The Analysis of the Consolidation Process by the Isotaches Method

L'analyse du Processus de Consolidation par la Méthode des Isotaches

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Summary

A graphical construction of consolidation curves for a saturated layer of arbitrary thickness (h) is presented. The following conditions and assumptions are considered:

(a) The sum of the intergranular and neutral pressures equals the total pressure. Assuming Darcy's law and parabolic isochrones, the neutral pressure is derived from the speed of consolidation and the measured coefficient of permeability (k).

(b) The position and slope of the consolidation curve in a given time must agree with the relationships between the speed of consolidation, the intergranular pressure and the porosity. These relationships are presented by a set of isotaches, derived from the consolidation curves of an oedometer test, carried out for various load increments; the logarithmic lines of secondary compression are extrapolated beyond the time of observation.

(c) The consolidation curve is continuous.

It is proved that for great values of the ratio h^2/k the Terzaghi consolidation curve combined with the logarithmic line of secondary compression represents a satisfactory approximation. The pore pressures in a thick consolidating layer can exceed appreciably the pore pressures in the consolidometer test.

Sommaire

Ce rapport présente une construction graphique de la courbe de consolidation d'une couche saturée d'eau et d'épaisseur (h) quelconque en tenant compte des conditions et des hypothèses suivantes:

(a) La somme des pressions intergranulaire et interstitielle doit être égale à la pression totale. Nous deduisons la pression interstitielle de la vitesse de consolidation et du coefficient de perméabilité mesuré directement en supposant valuable la loi de Darcy et en admettant la forme parabolique des isochrones.

(b) La position et l'inclinaison de la courbe de consolidation dans un temps donné doit satisfaire aux relations entre la vitesse de consolidation, la pression intergranulaire et la porosité. Nous présentons ces relations par un système d'isotaches dérivées des courbes de consolidation de l'essai œdométrique, executé pour différents intervalles de charge; nous en prolongeons les lignes logarithmiques de consolidation secondaire au delà du temps d'observation.

(c) La courbe de consolidation est continue.

L'étude montre que dans les cas où les valeurs h^2/k sont grandes, la combinaison de la courbe de consolidation de Terzaghi et de la ligne logarithmique de la consolidation secondaire présente une approximation satisfaisante. Les pressions interstitielles d'une couche épaisse peuvent dépasser de beaucoup les pressions interstitielles de l'échantillon œdométrique.

Introduction

The disagreement between the Terzaghi consolidation theory and the behaviour of samples in the laboratory consolidation test has been thoroughly investigated by TAYLOR (1940, 1942). In the first paper a theory was established—later called Theory A—based on Terzaghi's equation for one-dimensional consolidation, assuming that the speed of occurrence of secondary compression ($\partial e/\partial t$) is proportional to the undeveloped secondary compression (ρ_s):

$$\frac{\partial e}{\partial t} = -\mu \rho_s \qquad \ldots. \text{ (1)}$$

The ultimate amount of secondary compression is assumed to be known. A satisfactory analytical solution, made possible by the further assumption that $e^{-\mu t} = 1$, gives results only for small values of 'secondary compression factor'

$$F_a = \frac{\mu t}{r_p T} \qquad \ldots. \text{ (2)}$$

(t = time, T = time factor, r_p = primary compression ratio) as well as for $F_a \to \infty$ which corresponds to the Terzaghi case. Since the factor F_a increases rapidly with increasing layer thickness it may be concluded that the consolidation of thicker layers approaches to the values given by the Terzaghi's theory.

In Theory B (TAYLOR, 1940), all secondary compression occurring subsequent to the primary compression is disregarded and only the effect of secondary compression in the previous increment is considered. The total intergranular pressure σ' is assumed to consist of three parts:

$$\sigma' = p_g + p_b + p_v \qquad \ldots. \text{ (3)}$$

p_g is the part independent of the state of consolidation, p_b, called the 'bond', is the plastic resistance existing at the end of primary compression and p_v is the viscous structural resistance. In the basic consolidation equations a further assumption

$$p_v = -\bar{\eta}\frac{\mathrm{d}e}{\mathrm{d}t} \qquad \ldots. \text{ (4)}$$

is introduced with $\bar{\eta}$ = constant. The resulting consolidation ratio curves are plotted for different values of the 'ratio of time factors' J

$$J = \frac{k\bar{\eta}(1+e)}{h^2\gamma_w} \qquad \ldots. \text{ (5)}$$

At low values of the ratio J the consolidation curve comes close to the Terzaghi's theory, which is a special case of Theory B with $J = 0$. Thus Taylor presumes that the primary consolidation curves of thick layers approach to Terzaghi's curves and turn into the extrapolated secondary logarithmic lines of consolidometer tests.

With the same assumption a formula has been derived by KOPPEJAN (1948) combining the Terzaghi load-compression relationship and the Buisman secular time effect.

Recently the secular time effect has been interpreted by TAN TJONG KIE (1954) applying colloid-chemical concepts and rheological models. Three-dimensional differential equations have been derived for the deformation of clay, in which the effects of consolidation and of flow are taken into account. Solutions have been obtained for certain boundary conditions.

A previous report of the author's (1955) was based on Taylor's conception of plastic structural resistance, and on the relationship between the speed of consolidation and the neutral pressure. It was concluded that the consolidation curve of a natural layer lies between the extended secondary line of the laboratory sample and the oedometer consolidation curve displaced by the ratio of the squares of thicknesses of the layer and of the sample.

The limitations of the basic assumptions of Taylor's Theories A and **B** do not permit reliable conclusions concerning the consolidation of thicker layers. Further, the assumption, 1, of Theory A does not correspond to the logarithmic time law of secondary compression (cf. equation 7) and the resolution of the intergranular pressure into three parts in Theory **B** has no clear physical meaning.

Although it is impossible to give a general and precise solution of the problem, an attempt should be made to proceed from a clear set of assumptions which agree with experience in the laboratory and in nature. In order to avoid the limitation of such assumptions a combined analytical-graphical solution was preferred to a pure analytical solution requiring simplified assumptions.

Assumptions

The speed of consolidation depends on the mean values of void ratio ($\bar{e}$) and of total intergranular pressure ($\bar{\sigma}'$). This relationship can be derived from consolidation curves of consolidometer tests carried out for various load increments ($\Delta\sigma$) while the logarithmic lines of secondary compression are extended beyond the time of observation.

As complete saturation is assumed, the speed of consolidation must correspond to the speed of pore water flow in both the primary and secondary phase of consolidation.

Although the validity of Darcy's law is assumed, the variability of the coefficient of permeability with the development of consolidation is taken into account.

The case treated is one of uniformly distributed total pressures along the whole layer thickness and of one-dimensional water flow, assuming a uniform consolidation process in all parts of the layer. The corresponding pore water isochrones are parabolic. The midplane pore pressure values for double drainage layers or the boundary values for single drainage layers can be obtained from the consolidation speed and the coefficient of permeability.

No account is taken of plastic deformations which occur under the influence of shearing stresses without change of volume.

Determination of Consolidation Speeds and of Pore Pressures

Consolidation curves will be presented in diagrams $\log t - \bar{e}$. The logarithmic line of secondary consolidation is expressed in the form

$$\bar{e} = e_1 - (\Delta e_1 + \alpha_e \log_{10} t) \quad \ldots\ldots (6)$$

(see Fig. 4) and its speed by the relation

$$\frac{\partial \bar{e}}{\partial t} = -\frac{\alpha_e}{2{\cdot}3026t} \quad \ldots\ldots (7)$$

Thus the resulting specific speed of consolidation per unit of layer thickness is given by

$$v = -\frac{0{\cdot}434}{t}\frac{\alpha_e}{1+\bar{e}} \quad \ldots\ldots (8)$$

Applying the Buisman equation for settlement

$$\rho = h_1(\alpha_p + \alpha_s \log_{10} t)\Delta\sigma \quad \ldots\ldots (9)$$

the boundary pore pressure of a single drainage layer (Fig. 3) has been expressed in a previous paper (1955) by the formula

$$u_0 = \frac{\Delta\sigma\gamma_w h_1}{4{\cdot}6052}\frac{\alpha_s h}{kt} \quad \ldots\ldots (10)$$

where the factor h^2 of the original expression is replaced by the product hh_1, the initial height h_1 being distinguished from the height h in time t. Using the transformation

$$\alpha_s = \frac{h_s}{h_1\Delta\sigma}\alpha_e = \frac{\alpha_e}{(1+e_1)\Delta\sigma} \quad \ldots\ldots (11)$$

$$h_s = \frac{h_1}{1+e_1} = \frac{h}{1+e} \quad \ldots\ldots (12)$$

equation 10 may be written as

$$u_0 = \frac{h_s^2\gamma_w}{4{\cdot}6052k}\frac{\alpha_e(1+\bar{e})}{t} \quad \ldots\ldots (13)$$

As explained in a previous paper (1955), equations 8 and 13 can be applied also to the determination of the consolidation speed and of the pore pressure at an arbitrary time (t) on the primary consolidation curve by substituting α_e for α'_e (Fig. 4), i.e. by the gradient of the logarithmic tangent at the point $t - \bar{e}$.

Construction of the Consolidation Curve of a Thick Layer

Using equations 8 and 13 the corresponding specific consolidation speed (v) and pore pressure (u_0) are calculated from the diagrams $\log t - e$ and $\log t - k$ (Fig. 1) for various times (t),

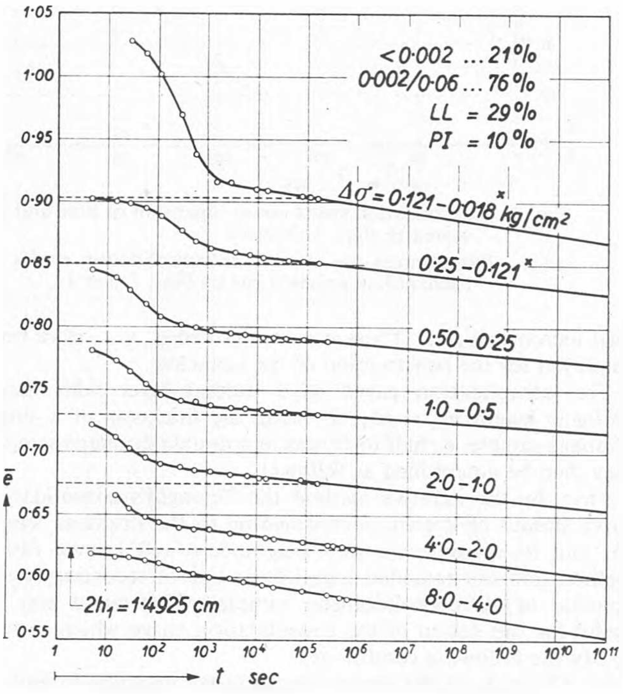

Fig. 1 Consolidation curves of a lacustrine chalk (sample deposited in the laboratory); + single drainage, otherwise double drainage

Courbes de consolidation d'une craie lacustre (échantillon sedimenté en laboratoire); + drainage unilateral, aux autres degrés $\Delta\sigma$ drainage bilateral

and the graphs of $\log t - v$ and $\log t - u_0$ (Fig. 2) are drawn for different load increments $\Delta\sigma$.

For certain constant values $v =$ const. the time t_1 corresponding to different load increments $\Delta\sigma$, are determined from the graph of $\log t - v$. The corresponding values u_0 and $\bar{e}$ can be found from the graphs of $\log t - u_0$ and $\log t - \bar{e}$. The resulting mean intergranular pressure ($\bar{\sigma}'$) is given by the equation (see Fig. 3):

$$\bar{\sigma}' = \sigma - \frac{2}{3}u_0 \quad \ldots\ldots (14)$$

Thus isotaches $\bar{\sigma}' - \bar{e}$ may be constructed for various specific consolidation speeds v. In the same co-ordinate system (Figs. 3 and 5) the curves $\bar{\sigma}' - \bar{e}$ are drawn corresponding to different

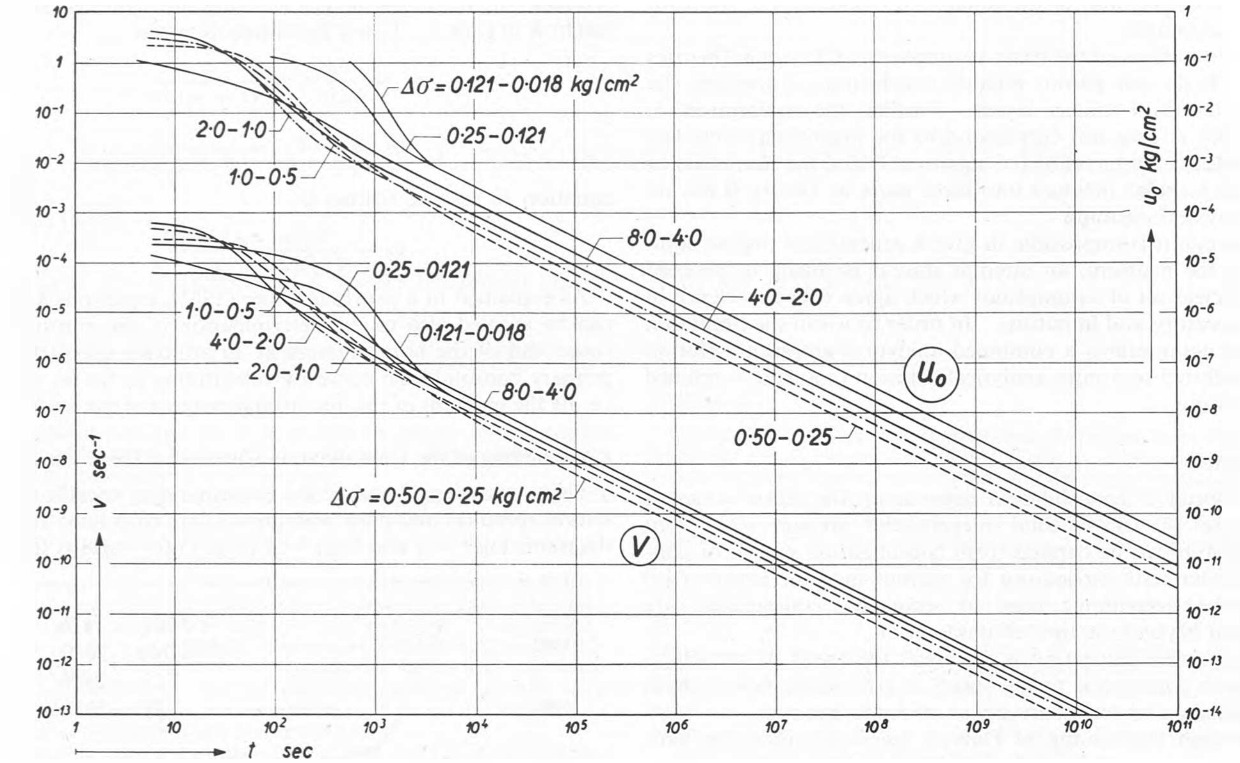

Fig. 2 Consolidation speed *versus* logarithm of time and pore pressure *versus* logarithm of time curves for the sample represented in Figs. 1, 3 and 4

Diagrammes des vitesses de consolidation et des pressions interstitielles en fonction du logarithme de temps pour l'échantillon présenté pas les Figs. 1, 3 et 4

load increments $\Delta\sigma$. These curves, marked $n_c = 1$, have been employed for the construction of the isotaches.

The consolidation curve of a natural layer with initial drainage length $h_{1n} = nh_1$, h_1 being the thickness of a single drainage sample, or half thickness of a double drainage sample, may then be determined as follows:

First, by the iterative method the Terzaghi's consolidation curve should be drawn, corresponding to the drainage length nh_1 and in agreement with Casagrande's well known fitting method and the extended logarithmic line of secondary compression of the consolidometer sample. This curve may be useful for the design of the consolidation curve which should satisfy the following conditions:

(a) The sum of the mean intergranular pressure $\bar{\sigma}'$ and of the average pore pressure must equal the total pressure:

$$\bar{\sigma}' + \frac{2}{3}u_0 = \sigma \qquad \ldots . \ (14')$$

(b) The specific consolidation speed (v), as well as the mean intergranular pressure ($\bar{\sigma}'$), at a given time (t) are determined from equations 8, 13 and 14 from the slope (α'_e) of the consolidation curve. From these calculated values of v and $\bar{\sigma}'$ a certain void ratio $\bar{e}$ results from the isotaches chart $\bar{\sigma}' - \bar{e}$ (Figs. 3 and 5).

The void ratio obtained must agree with the value ($\bar{e}$) of the consolidation curve $t - \bar{e}$ at time t.

(c) The consolidation curve must be continuous.

Considering the above conditions the consolidation curve may be constructed gradually (Figs. 4 and 6). In order to construct the curve near the initial point ($t = 0$, $\bar{e} = \bar{e}_1$), the consolidation curve should be presented as a square root of time plot (Figs. 4 and 6).

The method described is carried out in Figs. 1 to 4 for a lacustrine chalk sample, which has been gradually settled out

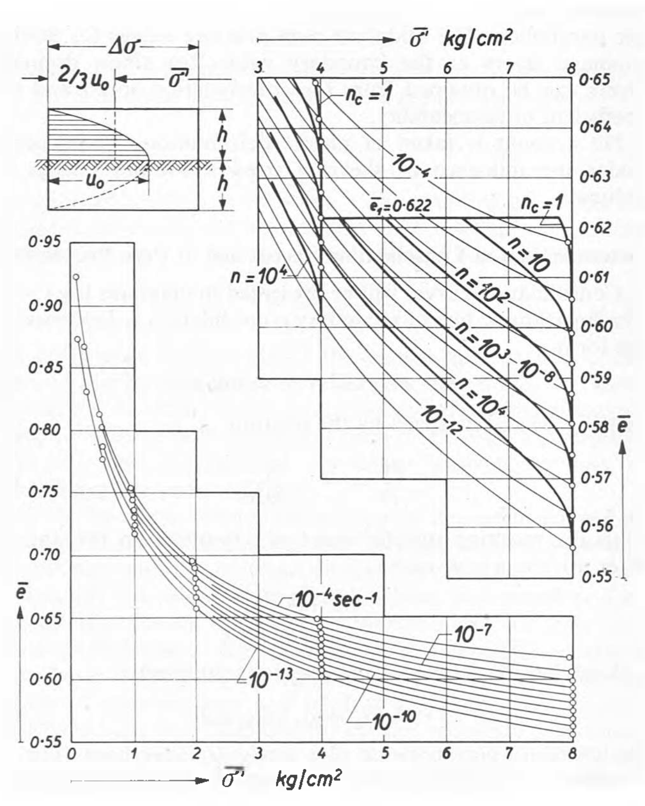

Fig. 3 Isotaches set with the curves $(\bar{\sigma}' - \bar{e})_{n=\text{const.}}$ for the sample represented in Figs. 1, 2 and 4

Système d'isotaches avec des courbes $(\bar{\sigma}' - \bar{e})_{n=\text{const.}}$ pour l'échantillon presenté par les Figs. 1, 2 et 4

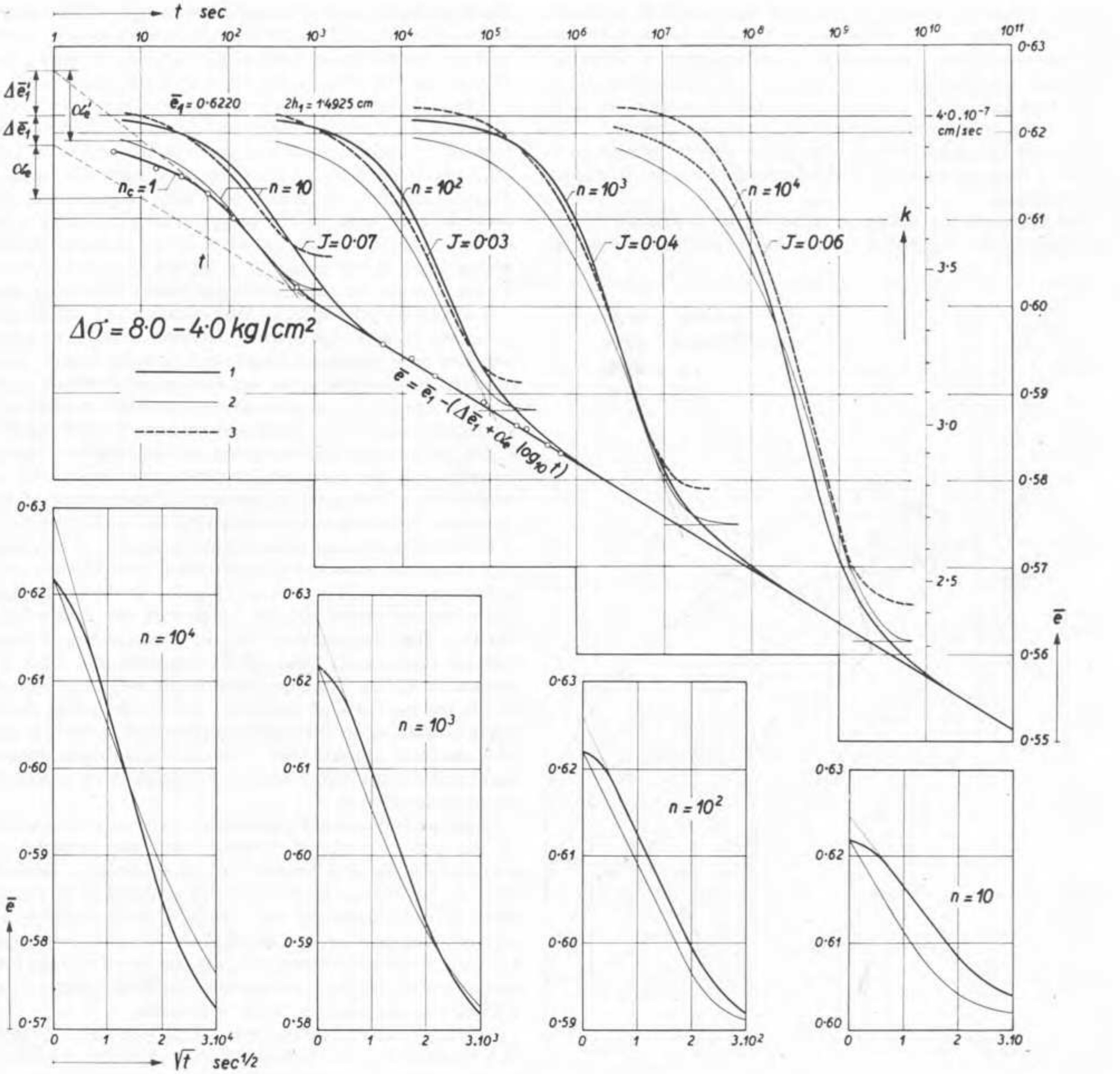

Fig. 4 Consolidation curves for the load increment $\Delta\sigma = (8 - 4)$ kg/cm² of the sample represented by Figs. 1, 2 and 3

n = ratio between the initial thicknesses of the layer (h_{1n}) and of the sample (h_1),
$n_c = 1$, corresponds to the consolidometer test,
$n = 1$, corresponds to the isotaches set,
1 .... consolidation curves corresponding to the isotaches set and to equation 13,
2 .... the corresponding Terzaghi's curves (Theory B),
3 .... the corresponding Taylor's curves.

Courbes de consolidation pour l'intervalle $\Delta\sigma = (8 - 4)$ kg/cm² de l'echantillon présenté par les Figs. 1, 2 et 3

n = rapport entre les epaisseurs initiales de la couche (h_{1n}) et de l'échantillon (h_1),
$n_e = 1$, correspond à l'essai oedométrique,
$n = 1$, correspond au système d'isotaches,
1 .... courbes de consolidation correspondant au système d'isotaches et à l'equation 13,
2 .... les courbes de Terzaghi correspondantes,
3 .... les courbes de Taylor correspondantes.

in vacuum. The classification details of the sample are given in Fig. 1; the small circles give the observed values.

For another undisturbed sample of smaller permeability (sediment from lake of Skadar) the isotaches and the consolidation curves for a load increment $\Delta\sigma = (2 - 1)$ kg/cm² are reproduced in Figs. 5 and 6.

Discussion of the Assumptions

In Fig. 7 some measured pore pressures u_0 are compared with values obtained from consolidation curves by means of equation 13, i.e. assuming Darcy's law and parabolic form of isochrones. Generally the corresponding values do not agree closely. In cases (a) and (b) when the plastic resistance at the beginning of the consolidation is small, the calculated initial values exceed the measured ones. This relation is reversed in the later phase of primary compression in cases (a) and (b) as well as during the whole primary compression in case (c), $\Delta\sigma = (4 - 2)$ kg/cm², showing a high plastic resistance already in the initial phase.

From this correlation it may be concluded that the correct sochrone form is closer to a parabolic curve with the exponent $n < 2$ in the first phase and $n > 2$ in the later phase of primary compression. This corresponds to a concentration of the consolidation process at the boundaries of the sample at the beginning and in the interior of the sample during the latter part of the primary compression as would be expected. The isochrones obtained both by Terzaghi's theory as well as by Taylor's theory B differ from the parabolic form ($n = 2$) in the same manner.

The effect on the apparent reduction of coefficient of permeability by gas trapped in the soil has already been discussed

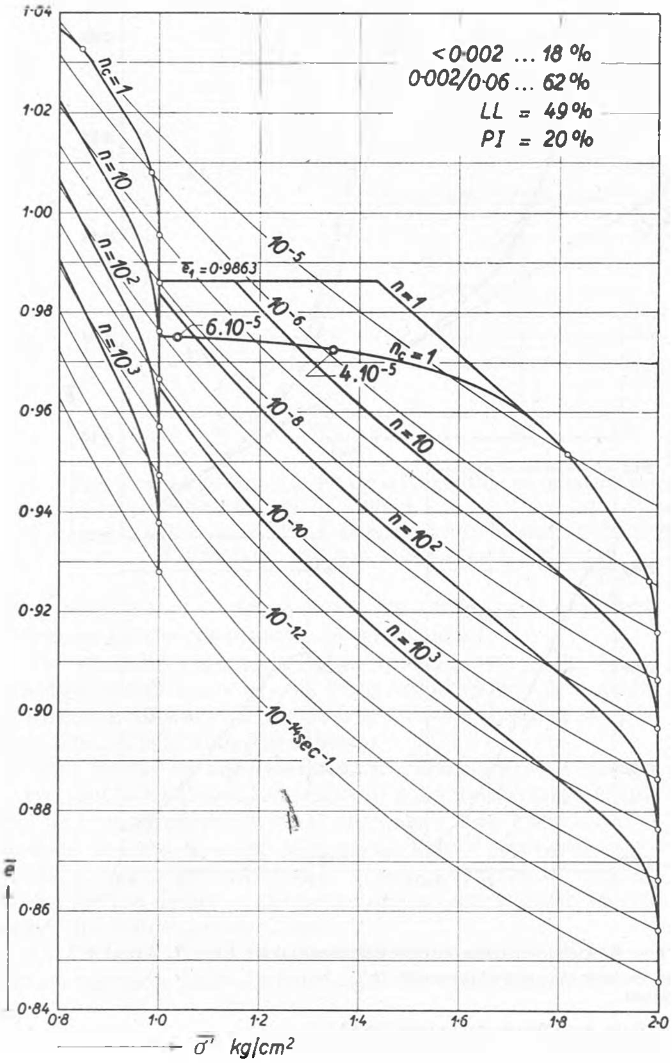

Fig. 5 Isotaches set with the curves $(\bar{\sigma}' - \bar{e})_{n=\text{const.}}$ for the load increment $\Delta\sigma = (2 - 1)$ kg/cm² of an undisturbed sample of a lacustrine clay

Système d'isotaches avec des courbes $(\bar{\sigma}' - \bar{e})_{n=\text{const.}}$ pour l'intervalle $\Delta\sigma = (2 - 1)$ kg/cm² d'un échantillon intact d'une argile lacustre

by Taylor (1942). Similar effect may be caused by the air trapped in the consolidometer below the sample or in the measuring device. Therefore, inaccurate measurements result showing apparently lower pore pressures at the commencement and higher values towards the end of the primary compression phase.

It may be pointed out that the differences between the measured and calculated values do not seem to be great for high values of plastic resistance (e.g. case (c), $\Delta\sigma = (8 - 4)$ kg/cm²) and that in Terzaghi's theory the results obtained by the assumption of parabolic isochrones do not differ appreciably from those obtained by the more elaborate solution. Therefore uniform consolidation process throughout the whole thickness of the layer may be accepted as a working hypothesis.

The validity of Darcy's law in the range of very small hydraulic gradients is a matter of dispute. From the observed increase of the consolidation speed at the transition from the unilateral to the bilateral drainage of the consolidometer sample during secondary compression it may be presumed that the coefficient of permeability decreases at very small gradients. On the other hand the influence of an eventual decrease of permeability at low gradients is limited in any case, as shown by the densities of older sediments which have been subjected to a secondary time effect of long duration (e.g. the post-glacial sediments of lacustrine chalk in the Soča valley) as well as by the very low pore pressures established in some recent sediments (e.g. the measurements of the Geological Institute Ljubljana in some hundred years old marine sediments at Istra) or in the secondary compression phase of some earth dams (Esmiol, 1955).

The assumption that the speed of consolidation depends on porosity and on intergranular pressure is limited by certain conditions. Two cases of apparent disagreement of this assumption with observation must now be considered.

In the first case an unexpected increment of consolidation speed occurred at a sudden increment of pore pressure as shown in the curve $\bar{\sigma}' - \bar{e}$ for $n_c = 1$ in Fig. 5. In the initial phase the consolidation speeds did not agree with the drawn isotaches charts. The phenomenon may be explained by considering that the pore water, when taking the surcharge load, is compressed by an amount dependent on the surcharge load as well as on the modulus of elasticity. Thus the whole additional stress is taken up by the solid grain contacts at the first moment of a suddenly applied load. Thereby thixotropic phenomena result and the structural resistance is temporarily reduced by an appreciable amount.

In the second case a disagreement of the consolidation speeds of the pre-consolidated samples with the isotaches corresponding to the first loading can be observed. However, in spite of the change in the porosity subsequent to the release there is still conserved the structure corresponding to the previous load and this can account for the above phenomenon. Thus the structural resistance to the transition into the porosity corresponding to the previous load is smaller than at the first loading starting from the same void ratio.

Although the logarithmic time law of secondary compression may be supported by a number of observations in the laboratory and in nature, it is quite clear that its validity is limited. Some approximate idea of the ultimate value of the void ratio of secondary compression can be obtained by observations of long duration subsequent to a forced consolidation carried out by a temporary overloading.

General Discussion

Disregarding the above-mentioned divergencies, the isotaches $\bar{\sigma}' - \bar{e}$ illustrate satisfactorily the difference between the faster consolidation at a sudden increment of intergranular pressure and the slower consolidation associated with its gradual increase. They give also a clear interpretation of the influence of the previous secondary compression on the decrease of the consolidation speed associated with the additional loading.

In the extreme case of a pure consolidation following the Terzaghi's theory all isotaches condense to a single straight line corresponding to the speed $v = 0$. For each point $(\bar{\sigma}', \bar{e})$ of this line complete equilibrium is reached. As the structural resistance does not exist in this case the transition of grains to new equilibrium positions occurs with the speed of intergranular pressure increase as given by the hydrodynamic law. Thus this special case is also included in our general consolidation scheme.

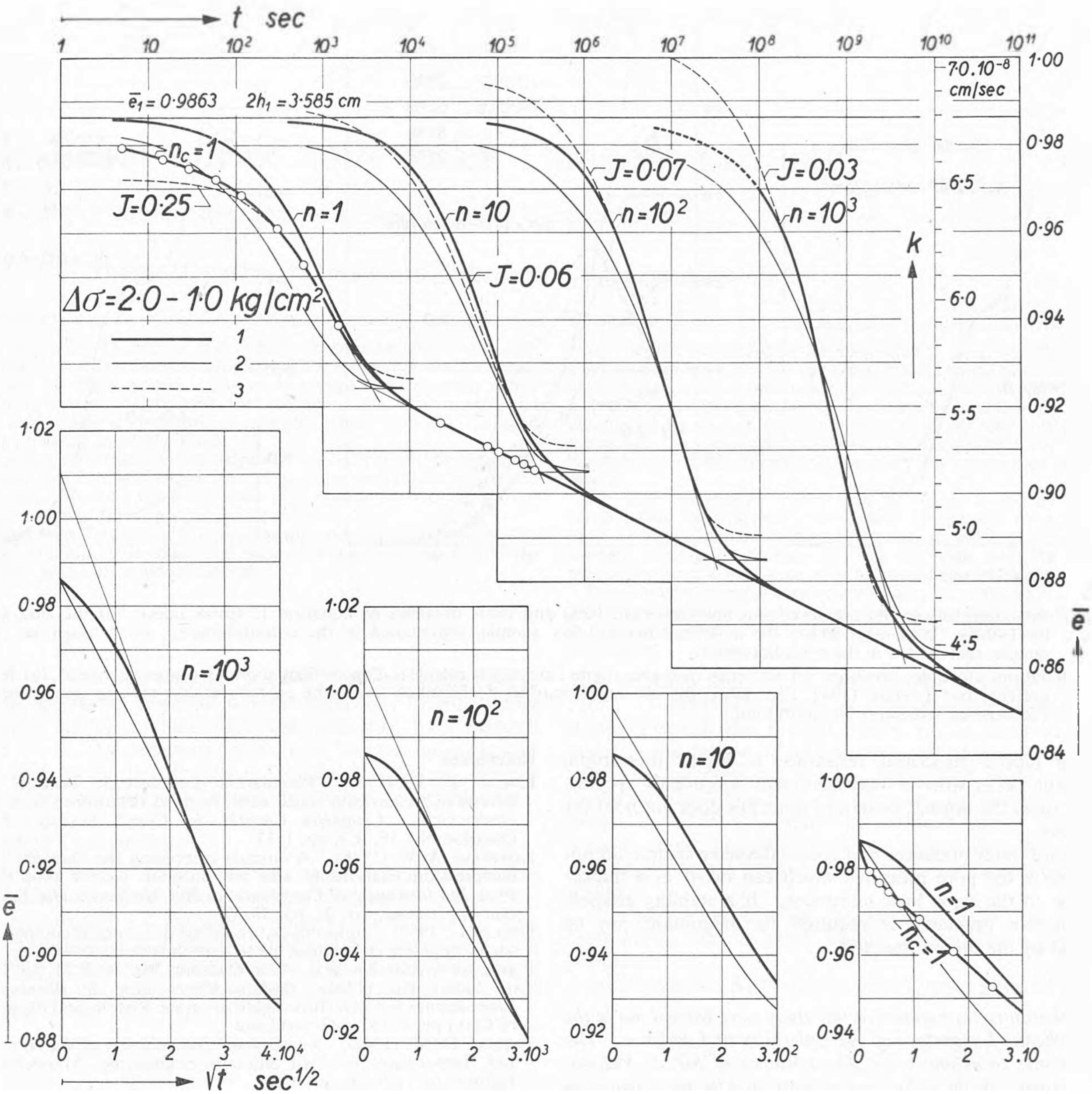

Fig. 6 Consolidation curves corresponding to the isotaches set of Fig. 5; for notations see Fig. 4
Courbes de consolidation correspondant au système d'isotaches de la Fig. 5; pour les notations voir Fig. 4

The dotted lines (3) in Figs. 4 and 6 represent the consolidation curves according to Taylor's theory B compared with the full lines which have been derived following the presented method. The corresponding ratios of time factors J are marked in the illustration. They are very small for thick layers as expected by Taylor, but with increasing layer thickness these ratios do not continue to decrease.

The transversal curves in Figs. 3 and 5, representing the consolidation of layers of various thickness ($n = 1$, 10, 10^2, 10^3, 10^4) for the load increment $\Delta\sigma = (8 - 4)$ and $\Delta\sigma = (2 - 1)$ kg/cm² respectively, suggest that high pore pressures can occur in thick layers, even in cases when they are small in the consolidometer sample.

Conclusions

The compressibility of an arbitrary saturated soil can be described by a system of isotaches ($\bar{\sigma}' - \bar{e}$). At the first loading this is a system of continuous curves, convex towards the $\bar{\sigma}'$-axis.

In the range of pre-stressing the isotaches of a pre-consolidated soil are less inclined than the isotaches corresponding to the first loading.

The isotaches of ideal soils following Terzaghi's consolidation theory meet in a single curve for $v = 0$, presented as a broken line, consisting of straight lines for each load increment.

Darcy's law being supposed valid, Terzaghi's consolidation curve combined with the logarithmic line of the secondary compression can present a satisfactory approximation in cases of great values of h^2/k (h = drainage length). However the coefficient of consolidation cannot be derived from the primary line of the odometer consolidation curve. The directly measured coefficient of permeability and the degree of development of secondary compression have to be considered. As it is probable that the coefficient of permeability will decrease at very low hydraulic gradients, the consolidation might be slower, especially in the secondary phase.

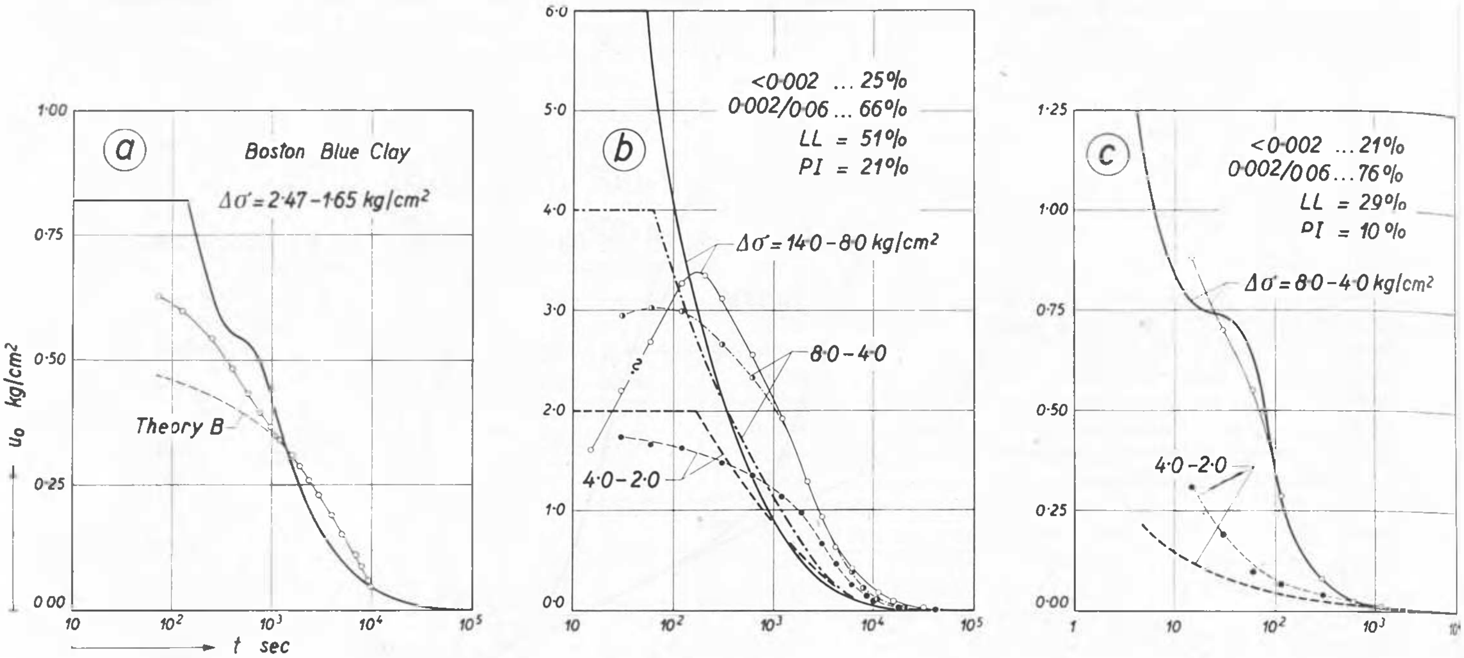

Fig. 7 Correlations between the measured pore pressures (thin lines) and those obtained by equation 13 (thick lines): (a) the case given by TAYLOR (1942, Fig. 50 b); (b) a tertiary marine clay sample, sedimented in the consolidometer; (c) a lacustrine chalk sample, sedimented in the consolidometer.

Relations entre des pressions interstitielles mesurées (ligne s minces) et calculées d'après l'équation 13 (lignes epaisses): (a) le cas présenté par TAYLOR (1942, Fig. 50b); (b) une argile marine, l'échantillon sédimenté en laboratoire; (c) une craie lacustre, l'échantillon sédimenté en laboratoire

If the plastic structural resistance is small, thixotropic phenomena occur when a load increment is suddenly applied. In these cases the normal system of isotaches does not hold for thin layers.

The pore-water pressures in a consolidometer sample do not give directly the pore pressures which can occur in a thicker layer due to the same load increment. If a stability analysis with effective pressures is required its magnitude can be estimated by the present method.

The laboratory tests quoted in this study were carried out in the Soil Mechanics Laboratory of the University of Ljubljana. The author wishes to acknowledge the assistance of Mr. S. Vidmar, Civil Engineer, in the experimental work and in the preparation of this paper.

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