The Possibility of Solving Soil Mechanics Problems by the use of Models

La Possibilité de Résoudre les Problèmes de Mécanique des Sols par les Études sur Modèles

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Summary

After the limitations of the analytical methods of soil mechanics have been emphasized, similarity conditions are presented which should be fulfilled by models to be used in studying the engineering problems of soil masses.

First conditions are given when the liquid phase can be ignored, considering the case in which the deformations can be taken as elastic and following this the general case of any type of deformation.

A consideration of the problems in which the liquid phase has to be taken into account follows. After dealing with the case of a solid phase with any type of deformation, the particular case of a solid phase without time effect is considered.

The similarity conditions to be fulfilled in the study of the rupture of soil masses are also given.

Some general indications are given as to the path to be followed in finding materials that fulfill the requirements for similarity. Emphasis is laid on the fact that, as a rule, it is pointless to be too insistent on the fulfillment of those requirements.

With regard to the construction of models attention is called to the case in which the materials of the prototype can be used.

Finally, attention is drawn to the value models can be as an aid in studying the problems raised by soil masses.

In this paper we will consider the prospects of using models in the solution of soil mechanics problems, a field in which, so far, they have not been used. Sometimes models are mentioned in this connection but with rather a different meaning, one that refers to the verification or investigation of laws by means of systems with reduced dimensions.

The first step we had to take in an attempt to study soil mechanics problems by means of models was to establish the similarity conditions.

It must be remembered that, in general, when considering the behaviour of a soil mass it is necessary to bear in mind that soils consist of two phases: a solid phase formed by the particles of the soil and a liquid phase which either partly or totally fills the space between these particles.

Similarity without Considering the Liquid Phase

Let us consider in the first place those cases in which the soil mass can be treated as a solid, i.e. the cases in which it is possible to ignore the liquid phase when studying the behaviour of the mass. This situation arises when the mass is unsaturated and changes in stress do not lead to the development of appreciable neutral stresses. In this case there will be no circulation of the liquid phase and the mass can be considered as a solid with a given stress–strain law. When dealing with saturated soils it is also frequent to assume the circulation of the liquid phase. In fact, for the low values of the coefficient of permeability which frequently occur, it is only at the end of considerable time, say days or weeks, that the circulation has any appreciable influence. Among the cases in which it is not necessary to consider the flow of the water in the pores, we mention those saturated soils with such a high degree of compactness that the water between the particles is so intimately joined to them that it is difficult to conceive of its circulation. In all the cases which we have just mentioned the soil masses behave like solid bodies in the sense already mentioned and the prediction of their behaviour can be made by means of models which obey the conditions of similarity established for solids.

Let us consider in the first place what is meant by similarity. Take the case of a footing whose behaviour we wish to study in a model to the scale of 1/λ (Fig. 1). The object is to establish

Conditions which any model should satisfy so that the stresses \( t_p \) and \( t_m \) acting on homologous elements of the surface, the strains \( \epsilon_p \) and \( \epsilon_m \) of homologous segments, and the displacements \( \delta_p \) and \( \delta_m \) of homologous points are proportional, i.e.

\[
\begin{align*}
\frac{t_m}{t_p} & = \left( \frac{1}{\lambda} \right) , \\
\frac{\epsilon_m}{\epsilon_p} & = \left( \frac{1}{\lambda} \right) , \\
\frac{\delta_m}{\delta_p} & = \left( \frac{1}{\lambda} \right) .
\end{align*}
\]

In the general case, it can be considered that these conditions are verified at moments \( \theta_p \) and \( \theta_m \) to a scale of 1/\( \lambda \):

\[
\theta_m = \left( \frac{1}{\lambda} \right) \theta_p
\]
The scale of displacement is \(1/\gamma = 1/\lambda \beta\) in the case where the deformations are small. For large deformations the above proportionality is only possible for \(1/\beta = 1\).

In order for the conditions of equation 1 to be satisfied, it is necessary in the first place to apply to the surface of the model, at times corresponding to the scale of \(1/\tau\), homologous forces per unit area \(f_p\) and \(f_m\) to a scale of \(1/\alpha\) or concentrated forces \(F_p\) and \(F_m\) to a scale of \(1/\lambda^2\alpha\):

\[
f_m = (1/\alpha) f_p, \quad F_m = (1/\lambda^2\alpha) F_p
\]

These are the boundary conditions which the forces applied at the surfaces of any model should obey. Let us consider the general conditions which the materials of the model will have to obey.

(a) Proportional deformations—Within the hypothesis which we laid down for the behaviour of those soils in which consideration of the liquid phase can be ignored, it is often legitimate to assume that the deformations of the soil are proportional to the stresses, or elastic as it is sometimes called.

In such cases the conditions of similarity demand only that the materials of the model have moduli of elasticity proportional to the homologous ones of the prototype, and equal Poisson’s ratios:

\[
E_m/E_p = E'/E'_p = \ldots = \text{const.,}
\]

\[
\nu_m = \nu_p, \quad \nu'_m = \nu'_p \ldots
\]

Thus, in the case of a footing, if the concrete and the soil have moduli of elasticity of 300,000 and 10,000 kg/cm\(^2\), in the model these materials can be reproduced by any elastic materials provided that the ratio of their moduli of elasticity is 1/30. As to the condition for Poisson’s ratio, it can be a rule be ignored since in practical problems there is no object in seeking very accurate solutions. If the footing is considered to be perfectly rigid, the soil can be reproduced by any elastic material.

Once the similarity conditions have been satisfied, the scale of stresses \(1/\alpha\) will be the arbitrary scale which is fixed for the forces applied to the surface and the scale of the strains will be \(1/\beta = (E_p/E_m)/(1/\alpha)\). There will be no time scale.

In the case of a footing it is not generally necessary to make a model study since satisfactory analytical methods are available, especially in view of the hypothesis of an elastic soil which we considered.

However, when dealing, for example, with an externally indeterminate complex structure, with numerous supports whose settlement can have important repercussions on the behaviour of the structure, a model study would be advisable even if it is assumed that the soil is elastic. If the soil consists of layers these can easily be reproduced with materials having appropriate moduli of elasticity. This will not present any difficulty in view of the wide range of elastic materials available today for the construction of models.

(b) General type of deformation—Suppose we are faced with the study of the behaviour of a soil mass in which we can ignore the liquid phase but for which we cannot presume, as is commonly done, that the deformations are proportional to the stresses. We can then use the general conditions of similarity which we established for solids with any law of deformation (ROCHA, 1952). These conditions of similarity will be presented here very succinctly, considering in the first place studies in which the weight of the mass itself can be ignored. This happens quite frequently in problems of shallow foundations.

Consider the soil of the prototype, for instance the soil on which the footing of Fig. 1 rests. Let a cylinder of this soil be submitted to a triaxial test without there being any change of water content in the sample, as we are working under the hypothesis of no flow of the liquid phase. Let us apply first a hydrostatic pressure \(\sigma_p\) (Fig. 2) and then a longitudinal pressure \(\sigma'_p\). Suppose we obtain the diagram of Fig. 3 for the relation between this stress and the longitudinal strain of the cylinder \(\sigma_p = \Delta p/L_p\). The diagram shows that an unloading and loading cycle was carried out. Let us see the conditions which the material of the model should obey in order that the conditions of equation 1 be satisfied. It was demonstrated that this material has to be such that, when submitted to a triaxial test analogous to the one above, it will show, for stresses to a scale of \(1/\alpha\), i.e.

\[
\begin{align*}
\sigma_m &= \frac{1}{\alpha} \sigma_p, \\
\sigma'_m &= \frac{1}{\alpha} \sigma'_p
\end{align*}
\]

strains to a scale of \(1/\beta\).

\[\epsilon_m = (1/\beta) \epsilon_p\]

This means that the material of the model must have a diagram which is obtained from that of the material of the prototype by multiplying the ordinates and abscissae respectively by \(1/\alpha\) and \(1/\beta\). It should be noted that a relation of the same type has to exist for any triaxial loading to which cylinders of the materials of the prototype and of the model be subjected and for the strain in any direction.

In the case where the deformations of the material are a function of the time, as is common in the practical conditions to which the soils are submitted, the evolution of the stresses with time has to be taken into consideration. If, for example, the stress is applied in time as shown in Fig. 4 the relation mentioned between the diagrams of deformations must be verified when the stress \(\sigma'_m\) is applied to the cylinder of the material of the model at a certain time scale \(1/\tau\).

Therefore, in brief, if the materials of the model obey the conditions laid down, i.e. if the scales \(1/\alpha, 1/\beta, 1/\tau\) exist and if we apply forces to the surface to the scale of \(1/\alpha\), the similarity relations of equation 1 will then be satisfied.
In the case of the footing of Fig. 1, the model of the soil and of the footing should be constructed with materials for which $1/\alpha$, $1/\beta$, $1/\tau$ exist and are the same. When forces $F_m = (1/\lambda\beta)F_p$ are applied, the settlements will be to the scale of $1/\lambda\beta$ at time intervals to a scale of $1/\tau$. The reactions of the soil, which must be known for the design of the footing, will be to the scale of $1/\alpha$. If the relation between the diagrams of deformation of Fig. 3 are maintained up to rupture, the loads which bring about rupture under the footings will be to a scale of $1/\lambda^2\alpha$.

But the crucial question which is raised is this: is it possible to obtain materials which satisfy the conditions mentioned?

When, as is the case under consideration, the stresses due to the weight of the soil itself can be ignored, the models can be constructed with the same materials as those of the prototype itself, the scales being in this case equal to unity $1/\alpha = 1/\beta = 1/\tau = 1$. Even when the mass consists of various layers, isotropic or not, complete similarity is obtained by reproducing these same layers in the model with the materials of the prototype and similarly for the structure resting on the soil mass.

There is particular interest in making use of model studies under the conditions just described in the case of foundations having special shapes, of complex structures externally indeterminate, of oblique forces or forces varying with time, when the distribution of the reaction of the soil on the surface of the foundation, etc., is required.

We will now consider problems in which the liquid phase can still be ignored but in which the weight has to be considered. The majority of soil mechanics problems such as deep foundations, stability of slopes, retaining walls, etc., demand the consideration of the weight of the soil mass.

In this case the similarity conditions to be satisfied are those already given, simply having to add that the stress scale, $1/\alpha$, has to be equal to $(1/\lambda). (1/\rho)$, $1/\rho$ being the scale of the specific weights of homologous materials, $1/\rho = \gamma_m/\gamma_p$. In the case of equal specific weights $1/\alpha = 1/\lambda$. The scales of the strains, $1/\beta$, and of the times, $1/\tau$, can be of any value. As $1/\alpha$ cannot be equal to unity unless $1/\lambda = 1$ it follows that when the weight of the mass itself is taken into account the materials of the prototype cannot be used for the construction of the models.

As an example, consider the behaviour of the reinforced concrete supporting wall of Fig. 5. Suppose that it is built on a very permeable sand and that the backfill consists of a clayey soil compacted in the best conditions in which, as is known, soils are not saturated. Suppose it is required to know during and after the construction the stresses set up in the wall and its displacements, and the displacements due to concentrated loads applied to the surface of the backfill. Take the wall as 20 m high and a model to a scale of $1/\lambda = 1/10$.

The material of the foundation can be reproduced in the model by a sand for which there are the scales $1/\alpha = 1/\lambda = 1/10$, and $1/\beta$, i.e. such that for the loading $\sigma_p/10$, taken for instance as constant, and $\sigma_p/10$, the strains are at a given scale, for example, unity $1/\beta = 1$ (Fig. 6). It will not be difficult to find a sand satisfying these conditions. In the case under consideration the material is very permeable and it is not necessary to consider a time scale.

The material of the backfill has also to be reproduced to the same scales

$$\frac{1}{\alpha} = \frac{1}{\lambda} = \frac{1}{10} \quad \text{and} \quad \frac{1}{\beta} = 1$$

and it may be necessary to consider a time scale $1/\tau$ for this material.

As to the wall, it will have to be reproduced by a material for which the scales $1/\alpha = 1/\lambda = 1/10$, $1/\beta = 1$ and $1/\tau$ hold. As the wall is of concrete we assume it to be elastic. Then from the equation

$$\frac{1}{\beta} = \frac{(E_p/E_m). (1/\alpha)}{\gamma_m/\gamma_p},$$

we obtain

$$E_m = \frac{(\beta/\alpha)}{(E_p)}.$$

Supposing that $1/\beta = 1$ and $E_p = 300,000$ kg./cm$^{-2}$, $E_m$ will be equal to 30,000 kg./cm$^{-2}$. In the model the wall could be constructed, for example, with plaster of Paris or certain plastics. With a model built in the way we have just described the conditions in equation 1 are fulfilled for any stage of the construction of the mass, and to study the effect of any concentrated loads acting on the mass it will only be necessary to reproduce it to the scale of $1/\lambda^2\alpha = 1/\lambda^3 = 1/1000$.

It should be emphasized that in soil mechanics it is not possible to foresee the displacements of the wall and that the conditions of rupture of the mass are based on very simplified hypotheses such as: simple shape of the wall and its foundation, a long wall, i.e. plane equilibrium, loads distributed equally along the wall, homogeneous masses and finally the crude hypothesis that rupture takes place simultaneously along the whole sliding surface.

Hence model studies will often be needed when, as is common, these hypotheses cannot be accepted and always, when displacements are required: for example, walls having special shapes on the back and for the design of which it is of interest to know the real distribution of the pressures set up by the backfill, walls in three dimensional equilibrium under the action of concentrated loads or where the length of the wall is
small compared to its height, or where heterogeneous backfills such as are common in quay walls, etc., are present.

**Similarity when Considering the Liquid Phase**

We shall now consider the problem of predicting the behaviour of masses in which the flow of the liquid phase has to be considered (ROCHA, 1955).

(a) **General type of deformation**—We will present very briefly the similarity conditions which have to be fulfilled in the model study of the behaviour of soil bodies in which the solid phase has any law of deformation and the liquid phase flows through the solid phase in accordance with Darcy's law.

When we seek to establish these conditions of similarity we are faced with the difficulty of not having general differential equations which govern the phenomena of soil mechanics, as we have in the Theory of Elasticity and in Hydrodynamics.

Now, besides the conditions (1) described at the beginning, where the values of \( t \) represent total stresses, we now have to establish conditions of similarity such that the effective stresses \( f \) and the neutral stresses \( u \) are to the scale of \( 1/\alpha \).

\[
f_m = (1/\alpha)f_p, \quad u_m = (1/\alpha)u_p
\]

In order to achieve similarity it is necessary, just as before, for the forces distributed at the surface to be to the scale of \( 1/\alpha \), i.e.

\[
f_m = (1/\alpha)f'_m, \quad f_m = (1/\alpha)f_p, \quad v_m = (1/\alpha)v_p
\]

from which

\[
F_m = (1/\lambda^2a)F_p, \quad f \quad \text{and} \quad v \quad \text{being the values of the effective and neutral stresses at the surface.}
\]

We will begin by considering those problems in which the weight of the soil mass can be ignored. Let us see what the conditions are which the materials should obey.

It is necessary for the diagrams which relate the effective stresses and the deformations of the solid phases of the materials of the prototype and the model to satisfy the relation given in Fig. 3 and, besides this, for the coefficients of permeability of the solid phases \( K_p \) and \( K_m \) to be at homologous points to the scale of

\[
1/\alpha = K_p/K_m = \alpha \tau/\lambda^2
\]

that is, the scales \( 1/\alpha \), \( 1/\beta \) and \( 1/\tau \) imposed by the deformations of the solid phase cannot now take any value: they have to be related by the above condition. In the case which we are considering the materials of the prototype cannot be used for the models.

If the problem being studied calls for the consideration of the weight of the mass, we find ourselves in the most general situation, i.e. to forecast the behaviour of a mass subject to surface forces and its own weight and with circulation of the liquid phase through a solid phase with any law of deformation.

The stress scale has then to be \( 1/\alpha = 1/\lambda \), whilst \( 1/\beta \) and \( 1/\tau \) can have any value. The scale of the coefficients of permeability will then be \( 1/\mu = \tau/(\lambda \delta) \), assuming that \( 1/\rho = 1 \).

To take into consideration the buoyancy to which the solid phase is subject due to its immersion in the liquid phase, the porosities of the homologous materials have to be equal, i.e.

\[
n_m = n_p.
\]

It is clear that the condition \( 1/\alpha = 1/\lambda \) makes it impossible for the materials of the prototype to be used for the model.

A case which is worth while studying in the conditions just described is the behaviour of the upstream slope of an earth dam when there is a sudden drop in the water level in the reservoir.

(b) **Solid phase without time effect**—Let us assume that the deformations in relation to time, observed for example in the settlement of a footing, occur solely as a consequence of the slow drainage of the water through the pores, the stress-strain relations of the solid phase being assumed independent of time. This happens with a perfectly elastic solid phase or the deformations of which qua function of time take place more rapidly than do those corresponding to the flow of the liquid phase.

Consider in the first place that the weight of the mass can be ignored.

The solid phase does not call for a time scale \( 1/\tau \) which is fixed only the condition relating to the coefficients of permeability

\[
1/\tau = (\mu)/(\lambda^2 \delta)
\]

There is the possibility of using the same materials of the prototype in the model. Then \( 1/\alpha = 1/\beta = 1/\mu = 1 \) and consequently \( 1/\tau = 1/\lambda^2 \). This means that the forces to a scale of \( F_m/F_p = 1/(\lambda^2 a) = 1/\lambda^2 \) have to be applied to the model at the rate of \( 1/\lambda^2 \).

Results of model studies in which these conditions were assumed have already been published (ROCHA and FALQUE, 1955).

When the weight of the soil mass has to be considered the ratio \( 1/\alpha \) has to be equal to \( 1/\lambda \) and therefore \( 1/\tau \) will be equal to \( \mu/(\lambda \delta) \), when \( 1/\rho \) is assumed to be equal to 1. Just as in the general case of section (a) \( n_m \) will have to be equal to \( n_p \).

The models cannot be constructed with the materials of the prototype.

(c) **Rupture**—so far we have mentioned the conditions of similarity to be observed when considering the deformation of the soil.

However, it is rare in soil mechanics to consider the deformation of the soil masses (except very roughly) and much less the non-linear relations between stresses and deformations and the time effect on the deformations of the solid phase. In the study of the strength of a soil mass, soil mechanics is limited in rupture. This is to avoid the consideration of the deformations general to a search for the values of the loads which produce and to define a soil by the simple parameters of cohesion and angle of internal friction.

We note, in the first place, that if the relation between the diagrams of deformation of the materials of the prototype and of the model given in Fig. 3 is satisfied then the cohesions are to a scale of \( 1/\alpha \) and the angles of internal friction are equal. In fact, if \( \sigma_p, \sigma'_p \) is a state of stress which produces rupture in the material of the prototype, the material of the model will have to fail under the action of \( \sigma_p/\alpha, \sigma'_p/\alpha \). If we use Mohr's circle we obtain, for each state of rupture, circles geometrically similar whose envelopes are also geometrically similar (Fig. 7).

If we suppose these envelopes to be straight lines it follows that

\[
C_m = (1/\alpha)C_p, \quad \phi_m = \phi_p.
\]

Accepting the radically simplified hypotheses assumed by soil mechanics it is easy to show that the similarity conditions to be fulfilled in a model for rupture study are reduced to

\[
C_m = (1/\alpha)C_p, \quad \phi_m = \phi_p.
\]

When the weight of the mass has to be considered then \( 1/\alpha = (1/\lambda) (1/\rho) \).

In the case of soils without cohesion, such as sands, then the condition will reduce to: \( \phi_m = \phi_p \).

In the general case of soils with cohesion the materials of the prototypes can be used in the models once the weight can be
ignored, which results in a much closer similarity as we have already seen.

In the case of soils without cohesion, the same materials can be used in the models even when the weight of the material itself has to be considered. Hence, these conditions are extremely simple to satisfy and it is surprising that model tests under these conditions have not been carried out frequently.

Of the many cases in which such studies are of interest we can mention retaining walls. In fact, soil mechanics only resolves simple cases satisfactorily. In more complex cases, which we cited at the end of the section on similarity when considering the liquid phase, the difficulties are at times insuperable.

Consider the case of a quay wall (Fig. 8) consisting of blocks and supporting and resting on a heterogeneous mass. Suppose that the load on the surface which would produce rupture in the wall is required. In the case of masses of sand and stone it is sufficient for the materials of the model to reproduce the angles of internal friction of the prototype. As to the reproduction of the wall, a material of the same specific weight will have to be used, once it is assumed that the scale of specific weights for the material of the masses is unity. The blocks geometrically similar to those of the prototype can be made of the same concrete.

In the model the wall will overturn or slide for a loading per unit area of the surface of the backfill \( f_m = \left(1/\lambda\right)f_p \). For sliding along the joints between blocks of the wall it is assumed that the friction characteristics are the same between the faces of the blocks of the prototype and of the model.

In relation to the problem of predicting the rupture of soil masses, models can be used to determine the shape of the sliding surfaces, and hence for analogous cases the loading producing rupture can be determined by the current analytical methods. In relation to the behaviour of slopes, models can also render great service even within the framework of the simplifying hypotheses which we have considered.

The Value of Model Studies

The following objection may be raised to the subject under discussion: in view of the accentuated erratic variations of soil properties from one point to another in a mass, even in an artificial mass made as homogeneous as possible, will there be any interest in undertaking model studies which, after all, seek to obtain a closer picture of the behaviour of the masses? That is, in view of the small accuracy which in general can be legitimately assumed for numerical values which define the properties of soils, will not the present analytical methods of soil mechanics be sufficient?

Note however that soil mechanics is obliged to assume radically simplified hypotheses of the real phenomena. Thus, in spite of seeking to deal with soils as systems having two phases—the fact is that—with the exception of the theory of consolidation, all problems are dealt with without considering the real interdependence of the two phases. For this assumption to be valid it is assumed either that the forces are applied slowly so as to make the neutral stresses negligible, or that they are applied so rapidly that there will not be sufficient time for the water in the pores to begin to flow. However, what really happens in the majority of cases calls for the consideration of intermediate situations.

As to the theory of consolidation, except in very particular cases, only the problem of uni-directional drainage is dealt with, while in practice drainage in two and three dimensions nearly always occurs.

As to the relation between the states of deformation and stress, soil mechanics is obliged to consider radically simplified extreme hypotheses: either it is assumed that this relation is linear so as to make use of the Theory of Elasticity, or it is assumed that perfectly plastic deformations are being dealt with, which makes it possible to use the scanty results of the Theory of Plasticity. But in practice the masses are often in a state of deformation such that it would be advisable to consider the non-linear relations between stresses and deformations. However, the mathematical analysis becomes insuperably complicated when non-linear relations are to be considered.

Continuing the consideration of the difficulties which face soil mechanics there are also those which are met when dealing with masses with a regular heterogeneity, such as stratification, which is common, or with masses with a regular anisotropy, also very frequently met with.

It is to be noted, too, that soil mechanics is often obliged to consider problems of equilibrium in two dimensions when in fact they are far from this.

Finally, in soil mechanics, as in all theories, great difficulties must be faced whenever the boundary conditions, i.e. the shapes and the loadings applied to the surface, are complex, as in this case it is impossible to express these conditions analytically in a simple form.

After this brief description of the difficulties of soil mechanics, it is understandable that frequently the accuracy of the available analytical methods is not satisfactory; a situation which will occur more frequently as the erratic heterogeneity of the masses increases.

Carrying out Model Studies

As a rule in a model study of a soil mechanics problem the following steps will have to be followed: choice of materials satisfying the conditions of similarity, construction of the model, application of the loadings and observation of the model.

The construction of the model and the application of the loads will not generally present difficult problems. It should be borne in mind, however, that the reduction of the scale of a model has, as a rule, very important repercussions on the rapidity and the economy of the experimental study. As to the observation of the model, it is generally sufficient to measure displacements. In fact, what is after all the main interest in the majority of cases is to forecast the deformations which a certain structure will undergo. The constant attention which the analytical methods pay to stresses is simply because they serve to predict deformations, especially rupture.

As to the other step mentioned, the choice of materials which satisfy the conditions of similarity, very different situations can arise depending upon the degree of difficulty. Thus, in the case of weight of the mass being negligible, materials of the prototype can usually be used for the construction of the model.

In the case of the problem to be resolved which calls for the consideration of the deformation of the mass and where use of the materials of the prototype is not possible, the choice of the materials demands triaxial tests. It is clear that the triaxial test for verifying similarity should be carried out under states of stress representative of those to which the materials will be subject in the problem under study. Thus, the maximum values reached by the stresses will have to be taken into consideration and also whether the stresses developed will be always increasing, whether they develop slowly or rapidly in
relation to time, whether or not there will be circulation of the liquid phase, and whether there will be important variations in volume, ruptures, etc.

Of course, in view of the complexity of the properties of soil it will not be possible to adhere to the general conditions of similarity very rigidly, but this is not important in view of the accuracy demanded in the study of problems raised by soil masses.

In addition, it is often possible to accept simplified hypotheses from which conditions of similarity result that are more easy to satisfy. However, each case has to be judged upon its own merits.

In the case in which the deformability of the masses need not be taken into consideration, and if it is only required to predict rupture, this simplification is such that it is sufficient to consider only the parameters cohesion and angle of internal friction.

Conclusions

We cannot at this juncture present definite conclusions as to the value of the use of models in the solution of problems which soil masses present to the engineer, nor was this our intention, as we only intended to call attention to the possibilities which they raise. However, we are bold enough to foresee that models will come to render valuable service.

We are taking the first steps. But a number of years will be necessary to develop the techniques for these studies and above all to discover suitable materials for the construction of models. The experience we have of models studies of structures gives us some idea of the numerous difficulties which will have to be overcome.

In relation to the value, universally recognized today, of the study of structures by models, we consider that the value of models in soil mechanics will be even greater in view of the difficulties with which this science is faced to obtain satisfactory solutions. These are obviously much greater than those met with in strength of materials.

We hope to have the opportunity of studying concrete problems by means of models as this will be the best means of judging their value.

We would also stress that soil mechanics is today passing through a crisis recognized by all specialists and which is common to many other domains of civil engineering: the scarcity of information about the real behaviour of constructions. Models, however, especially those satisfying the more general conditions of similarity, will make it possible to obtain much of that information, which is very important, as the observation of constructions themselves raises many difficulties, both economic and technical, and besides this it is not possible as a rule to carry out experiments on them, i.e. subject them to the conditions we would wish.

In view, too, of the fidelity with which models can reproduce the real conditions they can be of great service in teaching soil mechanics.

As to the economic value of model studies it should be recognized that it is in the behaviour of soil masses that the civil engineer foresees the phenomena most roughly, from which results the corollary that it is in this field that there is the greatest waste, either because of excessive safety (which is so common) or because of insecurity, which leads to numerous accidents.

In conclusion we would emphasize as we always do when speaking about the use of models, that we do not wish to place the analytical and experimental methods in opposition but rather to show the services that the latter can render in the search for safe and economic solutions of technical problems. The analytical methods which, after all, contain the human knowledge which so far it has been possible to express quantitatively should be applied whenever it is assumed that they may give results with the approximation desired.

References

