Investigations on Electro-osmotic Flow in Soils in Relation to Different Characteristics

Étude sur le Rapport entre l'Écoulement Electroosmotique et les Différentes Caractéristiques des Sols

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Summary

The present paper contains results of the author's investigations on the dependence of velocity and efficiency of electro-osmotic flow in soils on several of their properties and characteristic values on applied electric current; also discussed is the phenomenon of 'critical moisture content'.

Sommaire

Le rapport contient les résultats des recherches effectuées par l'auteur sur le rapport entre la vitesse et le rendement de l'écoulement électroosmotique dans les sols et certaines proprietés des sols, ainsi que les valeurs caractéristiques du courant électrique appliqué. On discuté également le phénomène de la teneur en eau 'critique'.

Character of Investigations

In the last 5 years the author has been concerned with research into certain problems of electro-osmosis in soils relating to its employment for practical purposes (electro-osmotic draining) as well as for supplementing the theoretical studies of soils. The main emphasis was on explanation: how the kind and properties of soils, the characteristics of applied electric current and the hydrological conditions of the soil mass to be drained would influence the efficiency of applied electric energy and the velocity of electro-osmotic flow in soils.

In these investigations electro-osmometers were used with a horizontal axis similar to the apparatus described by several other scientists. A special feature made it possible to measure separately the quantities of water coming in at anode and out at cathode. The apparatus allowed measurements of flow to be made under the simultaneous action of a hydraulic head and electric potential and also determinations of the actual gradient of electric potential within samples of soils and the contact resistance between samples and electrodes. The average range of observed results were as follows:

for flow velocity measurements K_c $\pm$ 4·1 per cent
for flow efficiency measurements K_c $\pm$ 3·3 per cent

Model systems were also used with perforated tubular electrodes 30 cm long and 15 mm in diameter; the distance between electrodes varied to 150 cm. The investigations were made by applying current density ranging from 0·3 to 10·0 mA cm^{-2} on 247 samples of 21 various soils with a content of clay fraction (< 0·002 mm) within limits of 3 to 73 per cent and of an activity (Skempton) of 0·62 to 1·54. The degree of saturation S_r varied within the limits of 0·95 to 0·99.

Altogether 3550 velocity and flow efficiency observations were made, including 226 model measurements. More exact descriptions of methods used and some observed results were published in 1954 (PIASKOWSKI, 1954).

Efficiency of Electric Energy

The efficiency is defined and measured by the quantity of electricity in coulombs flowing through the sample of soil during a period of time required for electro-osmotic flow of 1 cm^3 of water. It can be called 'coulometric' coefficient K_c, which is equal to the relation of the amount of electric current to the amount of water. Thus, assuming that the flow of water does not affect electric conductivity of the soil sample, coefficient K_c is expressed:

$$K_c = \frac{\lambda . E}{H_e . E \pm K_w . i} \quad \ldots . \text{(1a)}$$

where: K_c = coulometric coefficient in coulombs cm^{-3}; λ = specific conductivity of soil in ohm^{-1} cm^{-1}; E = virtual gradient of electric potential within the soil sample in volts cm^{-1}; K_e = coefficient of electro-osmotic water permeability of soil in cm^2 sec^{-1} $volt^{-1}$; k_w = Darcy's coefficient in cm sec^{-1}; and i = hydraulic gradient.

The symbol of plus in the denominator of the equation 1a refers to the case of conformity of the direction of both flows, and the symbol of minus is applicable to the reversed case. In this equation the principle given by SCHAAD and HAEFELI (1947) was adopted which consists in the algebraic addition of velocity of both kinds of flow.

If the component of hydraulic flow $k_w i$ is equal to zero, then:

$$K_c = \frac{\lambda}{K_e} \quad \ldots . \text{(1b)}$$

The formula 1b has been experimentally proved for 16 various soils with a different liquidity index. In two cases only, a discrepancy between the calculated and experimental value was fairly significant (18 to 26 per cent) for unknown reasons; in the remaining 14 cases, on the contrary, the mean discrepancy reached only $\pm$ 2·2 per cent.

Having at our disposal the theoretically defined and experimentally confirmed coefficient K_c it is possible to compute the consumption of electric energy in kWh for a given electro-osmotic flow. Assuming that in a case when the consumption was equal to N kWh (with potential difference between electrodes of U volts) and a total flow of Q litres of water was obtained, then the balance of the electric quantity will have the following form:

$$3600.1000.N.\frac{1}{U} = 1000.K_c.Q$$

from which:

$$N = \frac{K_c}{3600}.Q.U = K_t.Q.U \quad \ldots . \text{(2)}$$

The quotient $K_c/3600$ was separated out as a 'technical' coefficient K_t, which denotes the consumption of electric energy in kWh necessary for displacing 1 litre of water with a potential difference between the electrodes equal to 1 V. The values of K_t obtained for 21 various soils of liquidity index over 0·20 were within the range: 0·0032 to 0·0579 kWh/litre V. These values can be compared with results of PRAUSNITZ's (1931) investigations carried out with other materials.

The above considerations show that high tensions are uneconomical, as in electrolytic processes, because the cost depends on energy consumption, and the produced effect depends on electric quantity flowing through the soil, in other words with the current's amperage and the duration of its flow. In this way, using the coefficient K_c, it is possible to compute the discharge of electro-osmotic flow of water to the cathode:

$$Q = \frac{3600 . J}{1000 . K_c} = 3 \cdot 6 \frac{J}{K_c} \text{ litres/hour} \quad \ldots . (3)$$

In this equation J denotes the amperage of the current. Formula 3 was applied to compare discharges of water with those obtained by the methods of SCHAAD and HAEFELI (1947). Results agreed well, but the formula 3 is simpler and easier to apply because it does not include the value K_e which is difficult to determine, whereas it includes the coulometric coefficient K_c which can be easily obtained.

The considerable consumption of electric energy during the

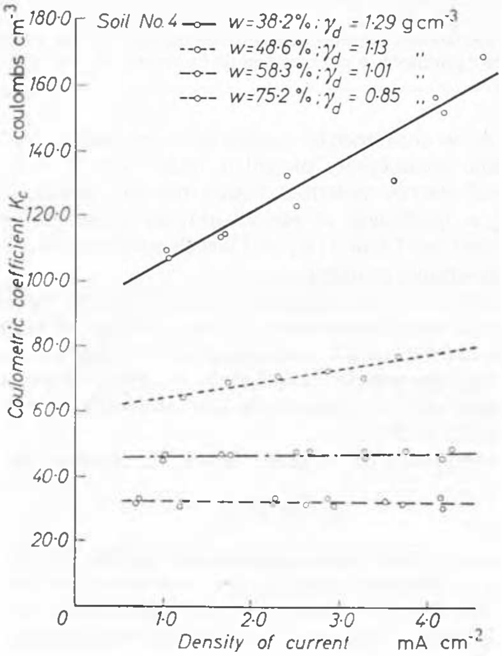


Fig. 1

electro-osmotic flow or drainage depends on a relatively high concentration of exchangeable cations in ground water. It can be seen, therefore, that electric energy consumption depends on: (a) content of clay fraction ($< 0 \cdot 002$ mm) in soils, and (b) their mineralogical character (CUMIENSKI, 1953). Some investigations carried out to find numerical relations on this matter confirmed some observations of MACLEAN and ROLFE (1945).

It was assumed in formulae 1–3 that K_c does not depend on the density of electric current or potential gradient which conforms to the Helmholts–Perrin–Smoluchowski principle and Ohm principle. In fact this assumption is only specifically true for soils of very high moisture content, and possessing the larger clay fractions; on the other hand, in most cases, it was determined that K_c is a linear function of current density S and can be expressed by equation:

$$K_c = K_c'[1 + \alpha(s - 1)]$$

where: K_c = the value of coulometric coefficient for current density s = mA cm^{-2}; K_c' = the above value but for current density 1 mA cm^{-2}; and α = the empirical factor expressed in a fraction of K_c' value.

An example of the above correlation is shown in Fig. 1. The factor α has a pronounced dependence (shown in Fig. 2) on volumetric moisture of soils ($w\gamma_d$), where w is the moisture content and γ_d dry density; from Fig. 2 it follows that α is equal to zero at a certain value of ($w\gamma_d$) which is different for various soils.

The linear relation, 4, is valid up to certain limits because

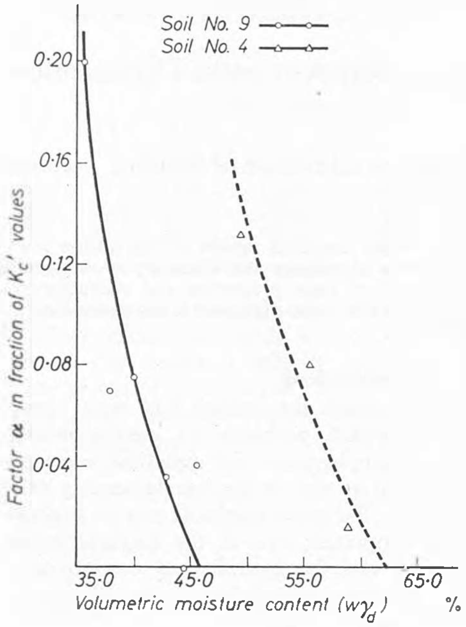


Fig. 2

over some current density coefficient K_c begins to increase rapidly (Fig. 3) tending towards infinity (in order to observe this phenomenon a bigger density of current is required for a larger soil moisture content). With a further increase of the current density sometimes a reversal was even observed of the flow direction associated with a gradual decrease of the K_c

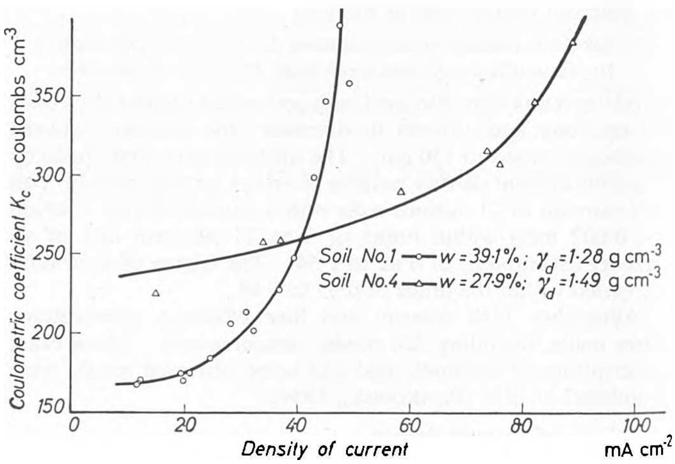


Fig. 3

values. The mentioned increase of K_c must be explained by the decrease of K_e since the author proved that λ over the whole range of the current density up to 10 mA cm^{-2} agrees with the Ohm principle maintaining a constant value. Only for small current density the resistance measured in soil samples increased a little.

It is important for practical purposes to know the dependence

Table 1

Soil number	Specific gravity G_s g cm^{-3}	Fraction content in %			Consistency limits		PI	Thixotropic moisture content W_t in %	Base exchange capacity in m.e./100 g of a dry soil mass	Electro-osmotic properties		
		0·05–2·0 mm	0·002–0·05 mm	< 0·002 mm (clay fraction)	LL	PL				Constants A (for S = 2·5 mA cm^{-2})	Constants B (for S = 2·5 mA cm^{-2})	The range of W_k % (for S = 2·5 mA cm^{-2})
10	2·68	23	57	20	36·4	15·2	21·2	109	12·7	748	0·0296	13–15
9	2·71	24	50	26	44·3	15·6	28·7	131	16·0	3,070	0·0389	19–21
19	2·68	21	47	32	48·6	16·2	32·4	125	17·3	1,500	0·0275	19–21
4	2·76	8	45	47	68·9	27·2	41·7	175	31·7	10,350	0·0390	25–29
21	2·78	7	41	52	83·7	22·2	61·5	192	30·8	5,870	0·0311	30–32
1	2·74	3	39	58	101·0	29·5	71·5	247	37·2	9,100	0·0337	29–31
26	2·71	8	25	67	104·6	31·4	73·2	303	—	8,700	0·0310	38–40
25	2·72	5	22	73	103·8	26·4	77·4	304	41·7	7,700	0·0333	39–41

Notes:—(a) Thixotropic moisture content W_t has been determined by 'reversal of test tubes' method 11. Diameter of test tubes 15 mm, the duration of thixotropic regeneration 5 min.
(b) The base exchange capacity has been determined using 1 N solution of barium chloride.

between K_c and soil moisture content (as shown in Fig. 4). This dependence can be shown in the following form:

$$\log_{10} K_c = \log_{10} A - B(w\gamma_d) \quad \ldots\ (5)$$

where: A, B are empirical constants (Table 1) unaffected by the moisture content of analysed samples but dependent on their physicochemical properties, on the density of applied current, and on properties of ground water (its specific conductivity), and $(w\gamma_d)$ = volumetric moisture content of soil per cent.

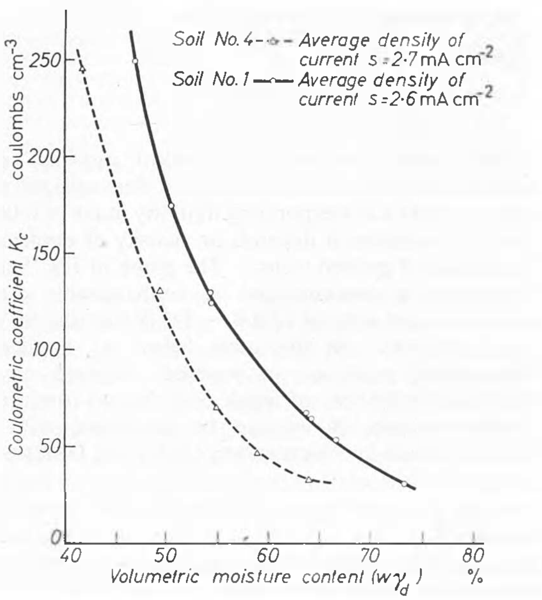


Fig. 4

The average discrepancy between K_c obtained from formula 5 and that determined by experiments was ± 3·5 per cent.

The last introduced constants A and B make it easy to compute the minimum of required electric energy consumption and the minimum space of time necessary for draining a given mass of ground under certain conditions (Piaskowski, 1956). These relations can be computed by using formulae 2, 3 and 5.

Coefficient of the Electro-osmotic Water Permeability of Soils K_e

According to the Helmholtz–Perrin–Smoluchowski principle the velocity of electro-osmotic flow should be directly proportional to the gradient of electric potential E. At the same time, the coefficient Ke should be independent of E. The values of coefficient K_e which were determined experimentally by the author reached the range up to 3·4 × 10^{-5} cm^2 sec^{-1}, which corresponds with results obtained by other investigators (Schaad and Haefeli, 1947; Bridgewater, 1950; Winterkorn, 1947), but represents smaller values than those given by Casagrande (1948). Moreover it was confirmed by the author by means of investigations carried out on soil samples with identical liquidity index, LI = 0·31 – 0·32, that K_e cannot

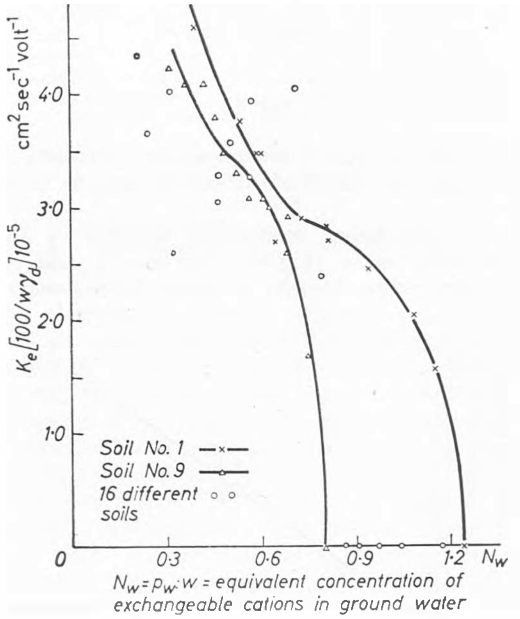


Fig. 5

depend decisively on granulometric composition, as can be seen on comparison:

	a	b
Percentage of clay fraction content	8–19	33–58
As above, mean percentage	13	43
Mean value of K_e	1·20 × 10^{-5}	1·53 × 10^{-5}

The ratio of K_e for these soils amounts to only 1·27, while that of Darcy's coefficient K_w would likely reach 100 or even 1000.

The values for $K_e 100/(w\gamma_d)$, determining the mean velocity of electro-osmotic flow in soil pores, show a notable dependence on equivalent concentration of exchangeable cations $N_w = p_w : w$ (where p_w is the exchange capacity in m.e./100 g of dry soil mass and w the moisture content in per cent) in ground water, as can be seen on Fig. 5 worked out assuming that the degree of dissociation of exchangeable cations is equal to one. Fig. 5 gives some confirmation of the theoretical approach.

A full applicability of the Helmholtz–Perrin–Smoluchowski principle has been confirmed only for soil samples whose moisture content exceeded certain limiting moistures (Fig. 2) when coefficient $\alpha = 0$ (formula 5). For different cases by combining the formulae 1b and 5 the following relation was obtained:

$$v_E = \frac{\lambda E}{K'_c[1 + \alpha(1000\lambda . E - 1)]} \qquad \ldots . (7)$$

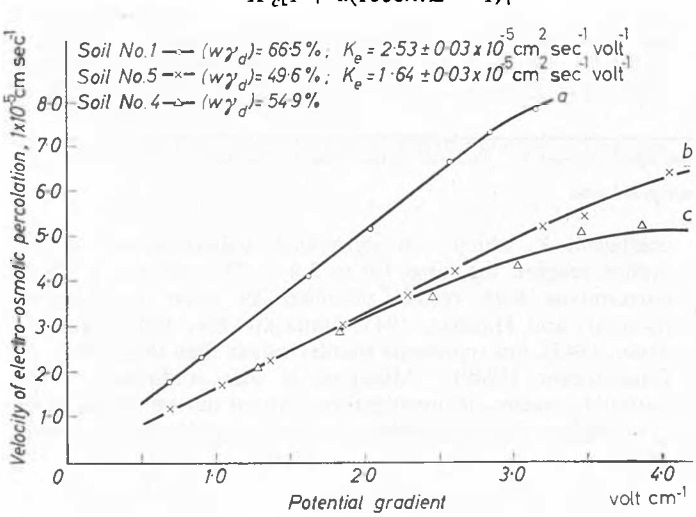


Fig. 6

where v_E is the velocity of electro-osmotic percolation in cm sec^{-1} and other denominations being the same as in formulae 1a, b and 5.

The above mentioned relationship between v_E and E is illustrated by the line 'c' (Fig. 6). The lines 'a' and 'b' represent examples, where initially a linear dependence may be

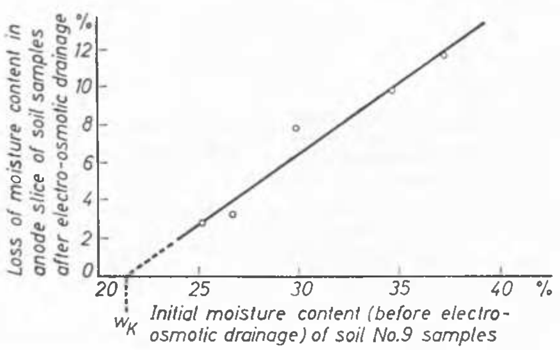


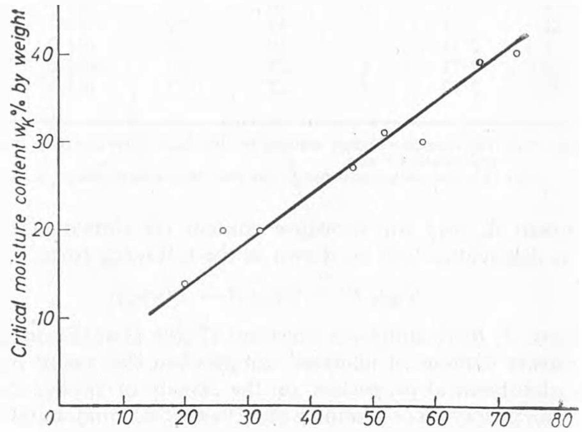
Fig. 7

observed, but afterwards the velocity of the flow gradually decreases and value K_e is diminishing.

Phenomenon of 'Critical' Moisture Content

Investigations carried out expose two limits of moisture content which are characteristic for the electro-osmotic properties of soils: (a) the moisture content at which the factor α becomes equal to zero (Fig. 2); and (b) 'critical' low moisture content w_k (or rather a certain range of moisture content) at which K_c tends to infinitely high values and consequently any electro-osmotic flow is impeded.

The 'critical' moisture content was determined in two ways: (a) as a limit to which $(w\gamma_d)$ is tending when K_c tends to infinity (Fig. 4)—it should be noted here that at values approaching this moisture content the coefficient K_c increases at a greater rate than that given by formula 6; (b) as a moisture content at which (Fig. 7) the loss of water through drainage by electro-osmosis in certain cases becomes equal to zero.



Percentage of clay fraction (<0·002) content
Activity of soils 0·89—1·23, average 1·08
The average density of current applied s=2·0—2·7mA cm^{-2}(mean 2·5mA cm^{-2})

Fig. 8

As can be seen from Fig. 8 the 'critical' moisture content is a function of clay fraction percentage. The 'critical' moisture content occurs at a corresponding liquidity index of 0·06 —0·18 (Table 1). Moreover it depends on density of electric current and properties of ground water. The graph of Fig. 5 shows, in addition, that a concentration of exchangeable cations in ground water over a range of 0·8 — 1·2 N is a sign of 'critical' moisture contents. At moistures below w_k application of electric current produces, in practice, drainage of soil by evaporation, electrolysis of water and thermo-osmotic migration. Phenomenon of w_k can be compared with certain observations made by Winterkorn (1947) and Dawson (1948).

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