Rheological Processes in Frozen Soils and Dense Clays

Procès Rhéologiques dans les Sols Gelés et les Argiles Denses

by S. S. Vialov, Candidate in Technical Sciences, Senior Research Worker, Frozen Ground Research (Merzlotovedenija) Institute of the U.S.S.R. Academy of Sciences, Moscow
and
A. M. Skibitsky, Civil Engineer, Research Division, Hydroproject, Moscow, U.S.S.R.

Summary

This report deals with the rheological laws of frozen soils and unfrozen dense clays. The laws were established experimentally. A hypothesis is given concerning the physical nature of the deformation of frozen soils and relationships are established between the stress, deformation and time for such soils; the condition for their limiting equilibrium is also considered. The creep of dense clays in shear is described and the deformation law of such soils in time is established.

Sommaire

Dans ce rapport on considère l'application des lois rhéologiques aux sols gelés et aux argiles denses. Ces lois ont été établies d'aprés les resultats de recherches expérimentales. On propose une hypothèse qui explique la nature physique de la déformation des sols gelés et on établit le rapport entre la tension, la déformation et le temps pour ces sols; on considère aussi la condition de leur équilibre limite.

Dans la seconde partie de ce rapport on décrit le procès de fluage pour l'argile dense et on établit la loi régissant sa déformation dans le temps.

Plastic soils, as is known, are capable of changing their stress-strained state in time. Continuous loads in such soils lead to creep and loss of strength. The present report deals with the laws governing these rheological processes as applied to frozen soils and to unfrozen dense clays. The experimental investigations of frozen soils described in §§ 1, 2 and 3 of the report (by S. S. Vialov) were carried out at the Frozen Ground Research (Merzlotovedenija) Institute of the U.S.S.R. Academy of Sciences, while those of dense clays, described in §4 (by A. M. Skibitsky), were carried out at the Research Division of Hydroproject.

(1) Strain Characteristics of Frozen Soils

Investigations of recent years (N. Tsytovich, M. Goldstein, V. Berezantsev, S. Vialov) have shown that rheological processes are specially intense in frozen soils, due to the presence of water in the solid and liquid phases.

Very many experiments (over 1000) were carried out to study these processes, including shear in a shearing apparatus, shear along rods frozen into soil, rupture, compression, and indentation by spherical and flat stamps. The experiments were carried out with various soils, from light sandy loams to heavy clays, at different temperatures, which were, however, kept constant during each experiment.

The results of some of the tests for creep in shear (of frozen in rods) are given in Fig. 1a, which gives curves of the deformation, γ, taking place in time, t, under constant stresses, τ. It can be seen that depending on the value of τ the deformations are of a damping or non-damping nature. Deformations of the latter type first develop at a decelerating rate, passing afterwards into a stage of steady creep characterized by a constant rate of deformation (plastic-viscous flow), represented by the linear section of the γ/τ curve. Further development of creep leads inevitably to a transition to the failure stage. The higher the stress, the sooner failure will occur. The relation between the failure stress and the time needed for failure to occur is shown in Fig. 1 b, where τ_0 is the instantaneous, and τ_∞ the continuous shear strength. With $\tau < \tau_\infty$ the strain practically reaches a limit and failure does not occur; with $\tau > \tau_\infty$ a non-damping deformation arises, resulting in failure.

Although the classification of deformations as damping and non-damping is somewhat relative, slow flow not being impossible even with $\tau < \tau_\infty$, the ultimate continuous strength may be regarded as a real characteristic since such slow flow is of very little practical importance. Moreover, this characteristic should be accepted as the relevant one in calculating the continuous load-carrying capacity of soils.

The reality of the ultimate continuous strength was confirmed by rupture tests, in which all loads σ, greater than $0{\cdot}085\ \sigma_0$ (where σ_0 is the ultimate instantaneous strength) resulted in failure; the load $\sigma = 0{\cdot}085\ \sigma_0$, however, did not result in failure, though the sample was under test for over 4 years (Table 1).

Table 1

σ (*in % of* σ_0)	100	50	30	25	20	13	10	8·5
Time before failure	9 sec	3 min	5 hours	4 hours	24 hours	140 hours	766 hours	*No failure*

It is characteristic that the process of relaxation takes quite a long time, but the main drop in strength occurs during the initial comparatively short period.

The following hypothesis (Vialov, 1955) is given to explain the physical nature of the rheological processes in frozen soils.

According to Tsytovich and others (1937, 1954, 1955, 1956) the strength of these soils depends on their inner bonds, i.e. on the forces of cohesion between the soil components, viz. the mineral particles, the ice and the unfrozen (bound) water.

In unfrozen soils, cohesion consists (according to N. Denisov) of: (1) intermolecular cohesion, due to the forces of attraction between solid particles separated by films of bound water and depending on the density of the soil, and (2) structural cohesion, which is a result of various processes taking place in the soil during its formation and is eliminated when the structure of the soil is disturbed. In frozen soils a third type of cohesion is most important, namely, cohesion due to ice cementation. This component of cohesion is the least stable, varying as it does in accordance with soil temperature fluctuations, and disappearing when the soil thaws.

A load applied to a frozen soil leads (Tsytovich and Sumgin, 1937) to a concentration of stresses at the points of contact between the solid particles, resulting in a disturbance of the equilibrium between the film water and the ice, and in the melting of the latter. Under the action of the stress gradient, the film moisture, which increases due to the thawing of the ice, migrates to a zone of lower stress, where it freezes again. This is accompanied by a squeezing out of the air and plastic flow of the ice. These phenomena have been established (Vialov, 1955) experimentally (Fig. 2). The melting and flow of ice is

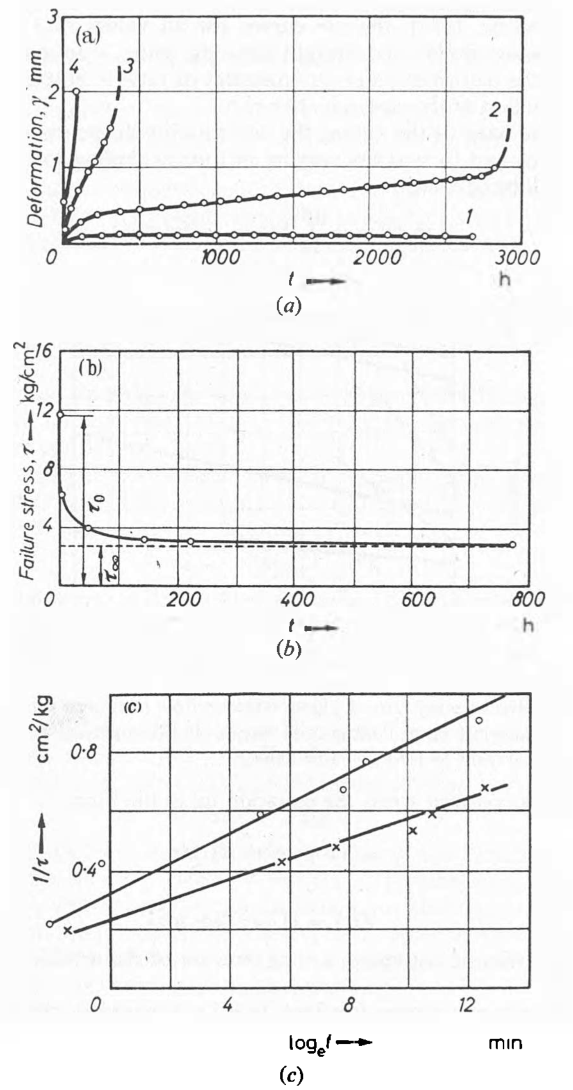

Fig. 1 Creep curves for frozen soil (a) and continuous strength curves in ordinary (b) and logarithmic (c) coordinates

Courbes de fluage du sol gelé (a); courbes de résistance; coordonnées ordinaires (b) et coordonnées logarithmiques (c)

accompanied by a disturbance of the structural bonds of the soil, i.e. by a weakening of the structural and ice-cementational cohesion. At the same time the molecular cohesion increases due to the closer packing of the solid particles, and the ice-cementational bonds are renewed as a result of reconstruction of the ice structure.

If the strengthening of the bonds predominates over their weakening, the deformation is of a damping nature. If, however, the weakening is not compensated for by strengthening, a non-damping deformation develops, resulting in failure.

Rheological processes in soils, as experimental data show (Vialov, 1955), do not obey the laws established for ideal elasto-plastic viscous bodies, particularly Kelvin's law of after-effect and Maxwell's law of relaxation. This discrepancy emanates from the very physical nature of the above phenomena and is due to volume changes in the soil and to variability of its rheological properties. For instance, the variability of the relaxation period t_r is evident from the fact that when plotted to a semi-logarithmic scale the experimental points clearly fall on a curve, while according to the relaxation equation expressed as

$$\log_e \frac{\tau_0 - \tau_\infty}{\tau - \tau_\infty} = \frac{t}{t_r}$$

these points should form a straight line.

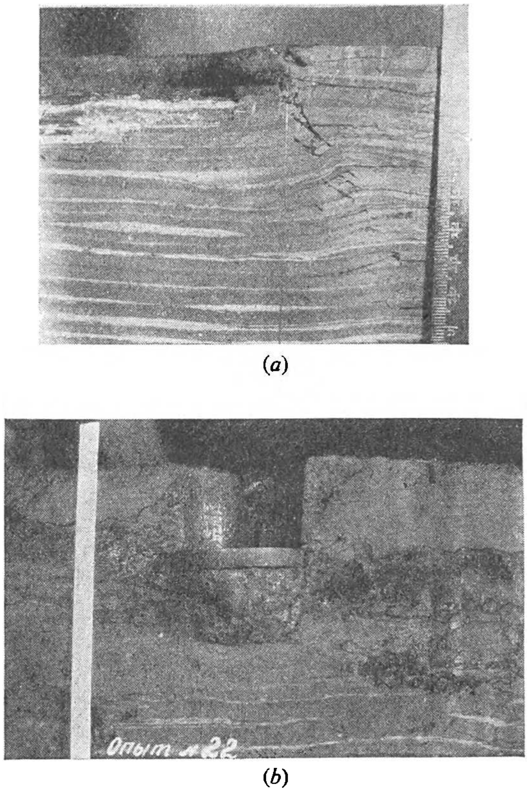

Fig. 2 Pressing a plate into frozen soil: (a) visible ice inclusions formed at the boundary of the stressed zone as a result of melting and squeezing out of the ice; (b) puncture of the ice layer by the dense core

Pénétration du coin dans le sol gelé: (a) inclusions de glace visibles formées à la limite de la zône contrainte en conséquence de la fusion et du serrage de la glace; (b) enfoncement d'une couche glaciale par le noyau consolidé

(2) Relation between the Stress, Strain and Rate of Flow of Frozen Soils

The deformation of frozen soils at a given moment of time is the sum of the recoverable and unrecoverable parts (Fig. 3a). The former is directly proportional to the stress; the latter is related to the stress by a power dependence (Vialov, 1956).

$$\gamma = \frac{\tau}{G(t)} + \left[\frac{\tau}{A'(t)}\right]^{1/\alpha'} \quad \ldots. \text{(1)}$$

Since both the unrecoverable and the recoverable deformations develop in time, the shape of the γ/t curves will also depend on the duration (Δt) of the loads (Fig. 3a), which is reflected in the variability in time of the parameters A' and G: with $0 < t < \infty$, $A_0 > A'(t) > A_\infty$ and $G_0 > G(t) > G_\infty$.

The effect of the recoverable deformation on the shape of the total deformation curve is relatively small, and therefore it may be accepted that

$$\tau = A(t)\gamma^{\alpha} \quad \ldots . (2)$$

where the coefficients A and α differ numerically from A' and α' in equation 1.

For a number of soils (especially at low temperatures) $\alpha \fallingdotseq 1$ and the equation becomes $\tau = G_{\Sigma}(t)\gamma$, where $G_{\Sigma} = G + A$.

The truth of equation 2 is confirmed by the fact that the curves straighten out if plotted in logarithmic coordinates (Fig. 3b). However, with stresses exceeding the yield point τ_s this relation loses its validity, as is proved by the bend in the γ/τ curves in the logarithmic graph. It can be seen from Fig. 3b that each t has its own point of inflection; hence, the yield point is a variable depending on the time of loading or action. The limit values are $\lim_{t \to 0} \tau_s = \tau_0$ and $\lim_{t \to \infty} \tau_s = \tau_{\infty}$.

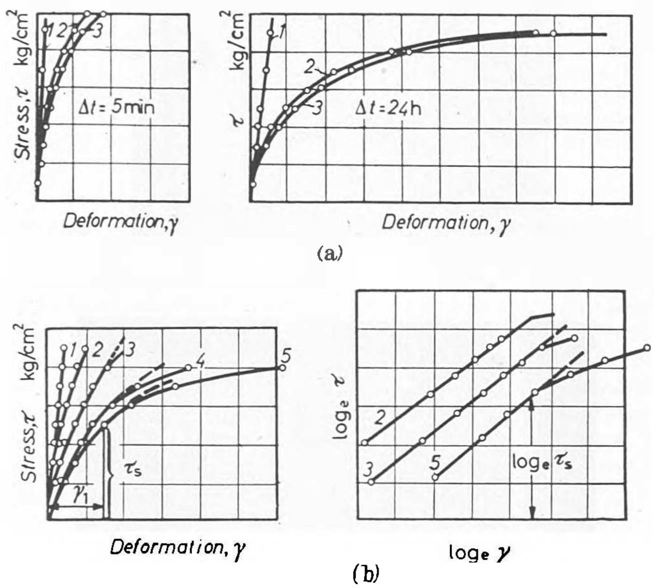

Fig. 3 Stress *versus* deformation: (a) division of the total deformation (*3*) into recoverable (*1*) and unrecoverable (*2*) deformations; (b) stress/strain curves with Δt equal to: (*1*) 1 min, (*2*) 1 h, (*3*) 24 h, (*4*) 120 h, (*5*) 240 h

Rapport entre l'effort et la déformation: (a) division de la déformation totale en déformation irréversible et la deformation réversible; (b) courbes effort/déformation pour Δt égal à: (*1*) 1 min, (*2*) 1h, (*3*) 24 h, (*4*) 120 h, (*5*) 240 h

For $\tau > \tau_{\infty}$ plastic viscous flow occurs; the relation between the rate of this flow and the stress (within a definite range of the latter) is given (VIALOV, 1956) by the equation

$$\frac{d\gamma}{dt} = \frac{1}{\eta}(\tau - \tau_{\infty})^{\beta} \quad \ldots . (3)$$

For plastic soils the parameter $\beta \fallingdotseq 1$, and then equation 3 becomes the Bingham–Schwedoff equation (Fig. 4).

Returning to Fig. 3b, it should be noted that the points of inflection of the γ/τ curves for all t lie on a common vertical. This means that the flow is initiated not by the magnitude of the stress, inasmuch as $\tau_s = f(t)$, but by the deformation reaching a certain critical value γ_1 not depending on the time factor. Taking into consideration that development of the flow leads to failure, the above condition $\gamma = \gamma_1$ = constant may be regarded as the first criterion of strength.

The second criterion of strength, which characterizes the beginning of the failure stage, is the attainment by the deformation of a new limit value $\gamma = \gamma_2$ = constant.

(3) Deformation and Strength Loss of Frozen Soils upon Continuous Shearing

The development of deformation with time may be described by the integral equation of Volterra–Boltzman as interpreted by Y. Rabotnov and M. Rosovsky, which makes it possible to take into account the non-linearity of the γ/τ relation and the variability of the relaxation time.

A damping deformation is the sum of the instantaneous deformation and that developing in time. From Fig. 3b it follows that the instantaneous deformation obeys Hooke's law, while that developing in time obeys the power law (TSYTOVICH and SUMGIN, 1937), the γ/τ curves for all values of t except $t \to 0$ being similar (the straight lines $\log_e \gamma / \log_e \tau$ are parallel). Hence the deformation at any moment of time is proportional to a function of the stress $\phi(\tau) = \tau^{1/\alpha}$.

On the basis of the above, the deformation at the moment of time t, caused by a stress varying in time according to the law $\tau(z)$, will be equal to

$$\gamma = \tau/G_0 + \int_0^t K(t - z)\phi[\tau(z)]dz \quad \ldots . (4)$$

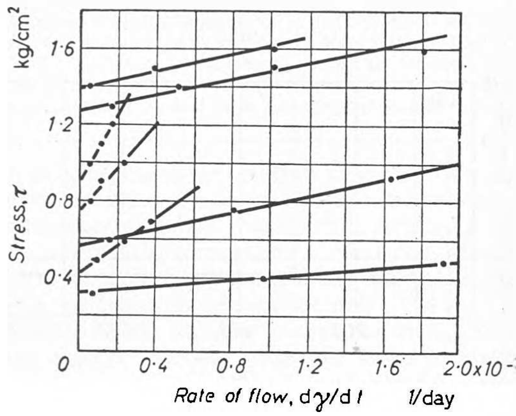

Fig. 4 Stress *versus* rate of plastic-viscous flow in frozen soils
Rapport entre l'effort et la vitesse de l'écoulement plastique-visqueux pour les sols gelés

With a constant stress the equation takes the form

$$\gamma = \gamma_0 + \tau^{1/\alpha}\int_0^t K(t)dt \quad \ldots . (5)$$

where

$$K(t) = [1/\phi(\tau)](d\gamma/dt)$$

is a function of the creep, i.e. of the rate of deformation with $\tau = 1$.

For soils the above function is of a hyperbolic character. Its simplest form is (after Boltzmann): $K(t) = B/t$. As will be shown in §4, this relation also describes well the time deformation of dense unfrozen clay. For frozen soils a more general expression may be employed

$$K(t) = Be^{-nt}/t^{1-m}$$

where B, m, n are parameters equal to $m = 0{\cdot}3$ and $n = 0{\cdot}2\ \text{hr}^{-1}$ for one of the soils investigated (sandy loam at a temperature $\theta = -0{\cdot}4°$ C.).

A comparison of the experimental data with the results of calculations made by the formula accepted for the experiment mentioned is given in Table 2.

Non-damping deformation can be expressed by means of equation 4 if we add a term for the plastic-viscous flow.

When τ = constant we get

$$\gamma = \gamma_0 + \tau^{1/\alpha}\int_0^{t_1} K(t)dt + \frac{1}{\eta}(\tau - \tau_{\infty})^{\beta}(t - t_1) \quad \ldots . (6)$$

where t_1 is the time required for the deformation to attain its

Table 2
Rate of settlement (V mm/hours) of a loaded plate
Vitesse de tassement (V mm/heure) d'une plaque chargée

	t_E hours								
	0·25	0·51	1·0	2·0	3·0	4·0	5·0	6·0	7·0
Theory	0·223	0·132	0·073	0·037	0·023	0·015	0·010	0·008	0·006
Experiment	0·250	0·140	0·075	0·035	0·022	0·015	0·013	0·011	0·008

limiting value γ_1; this time is as high as 100 to 500 hours for frozen soils.

For a number of cases it may be assumed that $\alpha \doteqdot \beta \doteqdot 1$.

Now let us consider the law governing the strength loss of frozen soils. From Fig. 1c it can be seen that continuous strength curves straighten out if plotted in the form $1/\tau$ against $\log_e t$. Hence, the relation between the failure stress and the duration of its action can be expressed by the formula

$$\tau^*/\tau = \log_e t/t^* \quad \ldots . \; (7)$$

where τ^* and t^* are parameters, and for one of the examples in question (Fig. 1c) $\tau^* = 31{\cdot}9$ kg/cm² and $t^* = 6{\cdot}5 \times 10^{-6}$ hours.

It should be noted that the above relation (of Glathard and Preston) corresponds (NADA, 1950) to the second strength criterion considered above.

The ultimate continuous strength corresponds to a certain sufficiently large time value t_n, after which the loss in strength is so small that it may be neglected in practice.

$$\tau_\infty = \tau^*/\log_e (t_n/t^*)$$

For instance, in the above example (Fig. 1c) the experiment gave $\tau_\infty = 1{\cdot}35$ kg/cm², which corresponds to $t_n = 115{,}000$ hours. If we take $t_n = 50$ or 100 years, the value of τ falls only to $\tau_{50} = 1{\cdot}28$ and $\tau_{100} = 1{\cdot}26$ kg/cm² respectively.

The condition of limiting equilibrium of frozen soils, which determines their stability under local load, is characterized (TSYTOVICH *et al.*, 1954, 1955, 1956) by the relation

$$\tau_t = c_t + \sigma \tan \phi_t \quad \ldots . \; (8)$$

which differs from the Coulomb equation in the variability of the parameters $c = f_1(t)$ and $\phi = f_2(t)$. The maximum values, c_0 and ϕ_0, correspond to the instantaneous shearing strength, and the minimum values, c_∞ and ϕ_∞, to the continuous shearing strength. It is the latter strength parameters that determine the stability of frozen soils under local load.

Experiments have shown that the failure of frozen soils when a plate is pressed into them is not due to upheaval of the soil, but to its being pushed apart by the densified core under the plate (Fig. 2b) and to the appearance of plastic viscous flow. The load which causes this is the ultimate load. Its value can be determined (VIALOV and TSYTOVICH, 1956) by the formulae of the theory of plasticity if allowance is made for the relaxation properties of soils*.

(4) Shear Deformation of Unfrozen Dense Clay with Time

Rheological processes, as is known, take place also in unfrozen plastic soils. In this case investigations show that the type of deformation varies appreciably, depending on the soil density.

For weak clay with a high moisture content, the main type of deformation is plastic-viscous flow and, according to GEUZE (1953), this stage is observed as early as five hours after application of the load. In the case of dense clay, however, which has a much higher cohesion and contains water only in a bound state, experiments which will be described below have shown a different relationship to exist. Creep at a decreasing rate, which is fairly important in frozen soils, is a factor of considerable significance in unfrozen dense clay.

Direct shear tests were made on Tertiary clay of natural structure taken from a depth of about 50 m; the thickness of overlying formations was as large as 200 m in the geological past.

The clay was of the beidellite–montmorillonite type with Ca predominating among the absorbed cations. The content of chlorides and sulphates was up to 0·6 per cent, carbonates up to 0·5 per cent, and amorphous silicic acid about 1 per cent. The granulometric composition was: sand up to 1 per cent, silt 37 per cent, clay 62 per cent. Up to 70 per cent of the clay component consists of colloidal particles (diameter less than 0·22 micron). The moisture content of the clay in its natural condition was $w = 34{\cdot}0 \pm 1$ per cent; $LL = 73{\cdot}5$ per cent; $PL = 34{\cdot}5$ per cent. The degree of water saturation $S_n = 0{\cdot}98$.

The shear stresses used in the experiments were 0·3, 0·5, 0·7 and 0·9 of the standard strength τ_f of the clay as measured by conventional short duration tests, with an average shear load increment time of about 1 hour. With a normal stress of 7 kg/cm² the average strength was 2·75 kg/cm².

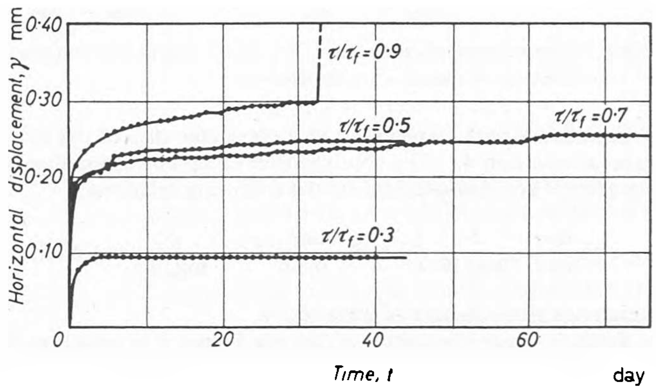

Fig. 5 Creep curves of dense (unfrozen) clays
Courbes de fluage pour d'argiles denses, non gelées

The graph in Fig. 5 shows the observed displacements with time. The displacements were measured from the state of deformation reached at the end of the increment period of the shearing load.

In the course of the 2·5 months' observations shown on the graph the deformation of the test specimens either stabilized ($0{\cdot}3\,\tau_f$, $0{\cdot}5\,\tau_f$ and $0{\cdot}7\,\tau_f$) or resulted in failure ($0{\cdot}9\,\tau_f$).

The total displacement attained at the end of the experiment for $0{\cdot}7\,\tau_f$, including the displacement during the time of applying the shear load, was 0·735 mm, compared to 1·470 mm in the experiment with $0{\cdot}9\,\tau_f$, which caused failure of the sample. Failure was observed at $0{\cdot}9\,\tau_f$ in the other Tertiary clays which have been investigated.

The results of the experiment show, therefore, that there exists a certain shearing stress $\tau_\infty < \tau_f$, which, under given conditions of loading, may be called the ultimate continuous resistance of dense clay. With $\tau < \tau_\infty$ the deformation stabilizes and with $\tau > \tau_\infty$ the deformation leads to failure.

It is a characteristic feature that both stabilization of the deformation and failure of the specimens were preceded alike by decelerated creep which did not change into continuous flow.

If the creep curves obtained are plotted on a semi-logarithmic coordinate system (Fig. 6), it can easily be seen that they are quite satisfactorily described by an equation of the type

$$\gamma = a + b \log_{10} t \quad \ldots . \; (9)$$

* For more details see report by N. A. TSYTOVICH in this volume.

i.e. they obey the logarithmic relationship established by Keverling Buisman's 'secular law' (1936).

Fig. 6 also shows the vertical settlement observed in the specimens studied. It can be seen that these settlements were insignificant and stabilized quickly. Hence it is concluded that the settlements have little effect on the horizontal displacements.

The latter circumstance, as well as the failure of the dense clay specimens in the process of deformation described by equation 9, justify the extension of Buisman's law to shearing deformations in the absence of densification. Such an extension was hinted at by Geuze (1948).

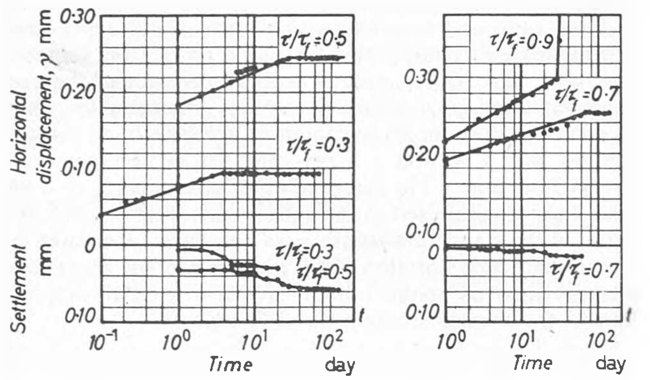

Fig. 6 Creep curves of dense clays in γ *versus* $\log_{10} t$ coordinates
Courbes de fluage d'argiles denses

Equation 9 may be regarded as a particular case of the more general equation 4. The conditions of creep corresponding to equation 9 are characterized by the following relations:

$$v = \frac{d\gamma}{dt} = \frac{b}{\log_e 10} \cdot \frac{1}{t} \quad \text{and} \quad \frac{v^2}{dv/dt} = -\frac{b}{\log_e 10} = \text{Const.}$$

which describe the core of equation 4.

Table 3 shows the values of the coefficient b in equation 9, according to the results of our experiments.

Table 3

τ/τ_f	b	τ/τ_f	b
0·3	35μ	0·7	35μ
0·5	40μ	0·9	55μ

The data of this table do not confirm the idea of the proportionality of the coefficient b to the applied load, and the question of the nature of this coefficient requires further study.

It should be mentioned that in their time Pokrovsky and Nekrasov (1934) suggested an equation of the form: $\epsilon = \mu \log_e (\rho t + 1)$ to describe secular settlement. In this equation, the deduction of which is based on the enthropy principle, the coefficients μ and ρ depend entirely on the initial rate of deformation, i.e. not so much on the magnitude of the load as on the conditions under which it is increased.

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