (20), the above c may be used with a straight line interpolation for intermediate values of H_1/L and m_1. The influence of the type of drainage on c factor was studied but no specific relation was established. This influence is, in general, very minor. The tabulated values c are averages for any type of drainage. From the same Table 1, it is seen that (20) gives the exact solution, $c = 1$, for average H_1/L ratios for upstream slopes $m = 2·5$ and for low H_1/L ratios for $m_1 = 3·0$.

The factors n_2 and m_2, Fig. 3, determining the assymetrical position of the seepage line were found dependant on the dam shape. They were related to the upstream slope cotangent, m_1, and to the ratio H_1/L on the basis of experimental data and are representad on Fig. 4. Factor n_1 may be computed from

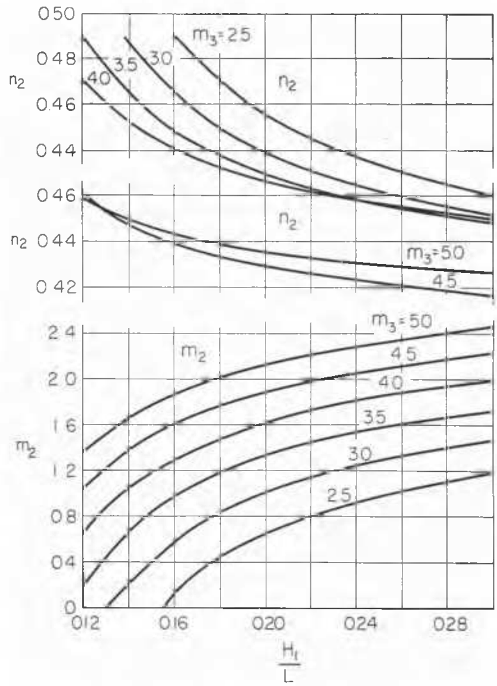

Fig. 4 Coefficients giving location of the origin of coordinats.
Coefficients déterminant l'origine des coordonnées.

the usual procedure for steady-state flow, but here it was obtained experimentally in function of H_1/L. Finally, k — factors (11) were expressed in function of only m_1, and H_1/L. Using k-coefficients, C_1, C_2 and C_3 (18) were also expressed in function of m_1, and H_1/L. The obtained formulas for coefficients C are too complicated, so they are represented here only graphically, Fig. 5. They may be used for the practical application of the equation (20).

When the time, t, after rapid drawdown of the reservoir for the given H/H_1, is computed from (20), the seepage line is obtained using (9) with h-axis passing through the point A, Fig. 3, determined by n_2 and m_2, Fig. 4.

We may denote the dimensionless depth function of (20) by

$$F\left(\frac{H}{H_1}\right) = C_1\left(\frac{H_1}{H} - 1\right) + C_2\log_e\frac{H}{H_1} + C_3\left(1 - \frac{H}{H_1}\right) \quad \ldots\ldots (21)$$

so

$$t = \frac{cn_e}{2k} LF\left(\frac{H}{H_1}\right) \quad (22)$$

The graph of the depth function $F\left(\frac{H}{H_1}\right)$ if plotted on Fig. 6 as an illustration for some experiments, from the series of over 30 performed, for the slopes, $m_1 = 2·5$, 3, and 5. The function itself is computed from (21) for conditions which correspond to these experiments.

On the same graphs are shown experimental points computed from (22)

$$F\left(\frac{H}{H_1}\right) = \frac{2k}{cn_cL}\, t \qquad (23)$$

for the time t observed during tests, with c indicated in Table 1. It is seen on the graphs that points having the values H/H smaller than 0·70 — 0·80 are located almost exactly on the graphs. The region with values above 0·70 — 0·80 corresponds to the initial stage of flow, when point A drops to A_o', Fig. 3.

Errors in depth, H/H_1, which are on ordinates, for a given time, are smaller than 10 per cent in 90 per cent of the tests and the maximum error from all tests is 14 per cent, which corresponds to a ratio $H/H_1 = 0{\cdot}9$. The function line of $F(H/H_1)$ is practically always above experimental points, which means that the function gives higher values of H than are actual values for the given time. This is on the safe side. The speed of lowering the phreatic line is given exactly by this function.

Taking H equal to the actual observed experimental values, the coordinates of the seepage line are plotted on Fig. 7. The encircled points are computed and the line itself is traced according to that observed in tests. Both figures represent the same dam with an upstream slope $m_1 = 3$ and a horizontal drainage with the drawdown beginning at $H_1/L = 0{\cdot}20$ and at 0·14. The depth functions $F(H/H_1)$ for the first two stages are ·504 and 6·64, respectively, and for the second two they are ·697 and 3·99. On the drawings the computed points are located exactly on the experimental seepage line; it is only a slight deviation from he line in the dowtream portion.

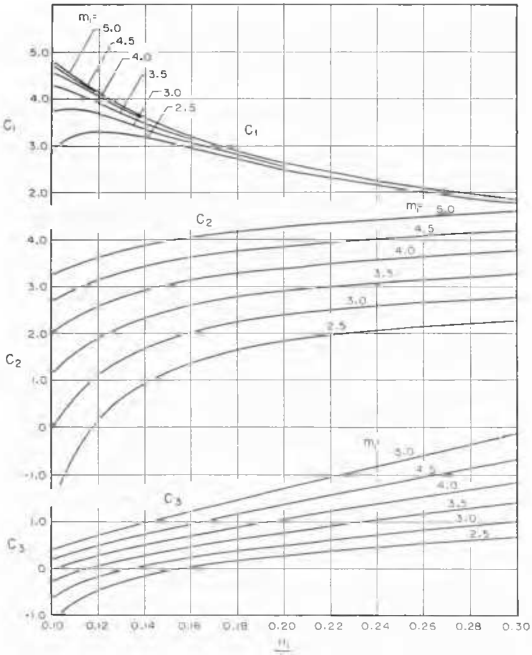

Fig. 5 Coefficients entering i ı the transient flow equation
Coefficients ae ı'équation de l'écoulement non permanent.

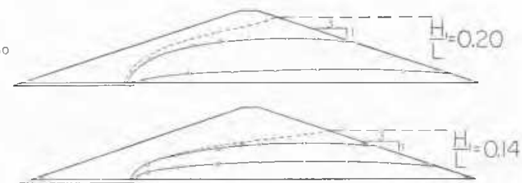

Fig. 7 Observed phreatic lines and points compared from the equation for these lines.

Nappes phréatiques observées et points calculés d'après l'équation de ces nappes.

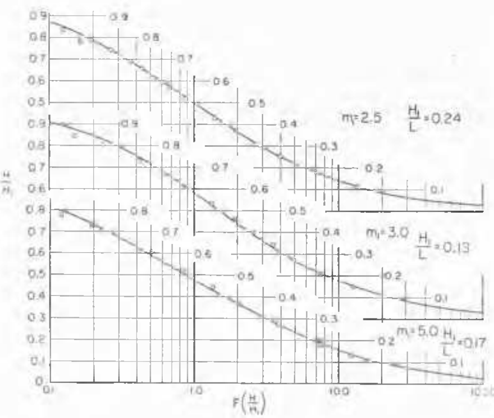

Fig. 6 Depth function compared from the equation of transient flow and points computed from experimental data.

Comparaison entre les profondeurs calculées suivants l'équation de l'écoulement non permanent et les point expérimentaux.

The experiments were conducted in the flume between parallel plates made of a plastic material with a space of 0·64 to 0·124 inches for different tests. The size of the dam base was from 30 to 80 inches, and the maximum head water was 7 1/2 inches. The liquid used was a silicone with kinematic viscosity of 0·011 cm. sec. at 25° C with an approximate change of viscosity of 0·002 cm sec· per 10° C. Using the curve viscosity versus temperature, a correction was introduced in the time computations, which was of over 10 per cent between some experiments. According to computations the capillary rise of the liquid in some experiments possibly attained a maximum of 2·5 mm. In most other cases it was smaller. Taking into account the size of the experimental dams, this was considered as tolerable.

Acknowledgement

This research was sponsored by the National Research Council of Canada and performed by the writer at Laval University. The treatment of experimental results was sponsored by Case Institute of Technology. It is the writer's pleasure to express his deep gratitude to these Institutions. The writer expresses his thanks to Mr. J.-C. Tremblay, P. Eng., for his help in conducting experiments.